Contents of December Lecture

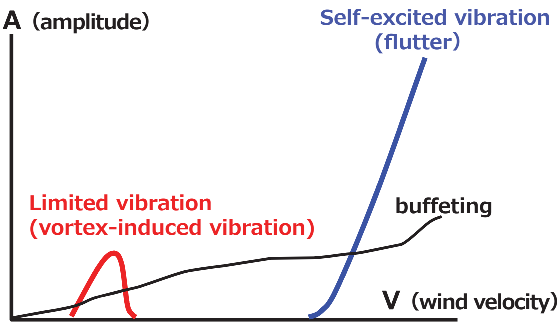
- Cable-stayed bridges -

[12-1-1]

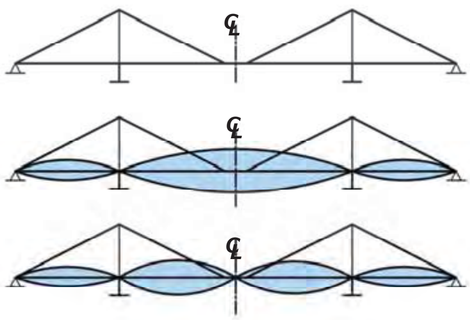
Fundamental of Vibration (1)

Flutter

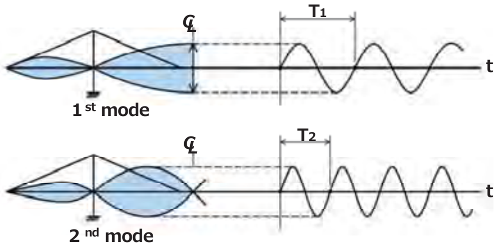
Dynamic response of structures due to wind



Vibration and Control



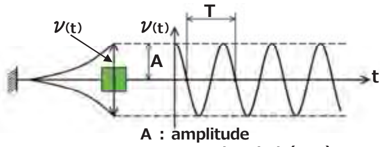
Mode shape and natural period



T_i : natural period (sec.)

f_i $(=1/T_i)$: natural frequency (cycle/sec., Hz)

Free vibration



$$\nu(t) = A\cos\omega t = A\cos\left(\frac{2\pi}{T}\right)t$$

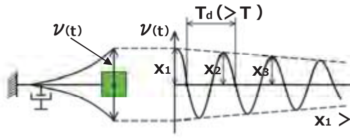
A : amplitude

T : natural period (sec.)

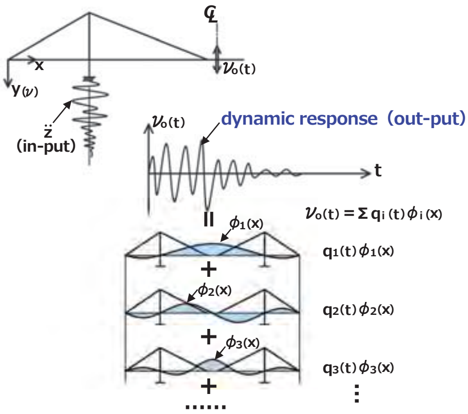
f $(=1/T)$: frequency (Hz, cycle/sec.)

ω $(=2\pi f)$: circular frequency (rad./sec.)

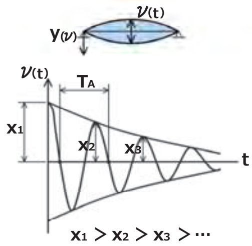
Free vibration with damping



$x_1 > x_2 > x_3 > \cdots$

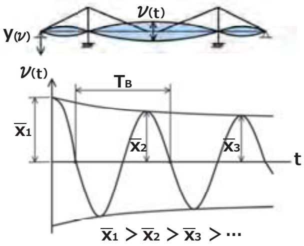


Short-span bridges



$x_1 > x_2 > x_3 > \cdots$

Long-span bridges



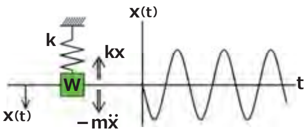
$\bar{x}_1 > \bar{x}_2 > \bar{x}_3 > \cdots$

$T_A < T_B$ $(f_A > f_B)$

$$\frac{x_1}{x_2},\ \frac{x_2}{x_3},\ \ldots\ldots > \frac{\bar{x}_1}{\bar{x}_2},\ \frac{\bar{x}_2}{\bar{x}_3},\ \ldots\ldots$$

Free vibration of 1 – DOF

Degree of Freedom



$$-m\ddot{x} = kx$$
$$m\ddot{x} + kx = 0$$
$$\ddot{x} + \omega^2 x = 0$$
$$\omega = \sqrt{k/m}$$

ω : natural circular frequency

W : weight (N, kN)

m $(=w/g)$: mass

g $(=9.8\text{m/sec}^2)$: gravitational acceleration

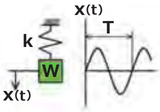
k : spring constant (N/mm)

$\ddot{x}$ $\left(=\frac{\partial^2 x}{\partial t^2}\right)$: acceleration

ex.1

W = 100kN

K = 20kN/mm

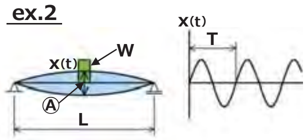


$$\omega = \sqrt{\frac{k}{W/g}} = \sqrt{\frac{20}{100/9800}} = 44.3$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{44.3} = 0.142 \text{ sec.}$$

$$f = \frac{1}{T} = 7.04 \text{ Hz}$$

ex.2



when load (P) is applied at Ⓐ

$$X_A = \frac{PL^3}{48EI}$$

EI : flexural rigidity of beam

$$K(\text{spring constant}) = \frac{P}{X_A} = \frac{48EI}{L^3}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{48EI}{L^3}\cdot\frac{g}{\omega}} = \sqrt{\frac{48EIg}{WL^3}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{WL^3}{48EIg}}$$

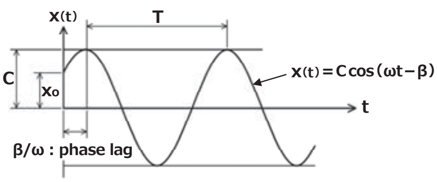
$$X = A\cos\omega t + B\sin\omega t$$

at $t = 0$ $\Rightarrow$ $X = X_o$, $\nu\left(=\frac{\partial x}{\partial t}\right) = \nu_o$ (← initial condition)

$$X = X_o\cos\omega t + \frac{X_o}{\omega_o}\sin\omega t$$
$$= C\cos(\omega t - \beta)$$

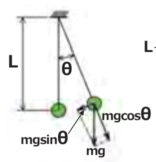
$$C = \sqrt{X_o^2 + \left(\frac{X_o}{\omega_o}\right)^2} = \sqrt{A^2+B^2},\quad \beta = \tan^{-1}(B/A)$$

$$\left(\omega = \sqrt{k/m},\ f = \omega/2\pi = \frac{1}{2\pi}\sqrt{\frac{k}{m}},\ T = 1/f = 2\pi/\omega\right)$$

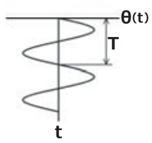


β/ω : phase lag

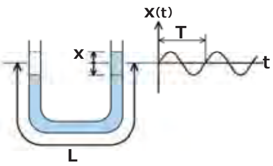
ex.3 Pendulum



$$L\frac{\partial^2\theta}{\partial t^2} + g\theta = 0$$
$$\omega = \sqrt{\frac{g}{L}}$$
$$T = 2\pi\sqrt{\frac{L}{g}}$$



ex.4 U-shape pipe with water



$$\frac{LAw_o}{g}\ddot{x} + 2w_oAx = 0$$
$$\ddot{x} + \frac{2g}{L}x = 0$$

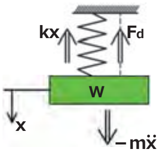
w_o : water weight per unit volume

A : diameter of pipe

L : Length of pipe with water

$$\omega = \sqrt{\frac{2g}{L}},\quad T = 2\pi\sqrt{\frac{L}{2g}}$$

Free vibration of 1 − DOF with damping



$$m\ddot{x} + F_d + kx = 0$$

1) Solid friction

$F_d = cx$

2) Coulomb friction

$F_d = -\mu N$

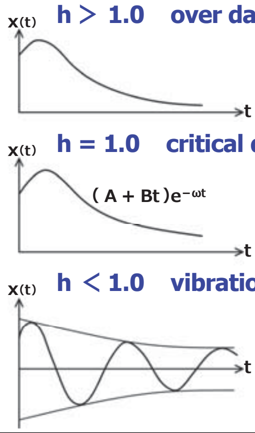
μ : friction coefficient

N : press force

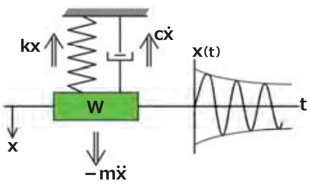
3) Viscous force

$F_d = c\dot{x}\ (= cv)$

v : velocity



Viscous damping



$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\ddot{x} + 2h\omega\dot{x} + \omega^2 x = 0 \text{ —(a)}$$

$$\omega = \sqrt{k/m}$$

$$2h\omega = \frac{c}{m} \quad \left(h = \frac{c}{2\sqrt{km}}\right)$$

C : damping constant h : damping coefficient

Assuming $x = Ce^{-pt}$, and substituting into eq. (a)

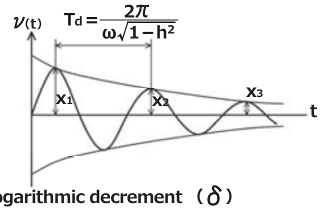
$$p^2 + 2h\omega p + \omega^2 = 0$$

$$p = -h\omega \pm \sqrt{h^2 - 1}$$

$h < 1.0 \longrightarrow p = -h\omega \pm i\omega\sqrt{1-h^2}$

$$\nu(t) = e^{-h\omega t}\left[A\sin(\omega\sqrt{1-h^2})t + B\cos(\omega\sqrt{1-h^2})t\right]$$

$$= Xe^{-h\omega t}\{\sin(\omega\sqrt{1-h^2})t - \phi_0\}$$



Logarithmic decrement (δ)

$$\delta = \ln\frac{X_1}{X_2} = \ln\frac{e^{-h\omega t_1}}{e^{-h\omega(t_1+T_d)}} = h\omega T_d$$

$$= \frac{2\pi h}{\sqrt{1-h^2}}$$

Since $h \ll 1.0$, $\delta = 2\pi h$

---

ex.

Damped vibration with $T_d = 1.0$ sec.

After 10 sec., an amplitude was reduced to 90%

$$\delta = \ln\left(\frac{X_m}{X_{m+10}}\right) = \frac{20\pi h}{\sqrt{1-h^2}}$$

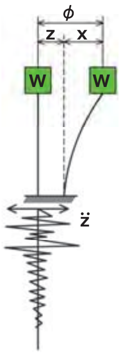
$$\text{from } \frac{X_m}{X_{m+10}} = \frac{1}{0.9}$$

$$\frac{20\pi h}{\sqrt{1-h^2}} = \ln\left(\frac{1}{0.9}\right) = 0.1054$$

$$20\pi h = 0.1054\sqrt{1-h^2}$$

$$\delta = 2\pi h = 1.678 \times 10^{-3}$$

In case of earthquake input



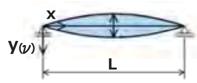
$$m\ddot{\phi} + c\dot{x} + kx = 0$$

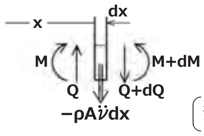
$$\ddot{\phi} = \ddot{z} + \ddot{x}$$

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{z}$$

[12-1-2]

Fundamental of Vibration (2)





$\left(\ddot{\nu} = \frac{\partial^2\nu}{\partial t^2}\right)$

$\rho = \gamma_s/g$

γ_s : weight per unit volume

g : gravitational acceleration

ρ : mass per unit volume

$-\rho A\ddot{\nu}dx + Q + dQ = Q$

$\rho A\ddot{\nu} = \frac{\partial Q}{\partial x}$ $\left\{Q = \frac{\partial M}{\partial x} = -\frac{\partial}{\partial x}\left(EI\frac{\partial^2\nu}{\partial x^2}\right)\right\}$

$\rho A\frac{\partial^2\nu}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2\nu}{\partial x^2}\right) = 0$

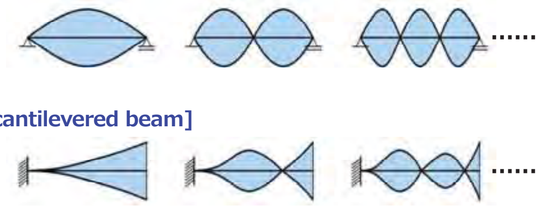
↓ EI : const.

$\frac{\rho A}{EI}\frac{\partial^2\nu}{\partial t^2} + \frac{\partial^4\nu}{\partial x^4} = 0$

Vibration of beams

[simple beam]

[cantilevered beam]



1 st mode　　2 nd mode　　3 rd mode

Assuming $\nu = X_{(x)}e^{i\omega t}$

$\frac{d^4X}{dx^4} - \omega^2\left(\frac{m}{EI}\right)X = 0$

$\frac{d^4X}{dx^4} - \lambda^4 X = 0$ $\left(\lambda^4 = \omega^2\left(\frac{m}{EI}\right)\right)$

$X = A\sin h\lambda x + B\cos h\lambda x + C\sin\lambda x + D\cos\lambda x$

in case of simple support bridges

at x = 0, $X = \frac{d^2X}{dx^2} = 0$

at x = L, $X = \frac{d^2X}{dx^2} = 0$

⇓

$\sin\lambda L = 0$

$\lambda L = i\pi$

$\omega_i = \lambda^2\sqrt{\frac{EI}{m}}$

$= \left(\frac{i\pi}{L}\right)^2\sqrt{\frac{EI}{m}}$

$X = \underbrace{\sin\frac{i\pi}{L}x}_{\text{mode shape}} \cdot (e^{i\omega t})$

Boundary conditions	λL		
	1st	2nd	3rd
Fix-Free	1.8751	4.6941	7.8548
Fix-Fix	4.7300	7.8532	10.9956
Fix-Pin	3.9266	7.0686	10.2102

$x = x_o + x_P$

$x_o = e^{-h\omega t}\left(C_1\cos\omega' t + C_2\sin\omega' t\right)$

$\omega' = \sqrt{1-h^2}\cdot\omega$

$x_P = A\cos\omega_p t + B\sin\omega_p t$ ——— (2)

Substituting eq.(2) into eq.(1)

$A = \frac{F_o}{m}\,\frac{\omega^2 - \omega_p^2}{(\omega^2-\omega_p^2)^2 + 4h^2\omega^2\omega_p^2}$

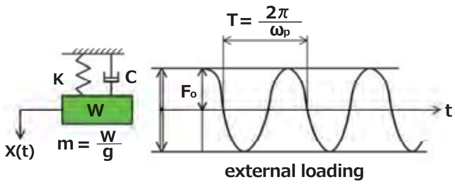
$B = \frac{F_o}{m}\,\frac{2h\omega\omega_p}{(\omega^2-\omega_p^2)^2 + 4h^2\omega^2\omega_p^2}$

$X = e^{-h\omega t}\left(C_1\cos\omega' t + C_2\sin\omega' t\right)$

$+\frac{1}{\omega^2}\frac{F_o}{m}\left\{\frac{1}{\sqrt{(1-\omega_p^2/\omega^2)^2 + 4h^2\omega_p^2/\omega^2}}\right\}\cos(\omega_p t - \varphi)$

$\frac{F_o}{\omega^2 m} = \frac{F_o}{K} = \delta_{st}$ (static response)

Forced vibration of 1-DOF



Under harmonic excitation

$m\ddot{x} + c\dot{x} + kx = F_o\cos(\omega_p t)$

$\ddot{x} + 2h\omega\dot{x} + \omega^2 x = \left(\frac{F_o}{m}\right)\cos(\omega_p t)$ ——— (1)

$x = x_o$ (general solution [← free vibration])

$+ X_p$ (particular solution)

$L = \frac{1}{\sqrt{(1-\omega_p^2/\omega^2)^2 + 4h^2\omega_p^2/\omega^2}} = \frac{D}{\delta_{st}}$

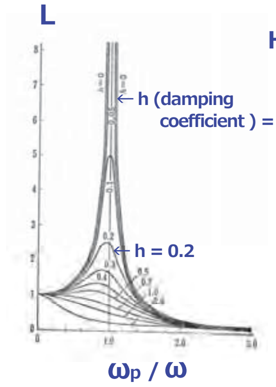
D : dynamic response

L : magnification factor (due to vibration)

Since first term (x_o: free vibration with damping) will distinguish as time passes , hence second term (x_p) exists, and peak value $L_{max.}$

$\frac{dL}{d(\omega_p/\omega)} = D \rightarrow \frac{\omega_p}{\omega} = \sqrt{1-2h^2}$

$L_{max.} \fallingdotseq \frac{1}{2h}$



How to avoid resonance

(1) Avoid ($\omega_p / \omega = 1.0$)

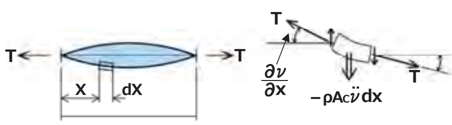
ω_p : frequency of force

ω : frequency of structures

change $\omega \left(= \sqrt{\frac{K}{m}}\right)$

(2) Add(increase) damping

Vibration of string（cable）



$$\frac{\partial \nu}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\partial \nu}{\partial x}\right) dx$$

T : tension force

$$\rho A_c \frac{\partial^2 \nu}{\partial t^2} - T \frac{\partial^2 \nu}{\partial x^2} = 0$$

$$\frac{\partial^2 \nu}{\partial t^2} - \lambda^2 \frac{\partial^2 \nu}{\partial x^2} = 0 \quad \left(\lambda^2 = \frac{T}{\rho A_c}\right)$$

Assuming $\nu = V(x) \cdot q(t)$

$$\frac{\partial^2 q}{\partial t^2} + \omega^2 q = 0 \quad \text{—— (1)}$$

$$\frac{\partial^2 \nu}{\partial x^2} + \frac{\omega^2}{\lambda^2} V = 0 \quad \text{—— (2)}$$

From eq. (1)

$$q = A \cos \omega t + B \sin \omega t$$

From eq. (2)

$$V = C_1 \cos\left(\frac{\omega L}{\lambda}\right)\frac{x}{L} + C_2 \sin\left(\frac{\omega L}{\lambda}\right)\frac{x}{L}$$

at $X = 0, L \rightarrow V = 0$

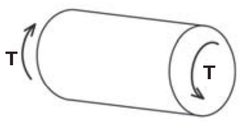
$$\sin \frac{\omega L}{\lambda} = 0 \rightarrow \frac{\omega L}{\lambda} = i\pi$$

$$\omega_i = \frac{i\pi}{L}\lambda = \frac{i\pi}{L}\sqrt{\frac{T}{\rho A_c}}$$

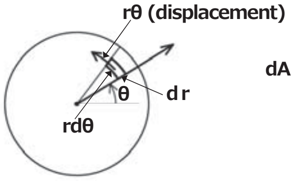
$$f_i = \frac{\omega_i}{2\pi} = \frac{i}{2L}\sqrt{\frac{T}{\rho A_c}} \quad (i = 1,2,\cdots)$$

A_c : cross sectional area of string

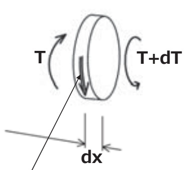
Torsional vibration



T : torsional moment



dA = rdθdr



$$-\rho \int_r \int_0 r^2 dr d\theta \left(\frac{\partial^2 \theta}{\partial t^2}\right)$$

$$= -\rho I_p \frac{\partial^2 \theta}{\partial t^2}$$

$$T = -\rho I_p \frac{\partial^2 \theta}{\partial t^2} dx + T + dT$$

$$\frac{\partial T}{\partial x} - \rho I_p \frac{\partial^2 \theta}{\partial t^2} = 0$$

$$T = GJ \frac{\partial \varphi}{\partial x}$$

$$\rho I_p \frac{\partial^2 \theta}{\partial t^2} - GJ \frac{\partial^2 \theta}{\partial x^2} = 0$$

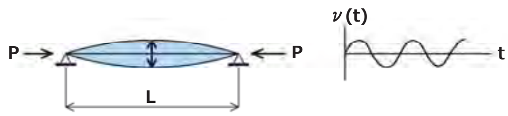
$$\frac{\partial^2 \theta}{\partial t^2} - \frac{GJ}{\rho I_p} \frac{\partial^2 \theta}{\partial x^2} = 0 \quad \left(\lambda^2 = \frac{GJ}{\rho I_p}\right)$$

$$\omega_i = \frac{i\pi}{L}\lambda = \frac{i\pi}{L}\sqrt{\frac{T}{\rho I_p}}$$

In case of round section

$$I_p = J = \frac{\pi^2 D^4}{32} \quad (D : \text{diameter})$$

Vibration of beams under compressive force

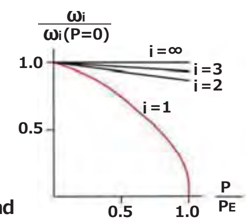


$$\rho A \frac{\partial^2 \nu}{\partial t^2} - P \frac{\partial^2 \nu}{\partial x^2} + EI \frac{\partial^4 w}{\partial x^4} = 0$$

$$\downarrow \quad \nu(x,t) = V(t) e^{\omega t}$$

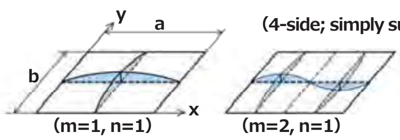
$$\omega_i = \left(\frac{i\pi}{L}\right)^2 \sqrt{\frac{EI}{\rho A}} \sqrt{1 - \frac{1}{i^2}\frac{P}{P_E}}$$

P_E : Euler (elastic)buckling load



Vibration of plate

(4-side; simply supported)



$$B\left(\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4}\right) + \rho t \frac{\partial^2 W}{\partial t^2} = 0$$

$$\Downarrow \quad W(x,y,t) = \Sigma\Sigma A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{i\omega t}$$

$$W_{mn} = \sqrt{\frac{B}{\rho t}}\left\{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right\}$$

$$B = \frac{Et^3}{12(1-\nu^2)}$$, t : plate thickness

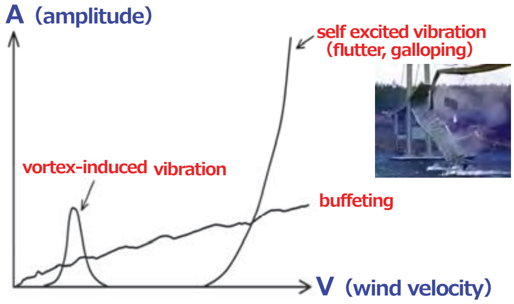
Rayleigh method

– approximate method for frequency –

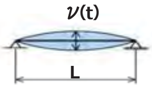
Assuming vibrational mode shape satisfying boundary conditions, and calculate kinematic energy (K) and strain energy (V),

$K_{max.} = V_{max.} \rightarrow$ natural frequency

Dynamic response due to wind



ex. beams



$$K = \frac{\rho A}{2}\int_0^L \left(\frac{\partial \nu}{\partial t}\right)^2 dx$$

$$V = \frac{EI}{2}\int_0^L \left(\frac{\partial^2 \nu}{\partial x^2}\right)^2 dx$$

Assuming $\nu(x,t) = X(x)\cos\omega t$

$$K = \frac{\rho A \omega^2}{2}\int_0^L X^2 dx \cdot \sin^2\omega t$$

$$V = \frac{EI}{2}\int_0^L X''^2 dx \cdot \cos^2\omega t$$

$$K_{max.} = V_{max.} \quad \rightarrow \quad \omega^2 = \frac{EI\int_0^L X''^2 dx}{\rho A\int_0^L X^2 dx}$$

We assume $X = \sin\frac{\pi}{L}x$

$$\omega = \left(\frac{\pi}{L}\right)^2\sqrt{\frac{EI}{\rho A}}$$

Control of vibration

(1) Aerodynamic means
control the wind flow by changing cross-sectional shape (or) by attachment

(2) Structural (Mechanical) means
add damping etc.

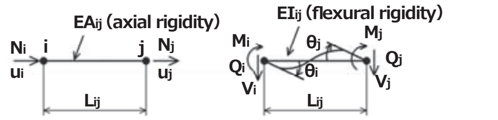
Structural (Mechanical) means

1) Change natural frequency
by mass or stiffness change
↑ we will face difficulty

2) Add damping (untuned type)
3) Add damping (tuned type)
} Passive type

4) Active damping

Truss and beam elements



$$\begin{Bmatrix} N_i \\ N_j \end{Bmatrix} = \frac{EA_{ij}}{L_{ij}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$$

↑Stiffness matrix

[truss element]

$$\begin{Bmatrix} Q_i \\ M_i \\ Q_j \\ M_j \end{Bmatrix} = \frac{EI_{ij}}{L_{ij}^3}\begin{bmatrix} 12 & 6L_{ij} & -12 & 6L_{ij} \\ & 4L_{ij}^2 & -6L_{ij} & 2L_{ij}^2 \\ & & 12 & -6L_{ij} \\ \text{sym.} & & & 4L_{ij}^2 \end{bmatrix} \cdot \begin{Bmatrix} V_i \\ \theta_i \\ V_j \\ \theta_j \end{Bmatrix}$$

↑Stiffness matrix

[beam element]

Multi degree of freedom by finite element analysis

[Static response]

$[K]\{d\}=\{P\}$

$[K]$: Stiffness matrix
$\{d\}$: displacement vector
$\{P\}$: force vector

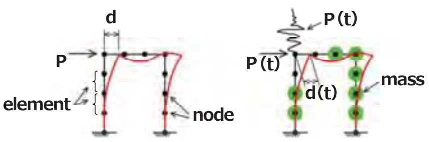
[Dynamic response]

$[M]\{\ddot{d}\}+[C]\{\dot{d}\}+[K]\{d\}=\{P(t)\}$

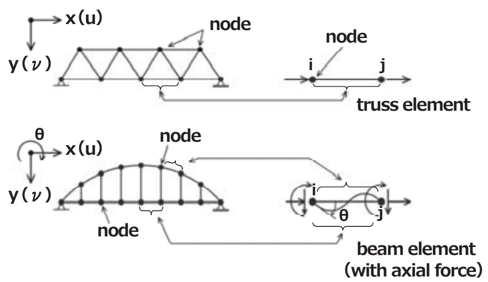
(or)

$\ddot{Z}[M]\{1\}$
earthquake input

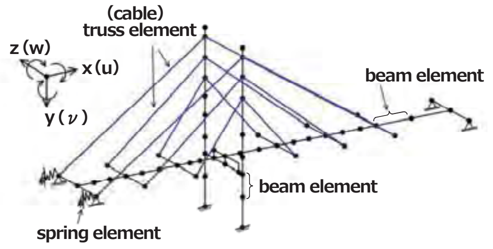
$[M]$: mass matrix
$[C]$: damping matrix



Plane (2D) model (by truss and beam elements)



Spatial（3D）model



a cable-stayed bridge modeled by
truss and beam elements

Eigenvalue analysis

$$[M]\{\ddot{d}\}+[K]\{d\}=\{0\} \quad ([C]=[0],\{P\}=\{0\})$$

$\downarrow$ putting $\{d\}=\{q\}e^{i\omega t}$

$$\det\left|[K]-\omega^2[M]\right|=0$$

$\Downarrow$

ω_i, $[\varphi]$ (←modal matrix) are obtained

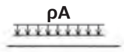
$\{d\}$ is expressed by

$$\{d\}=[\varphi]\{q\}$$

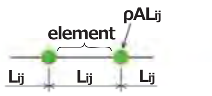
$\{d\}$: displacement vector

$\{q\}$: generalized displacement vector

Mass matrix



$\Rightarrow$

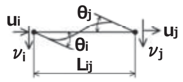


(lumped mass)

$\Downarrow$

(consistent mass)

$$[m] = L_{ij}\int_0^1 [A(\xi)]^T \rho [A(\xi)]\,d\xi$$



$$\begin{Bmatrix} u \\ \nu \\ \theta \end{Bmatrix} = [A(\xi)] \cdot \begin{Bmatrix} u_i \\ u_j \\ \nu_i \\ \theta_i \\ \nu_j \\ \theta_j \end{Bmatrix}$$

$$\xi = \frac{x}{L_{ij}} \; (0 \leq \xi \leq 1)$$

ex.

$$[M]\{\ddot{d}\}+[C]\{\dot{d}\}+[K]\{d\} = -\ddot{z}_0[M]\{1\}$$

z_0 : ground movement

$\{1\}^T$: $\{1,1,\cdots\cdots 1\}$

$[C]$: proportional damping

$$\{d\}=[\varphi]\{q\}$$

$$[\varphi]^T[M][\varphi]\{\ddot{q}\}+[\varphi]^T[C][\varphi]\{\dot{q}\}+[\varphi]^T[K][\varphi]\{q\}$$
$$= \ddot{z}_0[\varphi]^T[M]\{1\}$$

from orthogonality condition of each mode,
independent vibration of each mode is obtained

$$M_j^*\ddot{q}_j + C_j^*\dot{q}_j + K_j^* q_j = -\ddot{z}_0[\varphi]^T[M]\{1\}$$

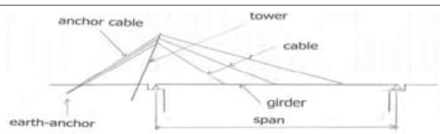
$$\ddot{q}_j + 2h_j w_j \dot{q}_j + w_j^2 q_j = -\frac{\ddot{z}_0[\varphi]^T[M]\{1\}}{M_j^*}$$
$$= -\beta_j \ddot{z}_0$$

[12-1-3]

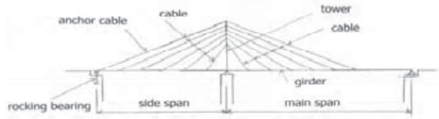
Vibration(DVD)
& Commentary

[12-2-1]

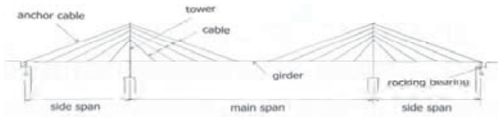
History & Name of cable-stayed bridge



Single span cable-stayed bridge

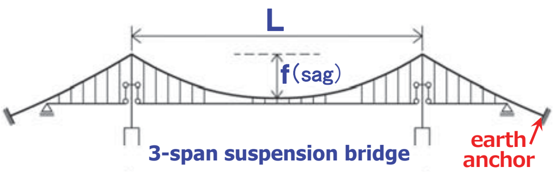


2-span continuous cable-stayed bridge



3-span continuous cable-stayed bridge

A cable-stayed bridge suspends the girder using diagonal cables.



3-span suspension bridge

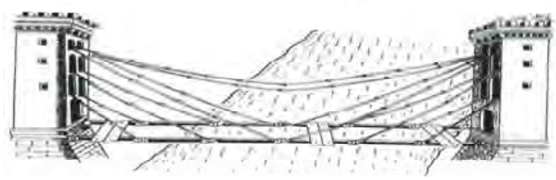
Single span suspension bridge

Continuous suspension bridge

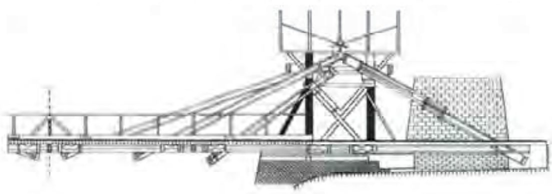
History

Cable-stayed bridges
self-anchored system
(↑compression in the girder)

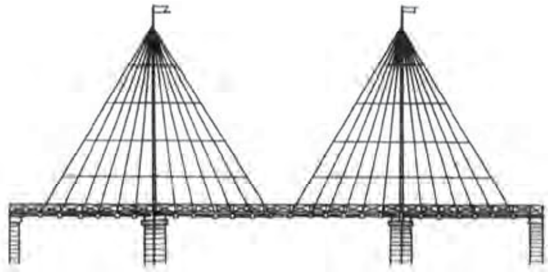
Suspension bridges
earth-anchored system
(↑good soil condition is requisite)



Cable-stayed bridge by Verantius, supporting the timber deck by chains (Venice, 1617)

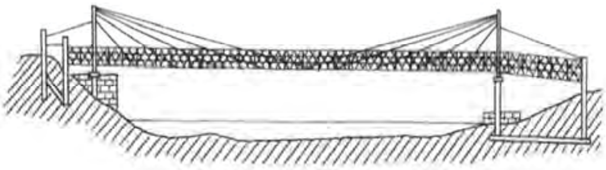


Timber cable-stayed bridge
by Loescher
(Loescher : German carpenter, 1784)



Fan type cable-stayed bridge
proposed by Poyet French (1821)

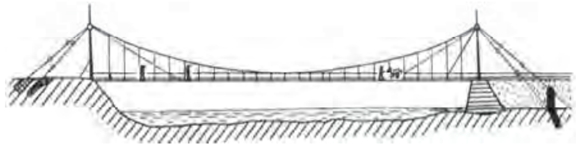
King’s Meadows Bridge



Tower : cast iron
Cable : wire
constructed by English engineers,
Redpath and Brown in 1817

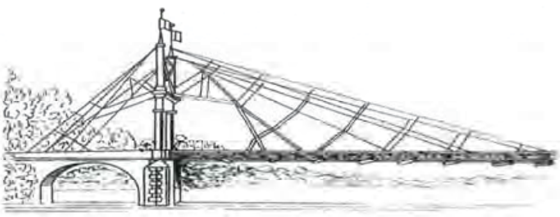
Dryburgh Bridge (span 79m, UK)

(pedestrian bridge crossing over Tweed river)



The bridge was collapsed in 1817,
a half year after opening
The reason is chain broken
due to wind-induced vibration

Cable-stayed pedestrian bridge, crossing over Saale river, Germany



In 1824, the bridge was collapsed.
The reason is the broken of chain due to walker loading

Harp-type cable-stayed bridge proposed by Hatley (1840)

Hatley pointed out that
in-plane flexural rigidity of Harp type
is inferior to that of Radial type

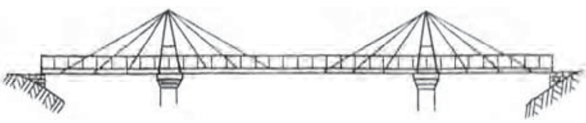
《in-plane flexural rigidity》
[Harp-type] < [Radial type]

At that time, famous scientist Navier made accident investigation, and concluded that

Suspension bridge is superior to cable-stayed bridge

Since then, this type of the bridge almost disappeared until the Stromsund bridge in Sweden was constructed in 1955, and it is called “ beginning of modern bridge”

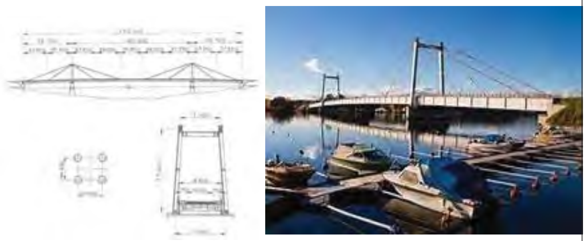
Cable-stayed bridge constructed over cannel at Manchester, UK (at around 1840)



Similar configuration to that of
modern cable-stayed bridge

Albert Br. over the Thames(1873, UK)

Stromsund Br. (Sweden, 1955)



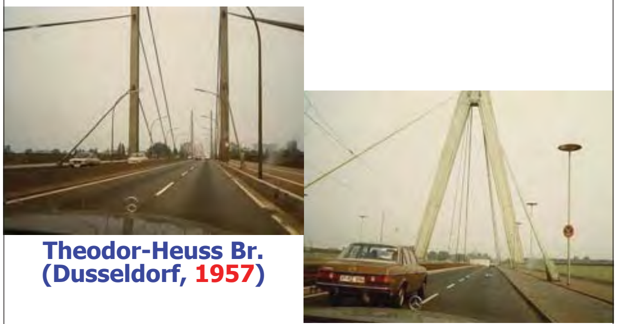
Beginning of
modern cable-stayed bridge

Cable-stayed and suspension bridge* proposed by Dischinger (1938) Hamburg, Germany

Proposal for a railway bridge with a span of 750-m over the Elbe river near Hamburg, Germany

*in order to increase in-plane flexural rigidity
*in order to get smaller deflection

Cable-stayed bridge crossing the Rhein river

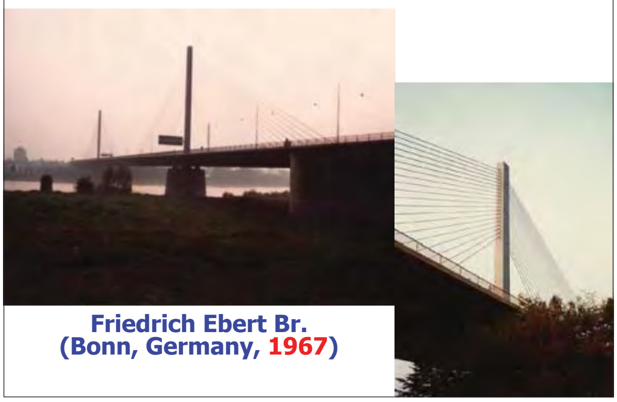


Theodor-Heuss Br.
(Dusseldorf, 1957)

Severins Br.
(Koeln, 1959)

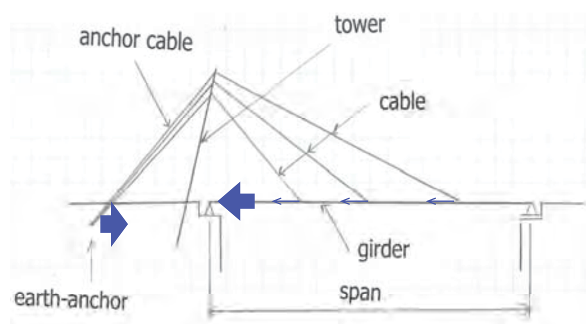
This time period,
number of cable is small.

First multi-cable type Cable-stayed bridge



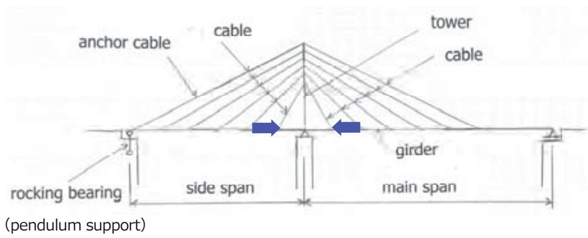
Friedrich Ebert Br.
(Bonn, Germany, 1967)

Simple span cable-stayed bridge

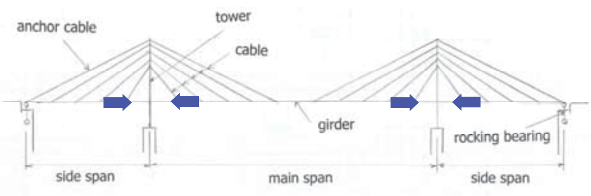


Number of span

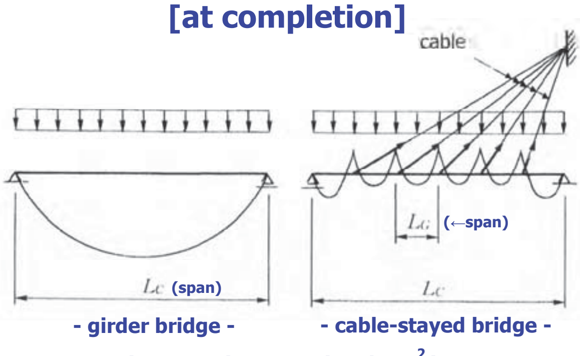
2-span continuous cable-stayed bridge



3-span continuous cable-stayed bridge

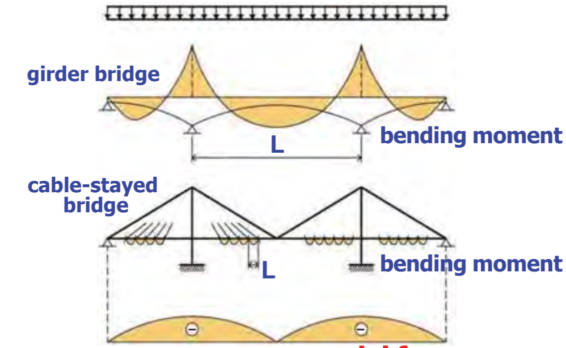


Girder bending moment [at completion]

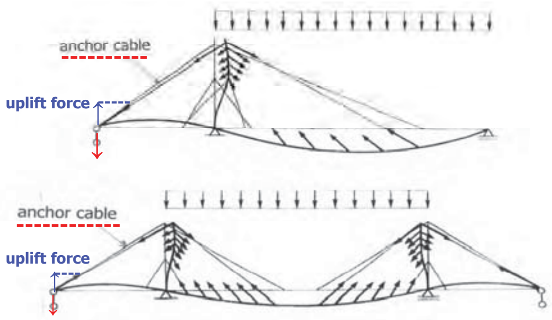


Under vertical and lateral loadings

Stress Resultants

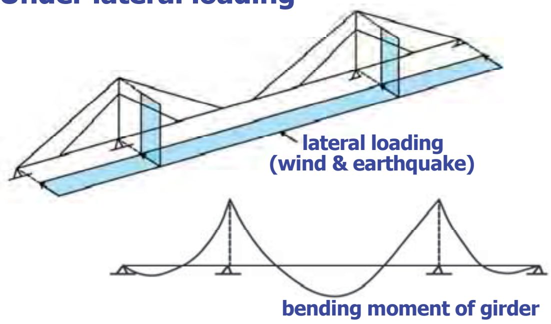


Mechanical behavior under live loading



Anchor cable plays an important role

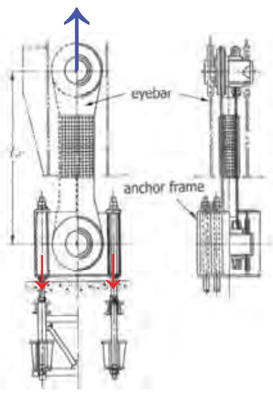
Under lateral loading



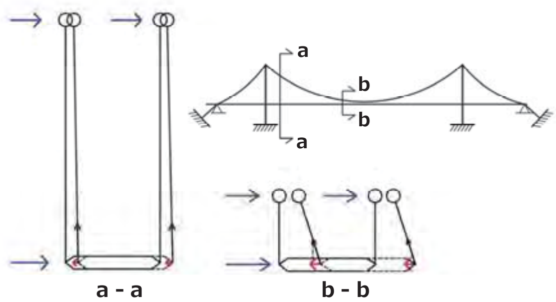
girder itself* has to resist lateral loading

*out-of-plane flexural rigidity

Pendulum support (up-lift force)

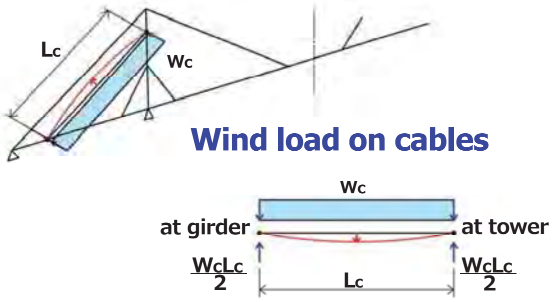


Suspension bridges



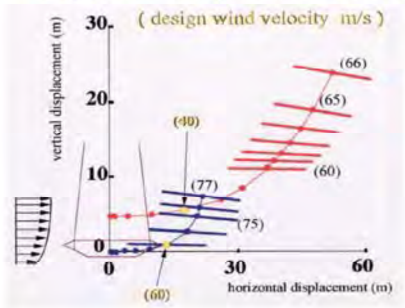
← restoring force
(expected)

Wind load on cables



Static instability -(2)-

Divergence under displacement-dependent wind loading



At the cable-stayed bridge design,

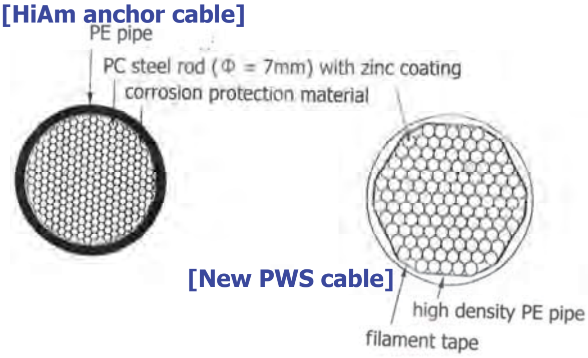
if span length (L) is large, and the width of the girder (B) is narrow*,

(L/B is large more than around 40)

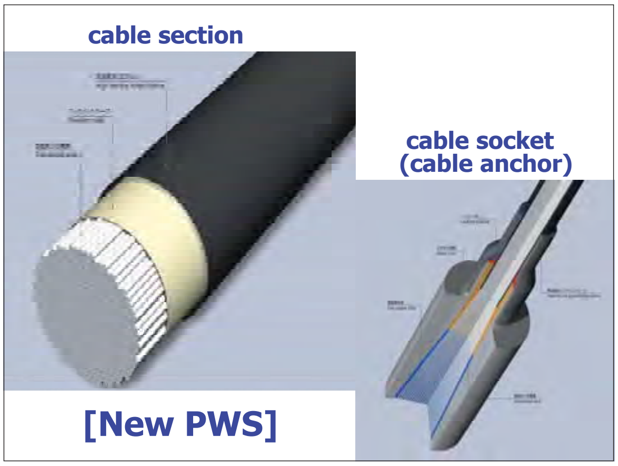
be careful about lateral instability!!

* 2-lane bridge (narrow width) with long span

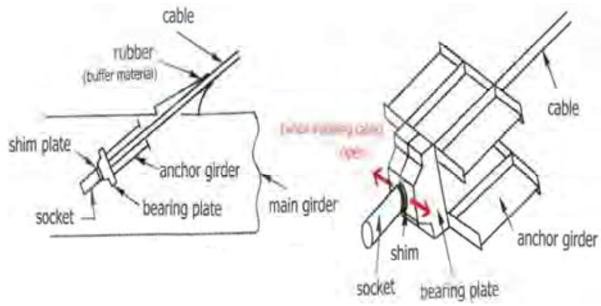
Cables



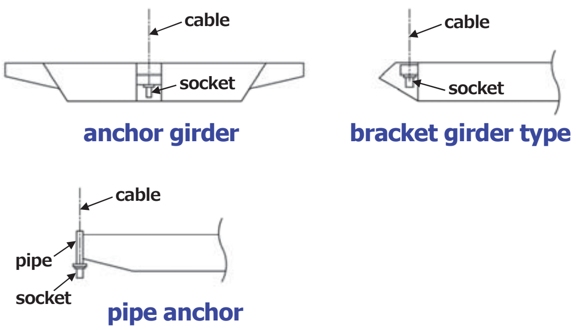
both have cable anchor (socket) with high fatigue strength



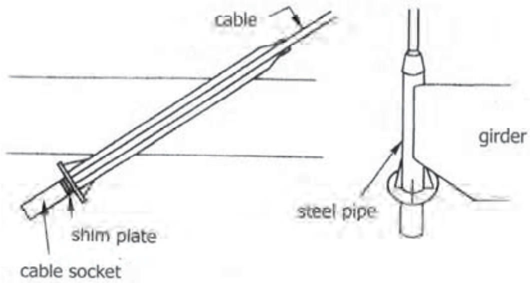
Anchor girder



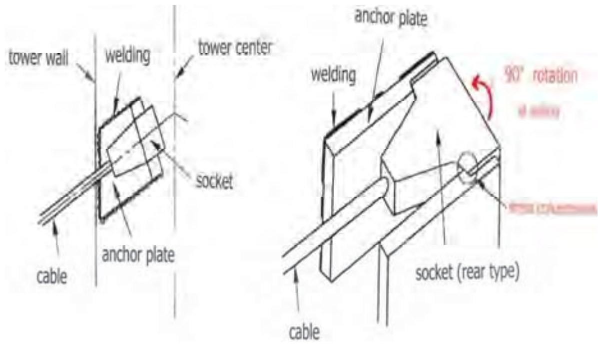
Design of Cable anchor in the girder



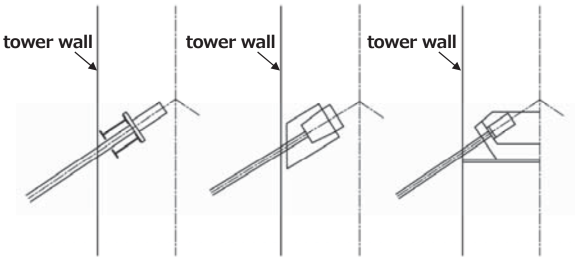
Pipe anchor



Anchor plate

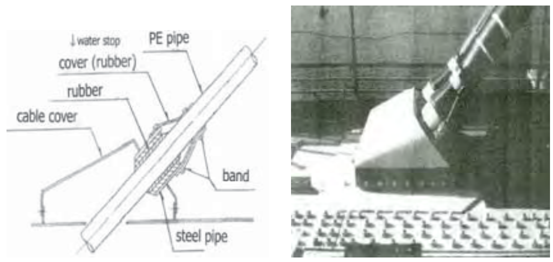


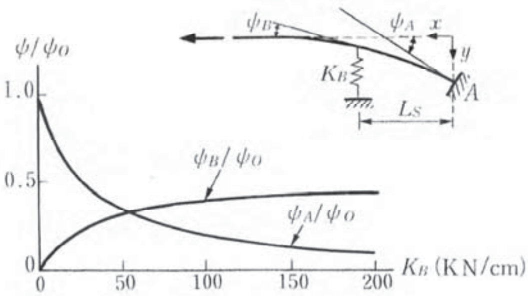
Cable anchor in the tower



anchor girder anchor plate saddle

Buffer (by rubber) To mitigate secondary moment





Ψ0 : without spring (KB)

Calculation of stress resultants $(N \rightarrow [\sigma_n], M \rightarrow [\sigma_b], Q \rightarrow [\tau_b], T \rightarrow [\tau_s])$ and deflection (δ) is carried out by Finite Element Analysis using fish-bone model (beam or fiber elements).

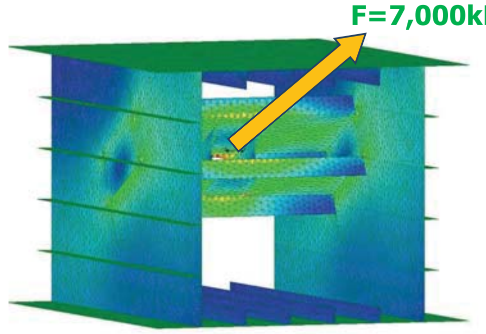
At structural details* accompanied by stress concentration, Finite Element Analysis (shell & solid elements) is carried out.**

* cable anchor structures etc.
** can not be caught by beam element

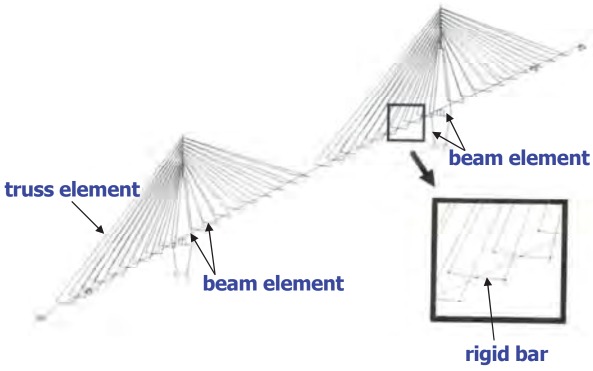
Stress resultants & deflection

FEA of the Cable Anchoring Section

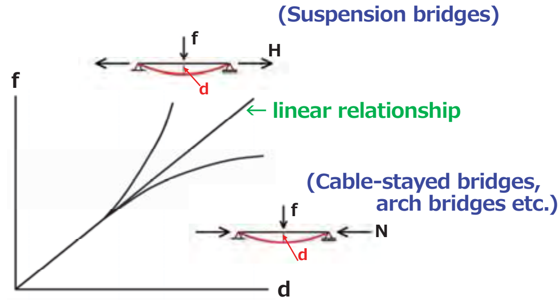
Cable Tension F=7,000kN



Fish-bone model for cable-stayed bridges



Non-linear analysis



Calculation theory for the design

$$[K_E]\{d\}=\{f\} \qquad (1)$$

linear analysis

$$[K_E+K_G(N_D)]\{d\}=\{f\} \qquad (2)$$

linearized finite displacement analysis

$[K_E]$: elastic matrix

N_D : initial axial force under dead load (given)

$[K_G(N_D)]$: geometrical matrix

$\{d\}$: displacement vector

$\{f\}$: force vector

Influence line analysis is possible!!

Geometrical non-linear analysis

$$[K_E+\underbrace{K_G(N_D+\triangle N)}_{\uparrow\ \text{large displacement}}]\{\triangle d\}=\{\triangle f\} \qquad (3)$$

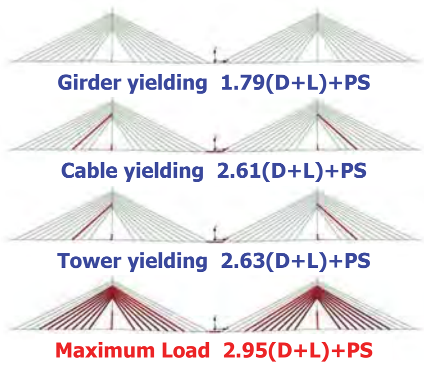
Material & geometrical non-linear analysis

$$[K_E+\underbrace{K_P}_{\uparrow\ \text{Plastic behavior}}+K_G(N_D+\triangle N)]\{\triangle d\}=\{\triangle f\} \qquad (4)$$

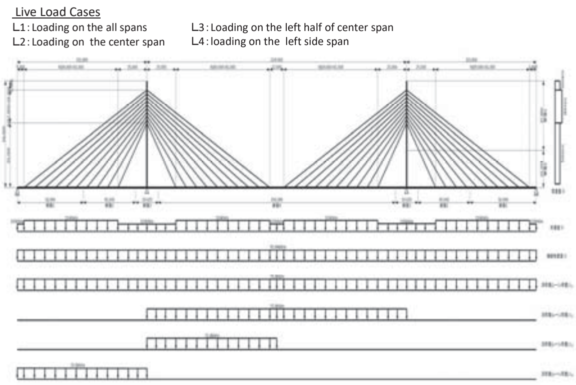
Since displacement under construction is large, eq. (3) has been used.

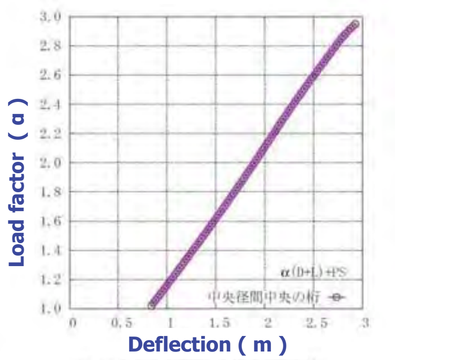
To obtain ultimate strength, eq. (4) has been used.

Analysis Case L1 [α(D+L)+Ps]



Load cases of Ultimate Strength Analysis





Load – deflection at the center of the girder

[12-2-2]

Design parameters & their selection (1)

---

[1] Mono or multi-cellular box [closed-section] girder
(from aerodynamic stability and maintainability)

[2] Truss girder
(mainly for double-deck type)

[3] Open-section [п -section] girder
(from economical viewpoint, however, shows poor aerodynamic stability compared to box section)

(   ) : reason of selection

---

[1] Girder

---

closed section

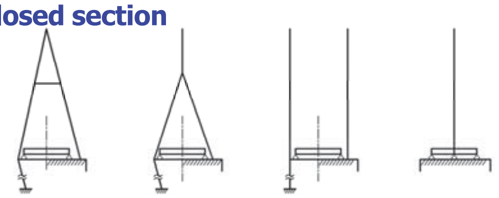
open-closed section

open section (←2-plane cable is inevitable)

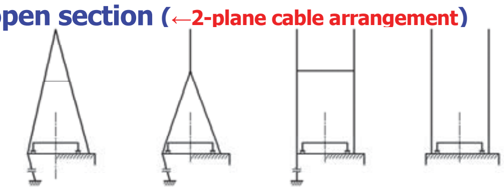
[п-section]   [multi-I section]

Combination with tower type

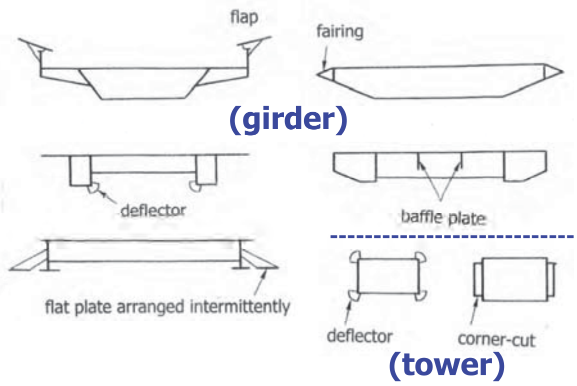
closed section



open section (←2-plane cable arrangement)



Aerodynamic means



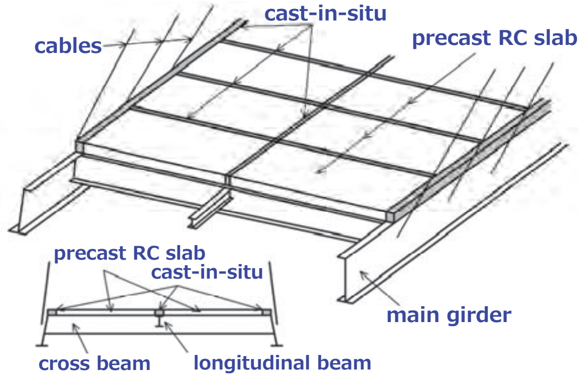
From aerodynamic stability viewpoint,
Streamlined box section has been selected.

Combination of
[box section] & [2-plane cable arrangement] & [A-shaped tower]
gives highest torsional rigidity

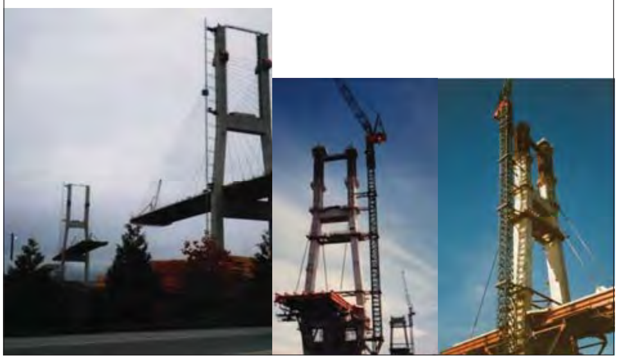
Truss girder
(double deck type)

Upper deck : roadway
Lower deck : railway

Steel-concrete composite I girder

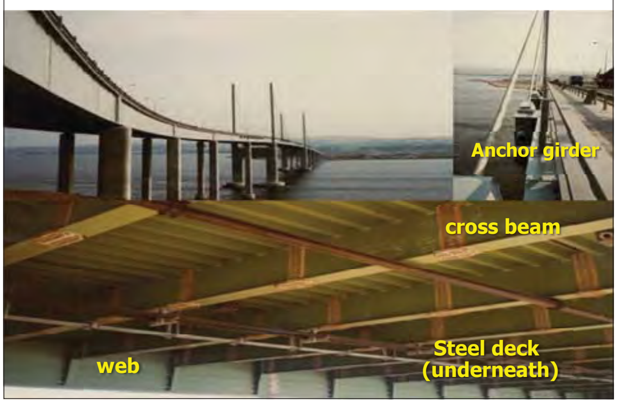


Girder with open section
(from economical viewpoint)



Twin Box type
(from aerodynamic viewpoint)

π -shaped (2-I) girder bridge with steel deck



[COST]

Open (π -shape)section ≦ Closed box section
{↑2-plane cable*}

- From aerodynamic stability viewpoint, closed section is preferably selected in Japan (typhoon attack)
- maintainability has to be taken into account

*since torsional rigidity of the girder is very low

(Steel-concrete) Hybrid girder

PC girder with steel corrugated web

PC girder with steel pipe truss web

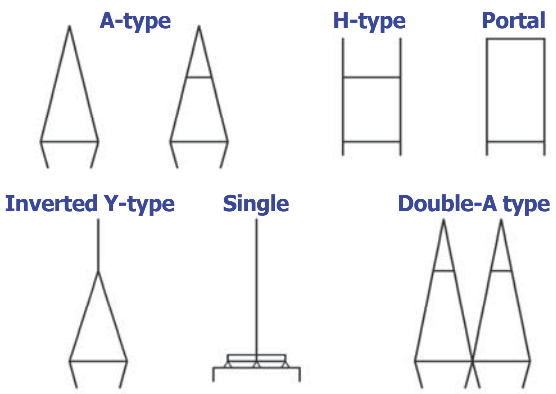
Curved Cable-stayed bridges

[2] Tower

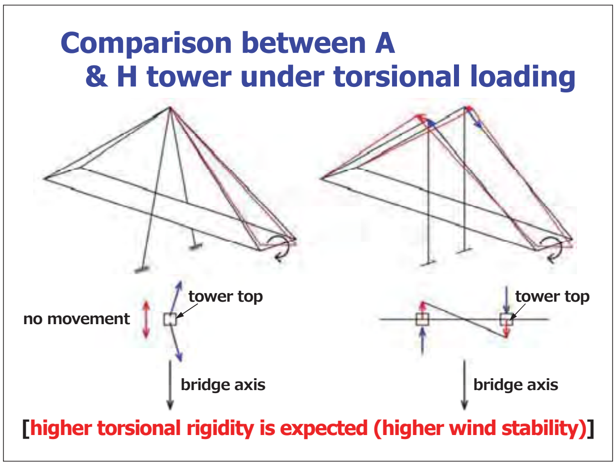
a)Type

b)Hight

"A" shaped Tower



"H" shaped Tower


Comparison between A
& H tower under torsional loading
tower top
no movement
bridge axis
tower top
bridge axis
[higher torsional rigidity is expected (higher wind stability)]

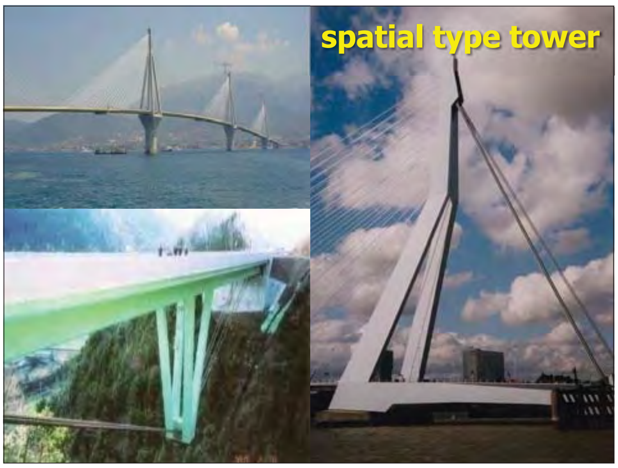

1,018m
(single tower)


1,104m
1,088m
890m
No.1 in Japan
856m
No.1 in Europe


Single tower


Two towers


Inverted Y-shaped tower
600m

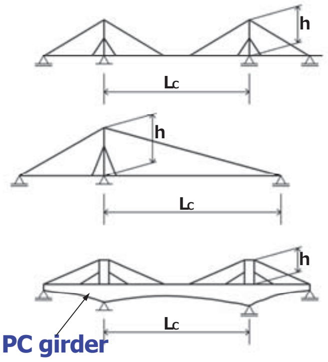

spatial type tower


Double-A type

Tower underneath the girder (king post type)

A-shaped tower (side view)

b) Height of the tower
(from the deck level)

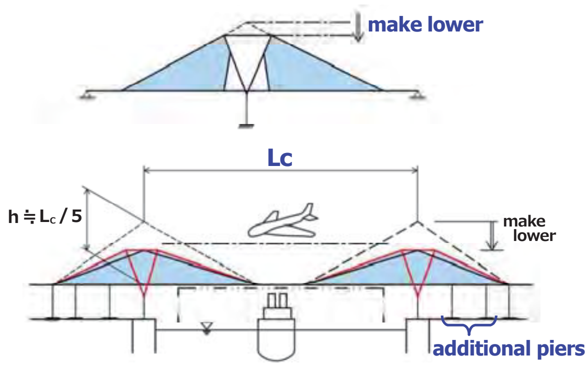


$L_c / h \fallingdotseq 5$

$(1.8 L_c) / h \fallingdotseq 5$

$L_c / h \fallingdotseq 10$
(extradosed-type)

side view



Optimal (economical) solution $L_c / h \fallingdotseq 5.0$

If difficult to set ($L_c / h \fallingdotseq 5.0$) at site,
don't exclude and check (compare!!)

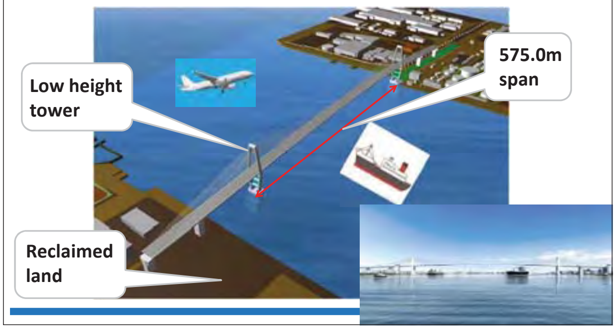
↓

[1] cable-stayed bridge with a lower tower
($h < L_c / 5.0$)

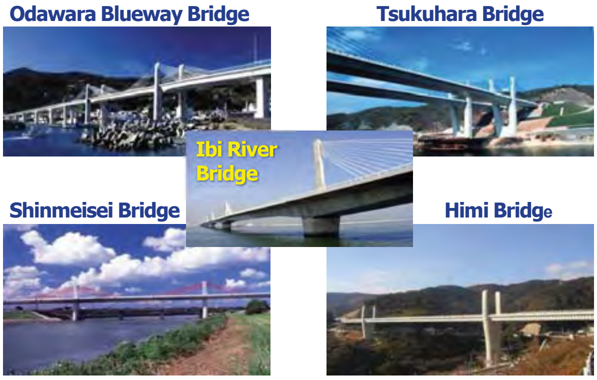
[2] Another solutions
such as truss bridges, arch bridges and so on

Check which one is economical !!

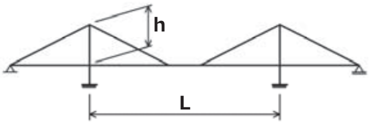
A large cable-stayed bridge with low height tower is planned crossing a channel near major airport in Japan.



Extradosed type bridge

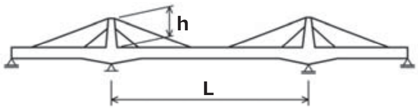


Cable-stayed bridges

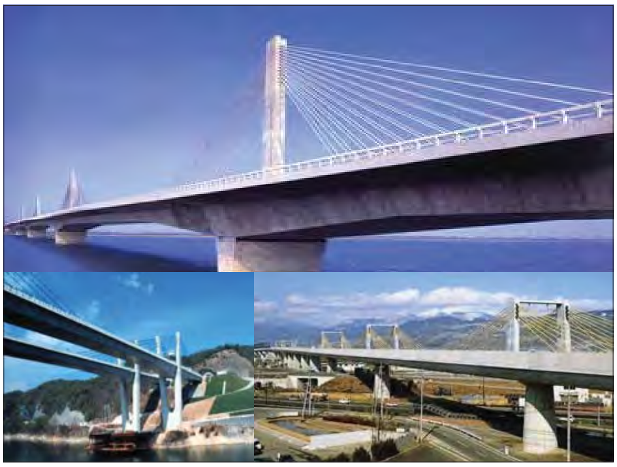


L / h ≅ 5.0 (economical)

Extradosed type



L / h ≅ 10.0



Under dead load,

cables support the girder.

However,

since cable inclination is small, &

flexural rigidity of the girder is large,

live load is carried by mainly girder.

➡ less possibility of fatigue in cables

[3] Cables

a) Cable arrangement – radial, fan, harp

b) Cable number –multi, a few

c) Cable plane –one & two

d) Cable type

In Japan,

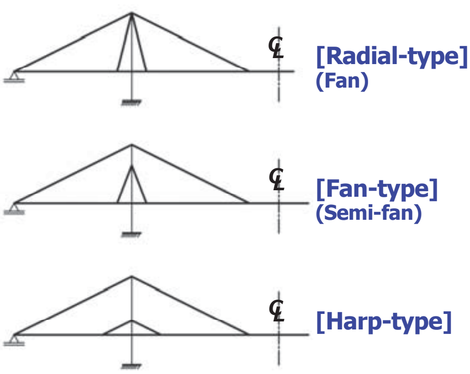
cable safety factor against breaking

is set 1.7 (for extradosed-type)

That for conventional cable-stayed bridge is 2.5*

*in USA, Europe, it is 2.2.

a) Cable arrangement



Radial-type

Fan-type

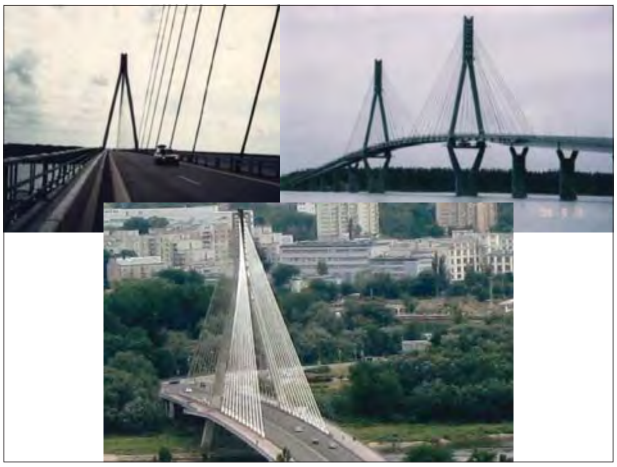
[From mechanical viewpoint], since steep inclination of cables can be obtained, [radial type is preferable].

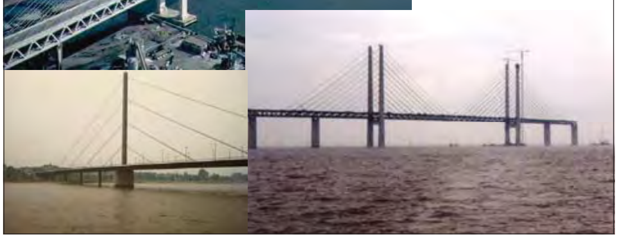
[However], since multi-cable has to be anchored at one point, [complex structural detail for anchoring] is requisite,

↓

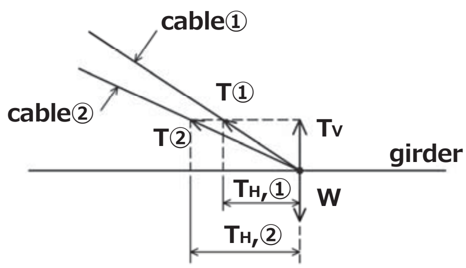
[Fan (or semi-fan) type] has been [preferably employed].

➡ **many practices is Fan type!!**





Harp-type



T① > T②

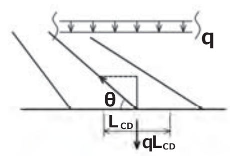
TH,① > TH,②

Even though mechanical efficiency* is a little bit inferior, because of smaller cable inclination,

In my private opinion and feeling,

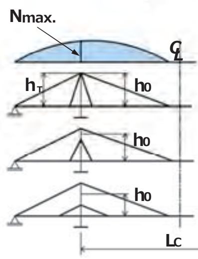
[Harp-type looks nice and gives a beautiful appearance].

*span up to 500m, difference of mechanical efficiency compared to fan-type is not so severe (please try to check).



$T\sin\theta = qL_{CD}$

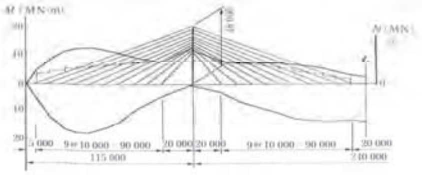
$T = qL_{CD} / \sin\theta$



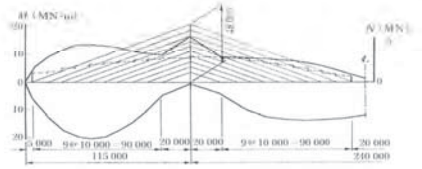
$$N_{max.} = \frac{qL_C^2}{8h_0}$$

Under live load

Fan-type

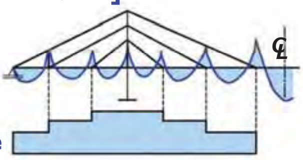


Harp-type

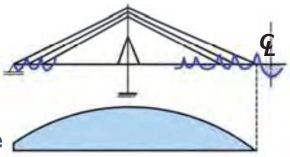


b) Number of cable

[under dead load]

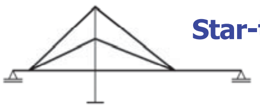


axial force

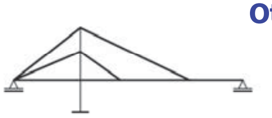


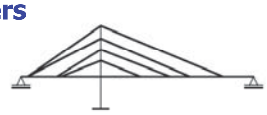
axial force

Star-type



Others

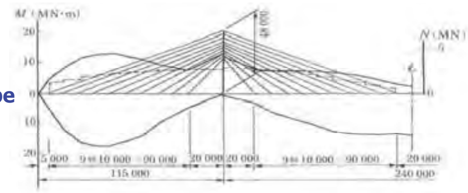




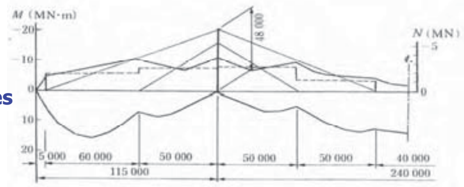
Multi-cable system

Under live load

multi-type



2 cables



Stayed by one (or) a few number of cables

Multi-cable

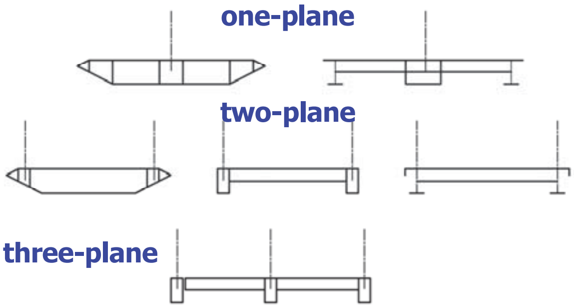
Cable size is smaller.

➡ easier to handle (design, fabrication, erection and maintenance viewpoints)

➡ easier to replace

➡ prone to vibrate (sometimes, need damper etc.)

c) Cable planes



One-plane

Two-plane

One-plane

- Box section with high torsional rigidity is requisite.
- Cable size is double compared to two-plane type.
- Bridge width becomes wider for central (single) tower and cable anchoring.

However, from aesthetic viewpoint, it is beautiful (my feeling)

[HiAm anchor cable] and [New PWS]

Cable strand with socket is made at shop and transferred to the site.

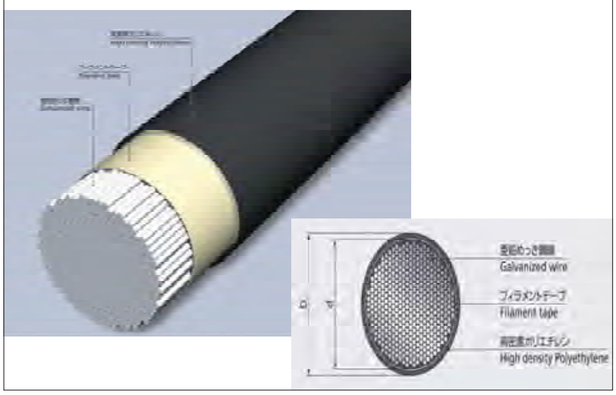
Anchor system (socket) has high fatigue strength.

Strand is consisted by Φ(diameter)-7* parallel wire (New PWS has a slight twist).

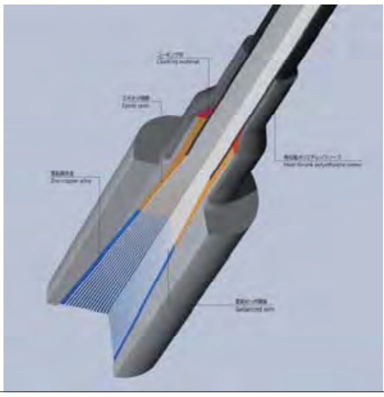
*7-millimeter diameter
(wire diameter of PWS for suspension bridge is around 5 millimeters)

d) Cable type

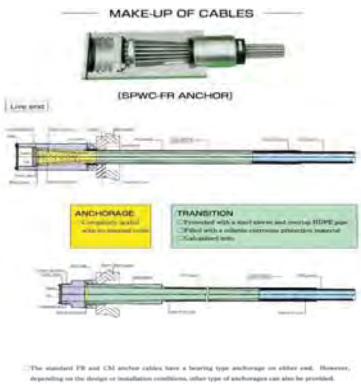
Cable section [New PWS]



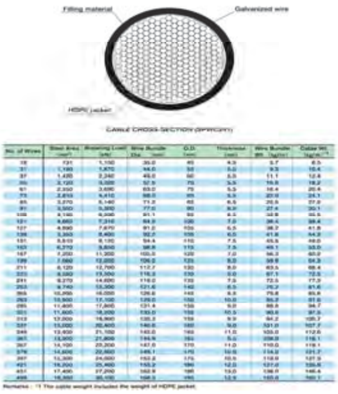
Cable socket (cable anchor)



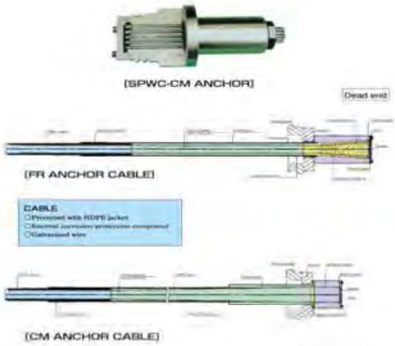
Cable anchor



Physical property [HiAm anchor cable]



Cable anchor

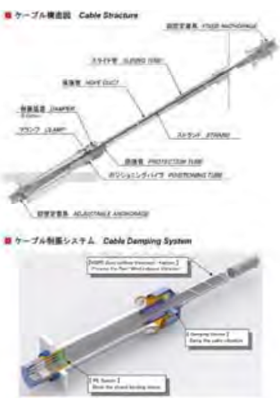


Strand ropes is transferred,

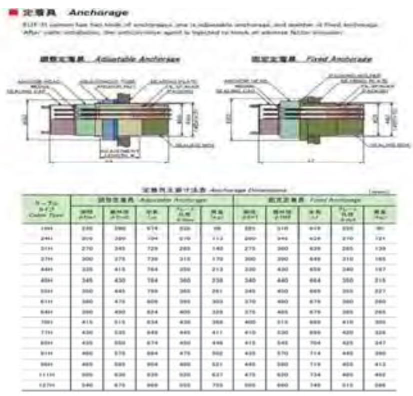
and cable is made <u>at site</u>.

Strand and section

Cable structure and damping system

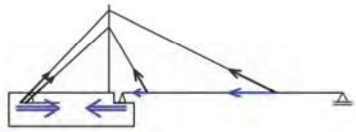


Anchorage

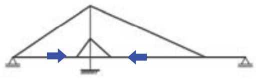


[12-2-3]

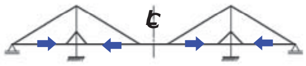
Design parameters & their selection (2)



simple span



two-span

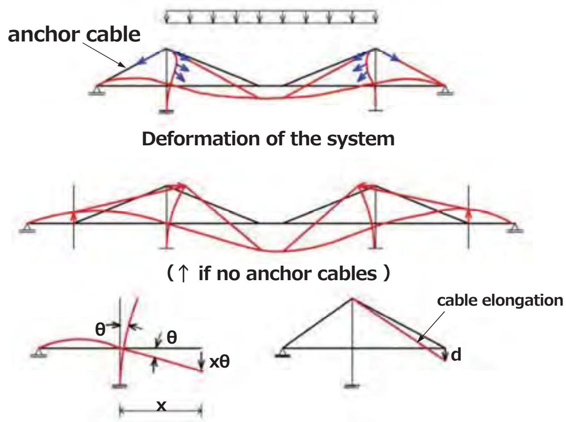


three-span

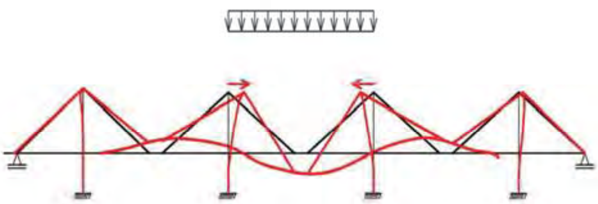
[4] Number of span

2-span

3-span



Multi-span cable-stayed bridge



due to tower top movement (→ ←) ➡ **flexible**

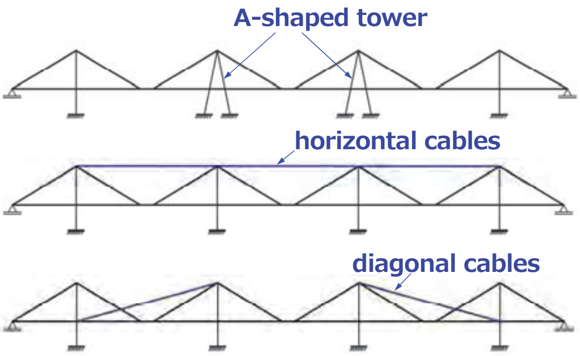
Installation of intermediate piers

in the side span is very effective

for increasing

in-plane flexural rigidity

Countermeasures

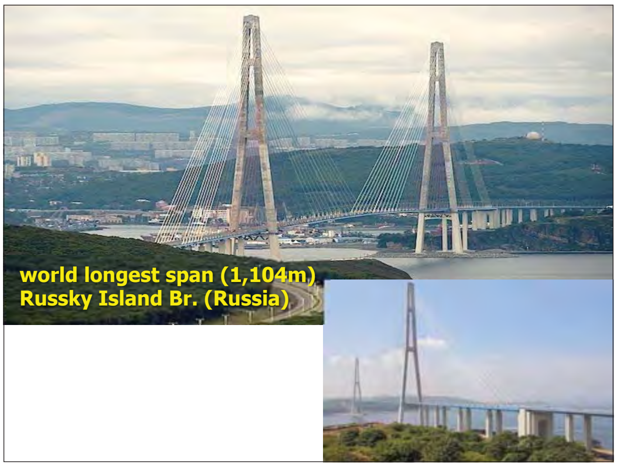


Nhat Tan Bridge (Vietnam, span = 300m)

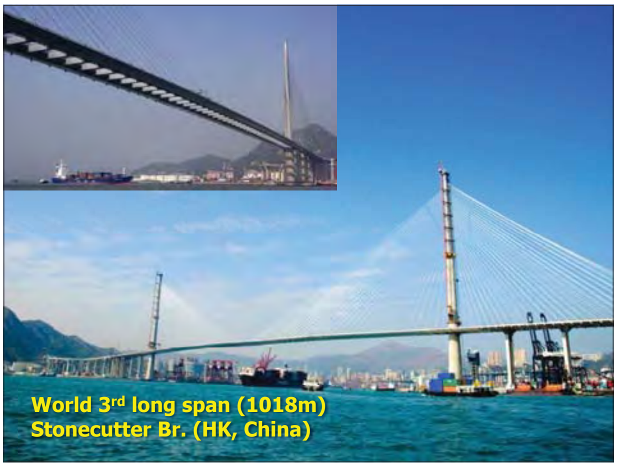
Without supplementary cables

Fudoh Br. (span = 135m) [Extradosed-type]

Yanba Br. (span = 135m) [Extradosed-type]



[5] Span length



Max. (possible*) span length of cable-stayed bridge will be

around 1,200-m (or 1,300-m)

Suspension bridge will be

3,500-m**

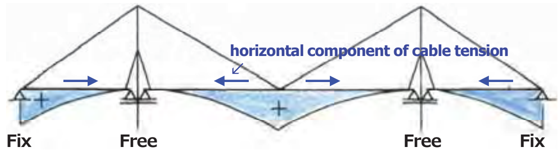
* From economical comparison with suspension bridge
**Using current cable material

[6] Cable system

(axial force in the girder,
compression ➡ tension)

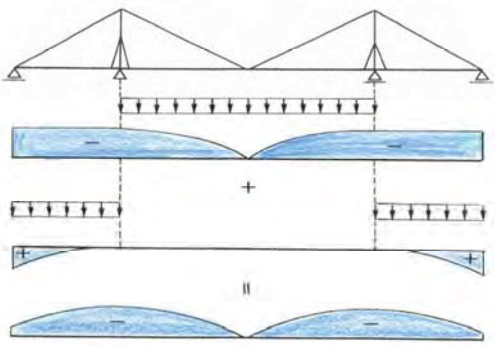
Perfect-anchored system

(Axial force in the girder is in tension)



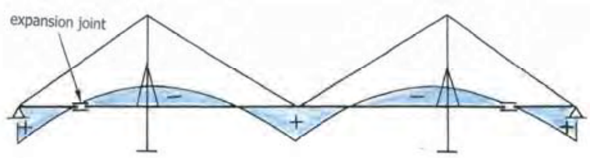
Axial force in the girder
(+) tension

Self-anchored system



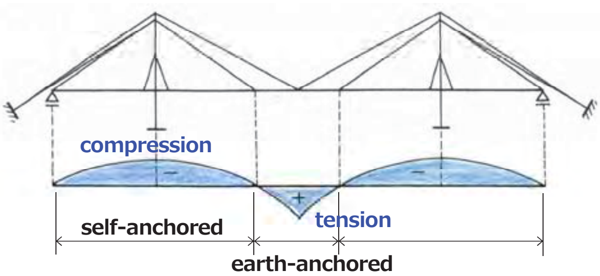
Axial force in the girder

Partial-anchored system



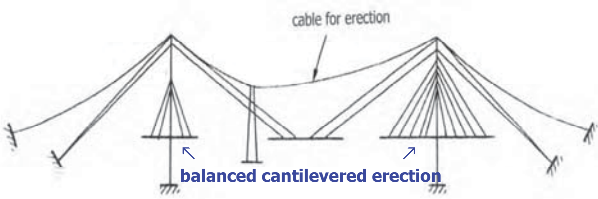
Axial force in the girder
(+) tension, (-) compression

Partially earth-anchored system

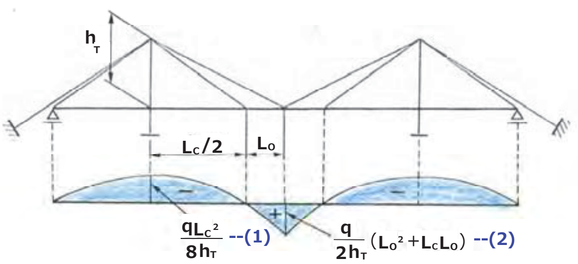


Erection method (1)

- cable erection + balanced cantilever erection -



Span extension by incorporating partially earth-anchored system

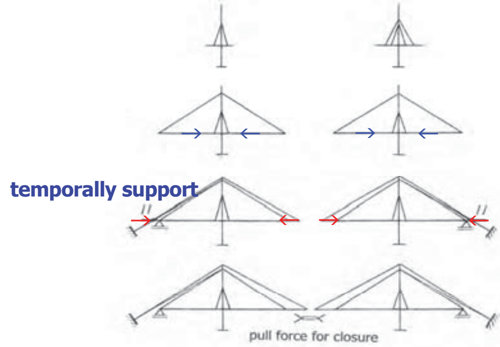


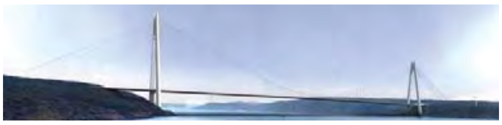
set eq.(1) and eq.(2) to be equal,

$$L_c + 2L_0 = \sqrt{2} \times L_c$$

Erection method (2)

- balanced cantilevered erection -

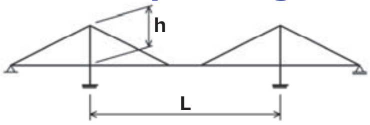




Combined [earth and self-anchored cable-stayed] bridge and [suspension] bridge

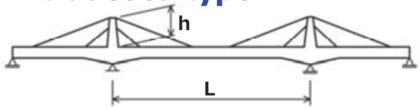
3rd Bosporus Strait Br. (Istanbul in Turkey)

Cable-stayed bridges



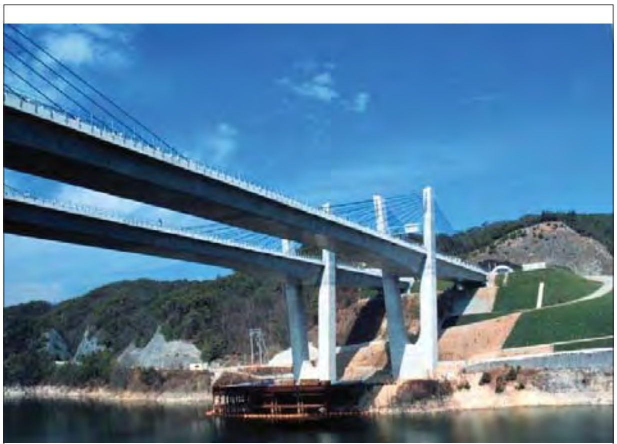
$L / h \cong 5.0$ (←economical)

Extradosed type



$L / h \cong 10.0$

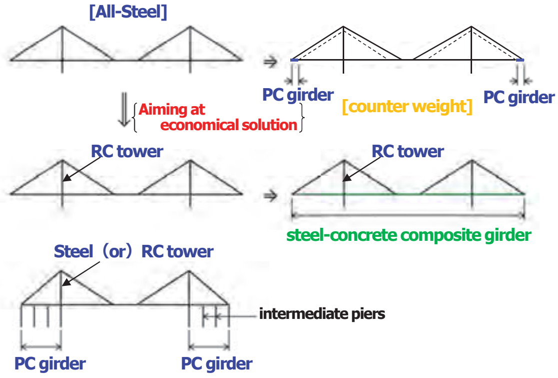
[7] Extradosed PC Bridges



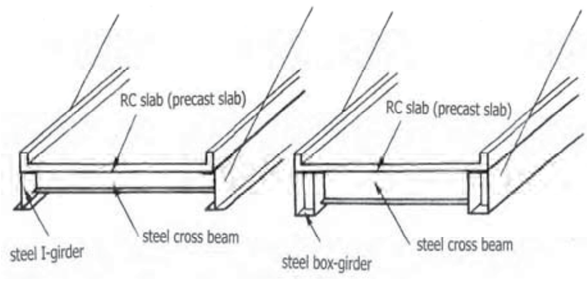
Ganter Bridge (Switzerland)

[8] Hybrid (composite & mixed) cable-stayed bridges

Hybrid type



Steel-concrete composite girder bridges



Basic concept

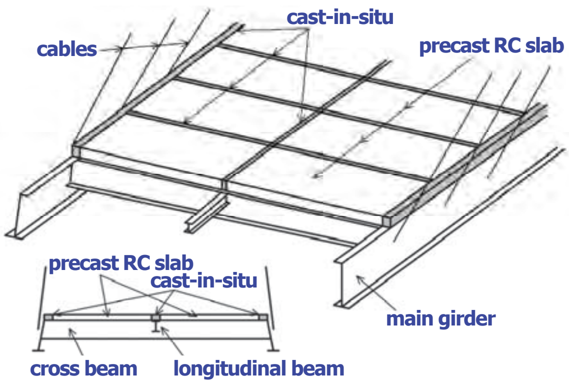
Steel : light but expensive

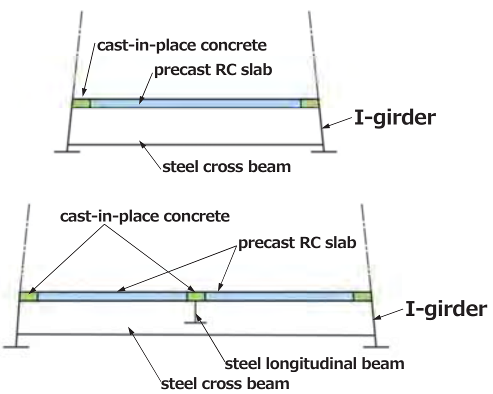
Concrete : heavy but cheaper

↓

Combination (How to combine)
of both merits &
lead to economical solution

Steel-concrete composite I girder

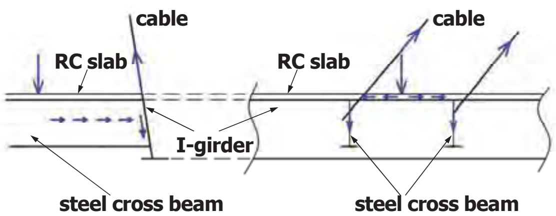




2-I-girder with composite RC slab
will be cheaper
than steel box girder

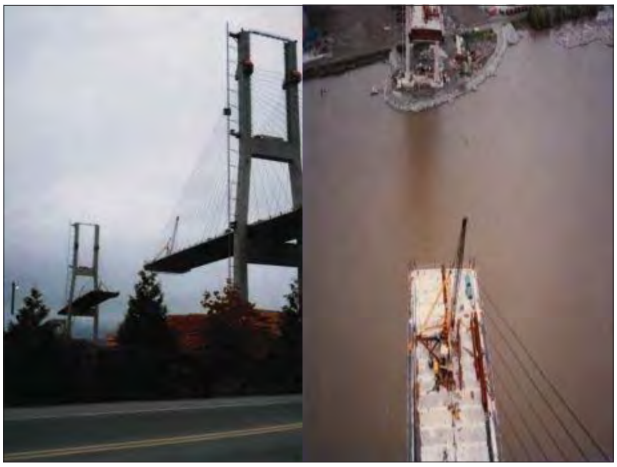
However, it shows
poor aerodynamic stability

Force flow

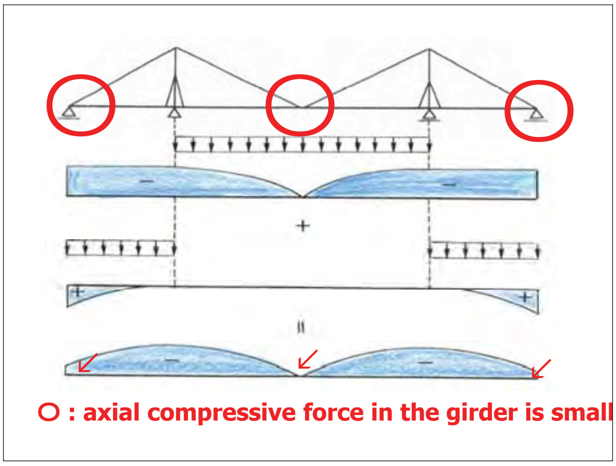


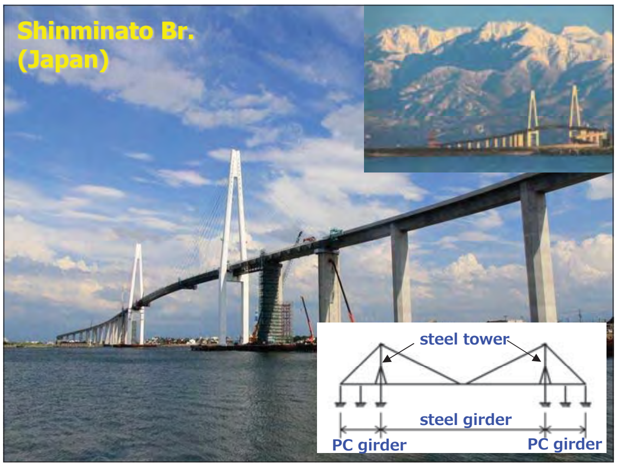
RC slab ⇒ Cross beam ⇒ Cable

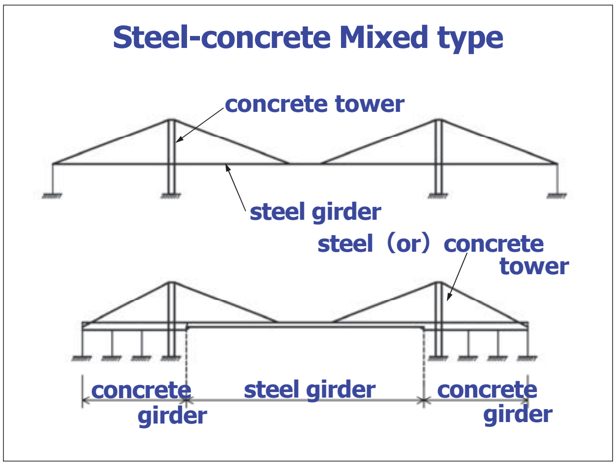

cracking



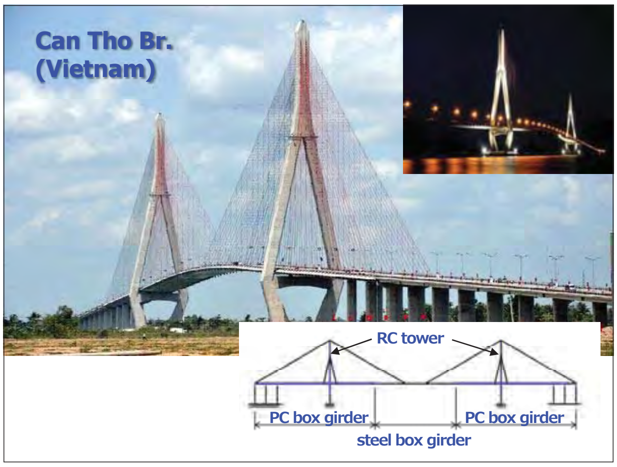

cracking
cracking


+
=
O : axial compressive force in the girder is small


Shinminato Br.
(Japan)
steel tower
steel girder
PC girder
PC girder


Steel-concrete Mixed type
concrete tower
steel girder
steel (or) concrete
tower
concrete
girder
steel girder
concrete
girder


Tatara Br.
(Japan)
Steel
PC girder
PC girder

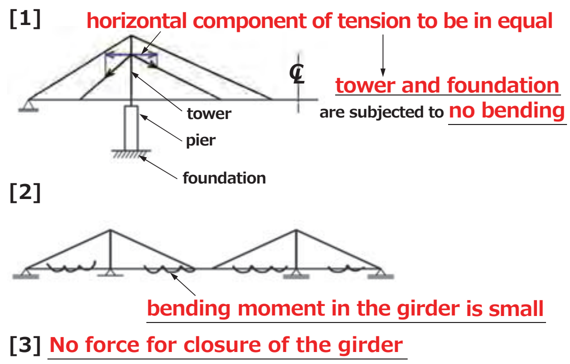


[9] Pedestrian cable-stayed bridge

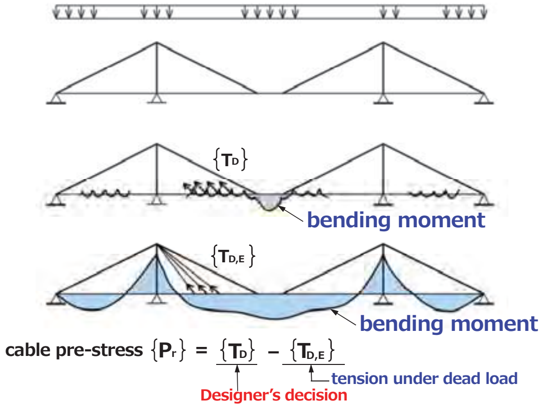
[12-3-1,2]

Estimation of stress resultants of cable-stayed bridge

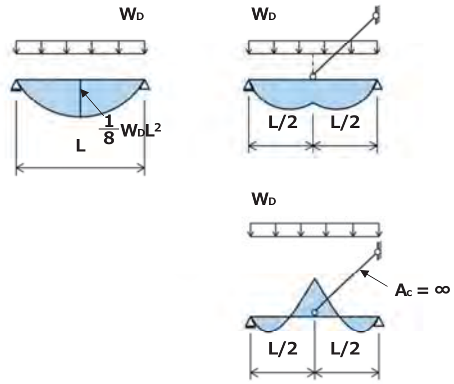
Tension in cables at completion under dead load

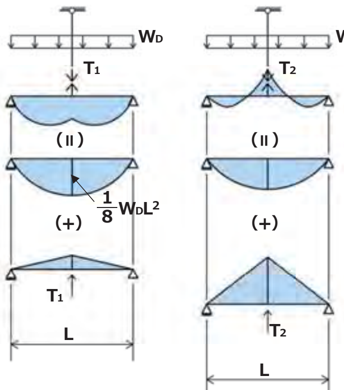


dead load



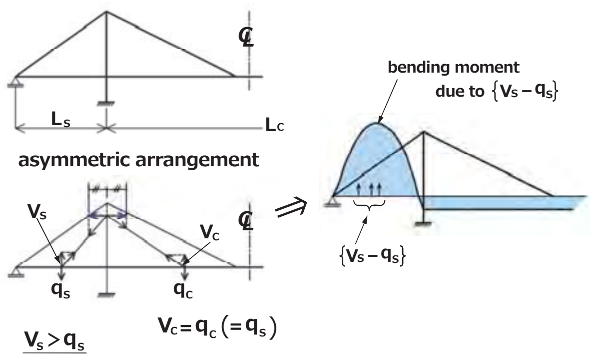
Bending moment in the girder



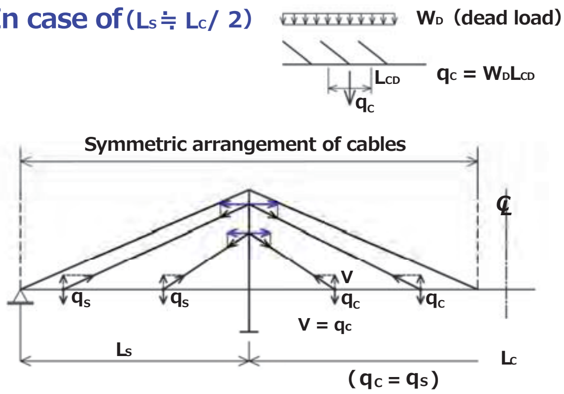


T_i : designer's choice

In case of ($L_S < L_C/2$)

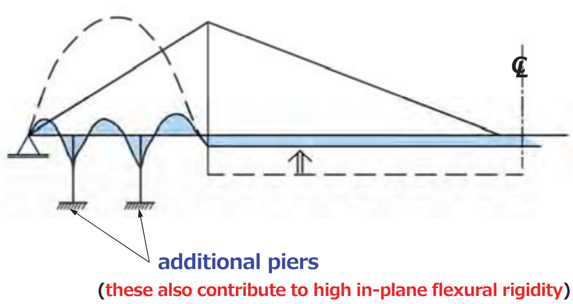


In case of ($L_S \fallingdotseq L_C/2$)



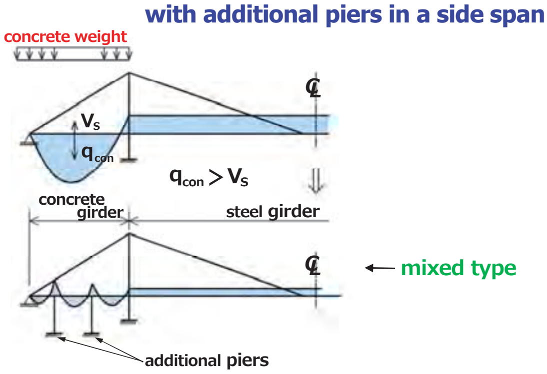
Countermeasure (1)

Installation of additional piers in a side span



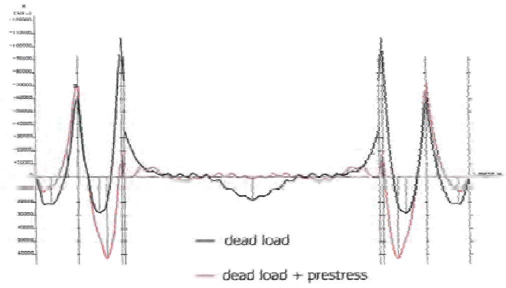
Countermeasure（2）

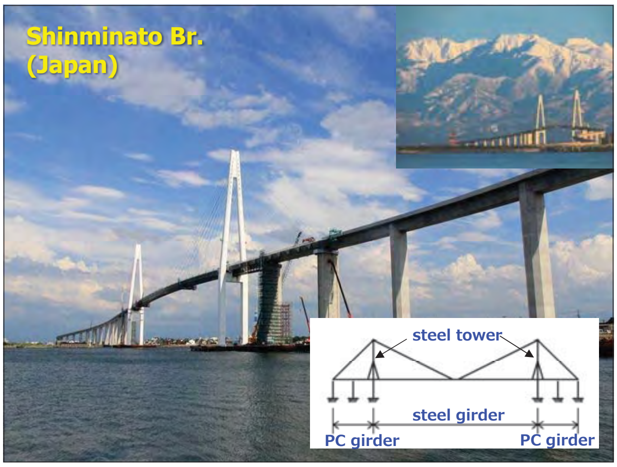
Installation of concrete weight(concrete girder) with additional piers in a side span



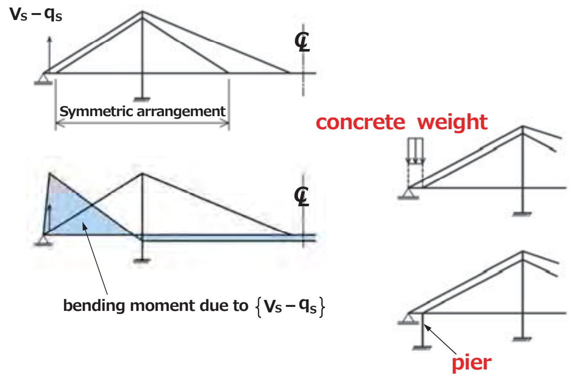
Ex. Bending moment at completion

(Shinminato Br.)

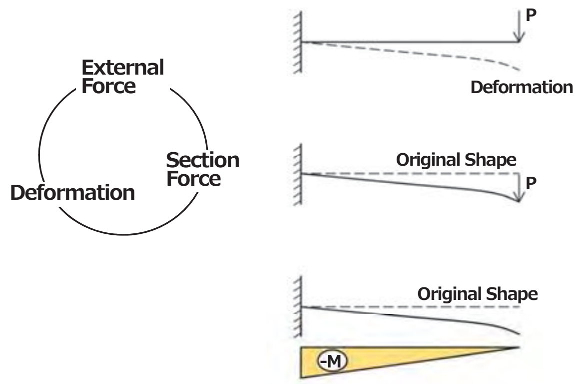




Countermeasure（3）



Fundamental rule

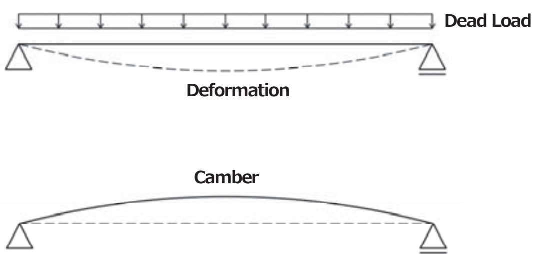


BRIDGE CLOSURE METHOD*

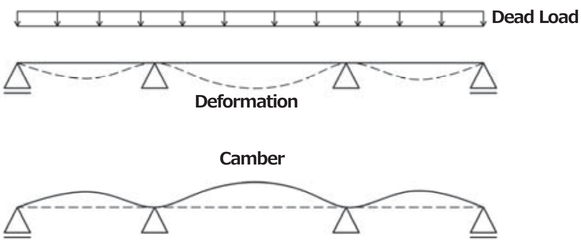
* By Mr. Tomoda (NIPPON KOEI)

Concept of camber

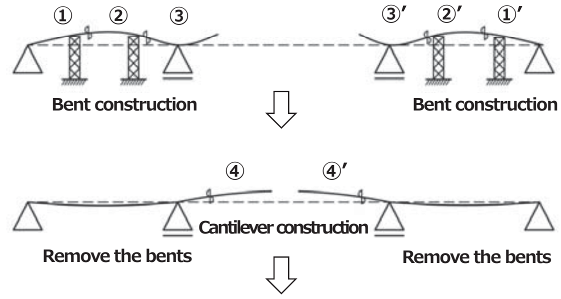
(Simple Girder)



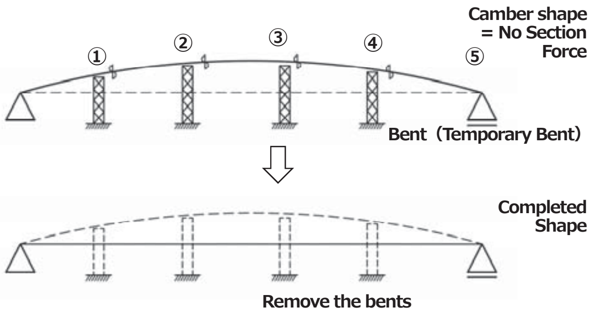
Concept of camber
(3-span continuous girder)

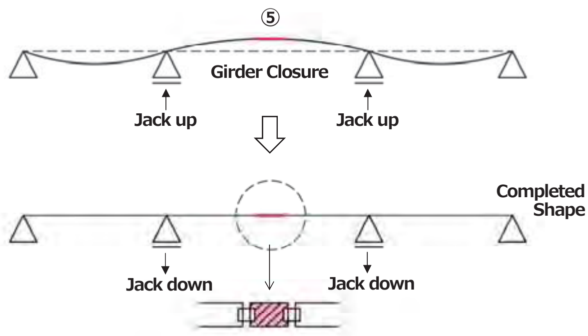


Cantilever construction method
(3 span continuous girder)

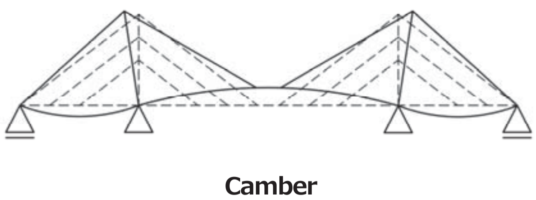


Bent construction method
(Simple Girder)



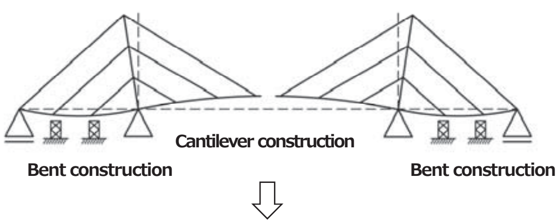


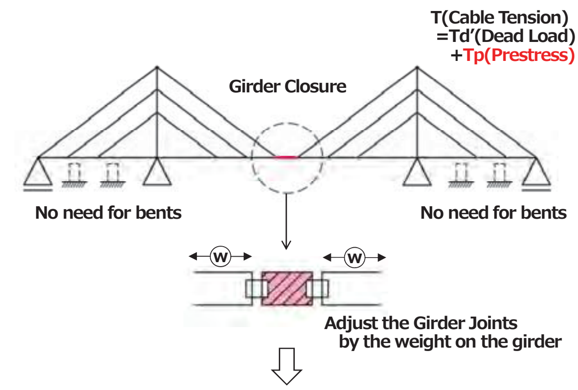
Concept of camber
(Cable-stayed Bridge)

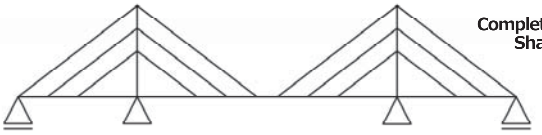


Cantilever construction

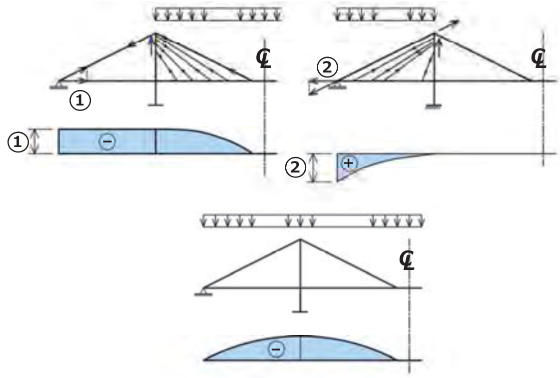
Cantilever construction method
(Center span)



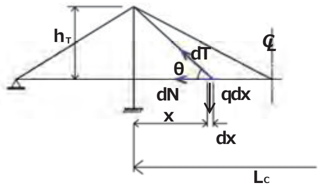




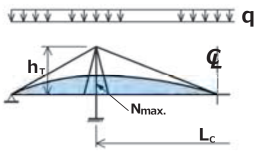
T(Cable Tension) = Td(Dead Load) + Tp(Prestress)



Axial force in the girder

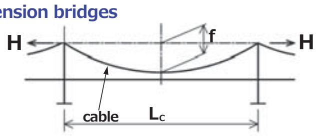


$$dT\sin\theta = qdx$$
$$dT\cos\theta = dN$$
$$dN = \frac{qdx}{\sin\theta}\cdot\cos\theta = \frac{qdx}{\tan\theta}$$
$$(1/\tan\theta = x/h_T)$$
$$dN = \frac{q}{h_T}xdx$$
$$N_{max.} = \int_0^{L_C/2} dN = \frac{q}{h_T}\left[\frac{x^2}{2}\right]_0^{L_C/2} = \frac{qL_C^2}{8h_T}$$



$$N_{max.} = \frac{qL_C^2}{8h_T}$$

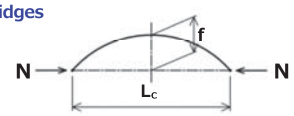
Suspension bridges



$$H = \frac{qL_C^2}{8f}$$

(f : sag)

Arch bridges

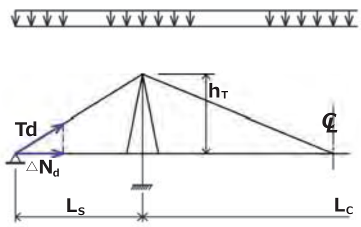


$$N = \frac{qL_C^2}{8f}$$

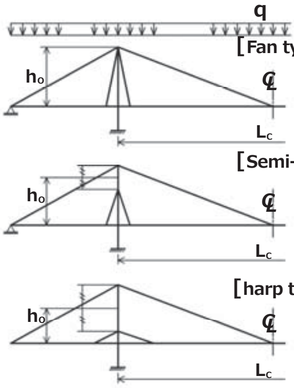
(f : raise)

($L_S < L_C/2$)

dead load (W_d)

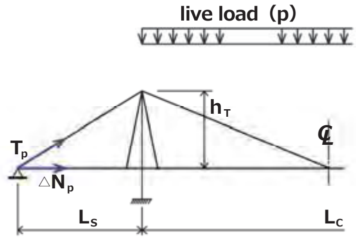


$$\triangle N_d = \frac{W_d}{h_T}\int_0^{L_C/2} xdx - \frac{W_d}{h_T}\int_0^{L_S} xdx$$
$$= \frac{W_d L_C^2}{8h_T}\left\{1 - \left(\frac{L_S}{L_C}\right)^2\right\}$$



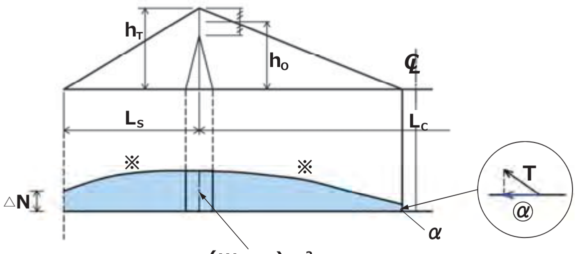
$$N_{max.} = \frac{qL_C^2}{8h_o}$$

live load (p)



$$\triangle N_p = \frac{pL_C^2}{8h_T}$$
$$\triangle N = \triangle N_d + \triangle N_p$$
$$= \frac{W_d L_C^2}{8h_T}\left\{1 - \left(\frac{L_S}{L_C}\right)^2\right\} + \frac{pL_C^2}{8h_T}$$

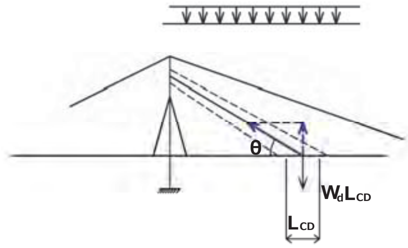
Axial force distribution



$\frac{(W_d + p) L_C^2}{8 h_O}$

※parabolic distribution

dead load (Wd)

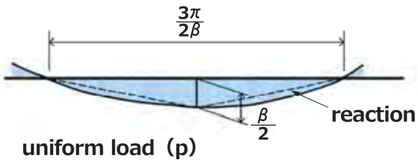


$T_D \sin\theta = W_d L_{CD}$

$T_D = \frac{W_d L_{CD}}{\sin\theta}$

Tension in cables

under live load



uniform load (p)

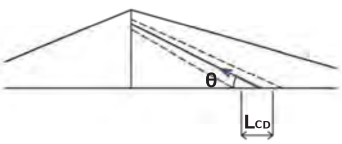
$$R_u = \frac{3\pi}{\frac{1}{2}\cdot 2\beta}\cdot\frac{\beta}{2}\cdot pL_{CD} = \frac{3\pi}{8}\,pL_{CD} \fallingdotseq 1.2\,pL_{CD}$$

concentrated load (P)

$$R_C = P\frac{\beta}{2}L_{CD} = \frac{P\beta}{2}L_{CD}$$

$$R_u + R_C = 1.2\,pL_{CD} + \frac{P\beta}{2}L_{CD}$$

Tension in cable for the design



$$T\sin\theta = W_d L_{CD} + \left(1.2\,p + \frac{P\beta}{2}\right) L_{CD}$$

$$T = \left\{ W_d L_{CD} + \left(1.2\,p + \frac{P\beta}{2}\right) L_{CD} \right\} / \sin\theta$$

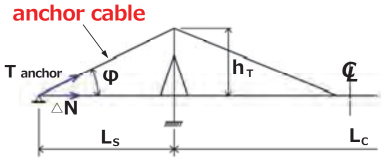
Cross sectional area (A_c)

$A_c = T / \sigma_a$

σ_a : allowable stress of cable

Bending moment in the girder

Tension in anchor cable and uplift force at end support



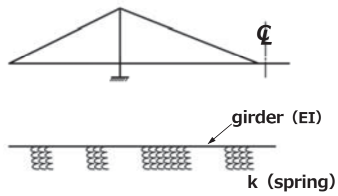
$$\triangle N = \frac{W_d L_C^2}{8 h_T}\left\{1 - 4\left(\frac{L_S}{L_C}\right)^2\right\} + \frac{pL_C^2}{8 h_T}$$

$T\cos\varphi = \triangle N$

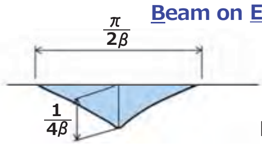
$T_{anchor} = \triangle N / \cos\varphi$

Uplift force

$R_{up\text{-}lift} = \triangle N \tan\varphi \quad (= T_{anchor} \cdot \sin\varphi)$



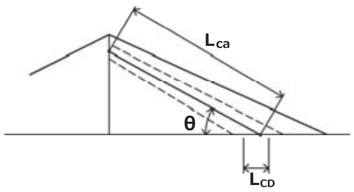
Beam on Elastic Foundation(BEF model)



$$\beta = \sqrt[4]{\frac{k}{4EI}}$$

EI : flexural rigidity of girder

Calculation of k

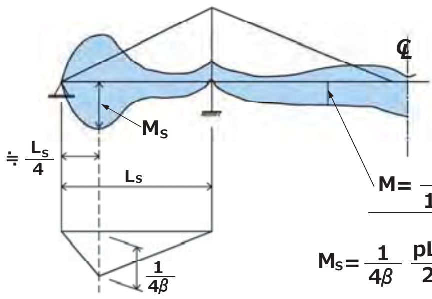


$$k = \frac{E_c A_c}{L_{ca}} \cdot \sin^2\theta \; / \; L_{cd}$$

$E_c A_c$: axial rigidity

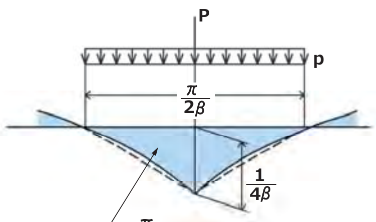
A_c : cross sectional area of cable

Bending moment distribution by live load



$$M = \frac{p\pi}{16\beta^2} + \frac{P}{4\beta}$$

$$M_S = \frac{1}{4\beta}\frac{pL_S}{2} + \frac{P}{4\beta} = \frac{pL_S}{8\beta} + \frac{P}{4\beta}$$



$$A = \frac{\pi}{16\beta^2}$$

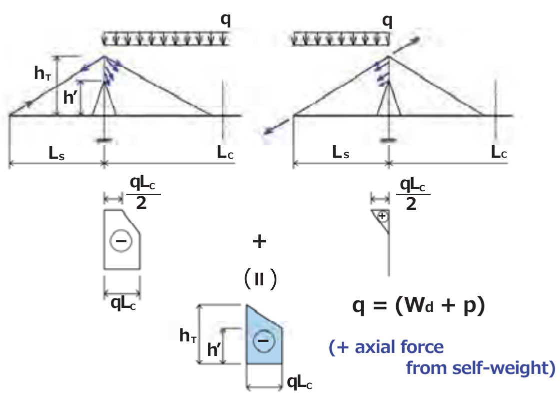
$$M = \frac{p\pi}{16\beta^2} + \frac{P}{4\beta}$$

p : uniformly distributed load

P : concentrated load

Axial force and bending moment in the tower

Axial force in the tower

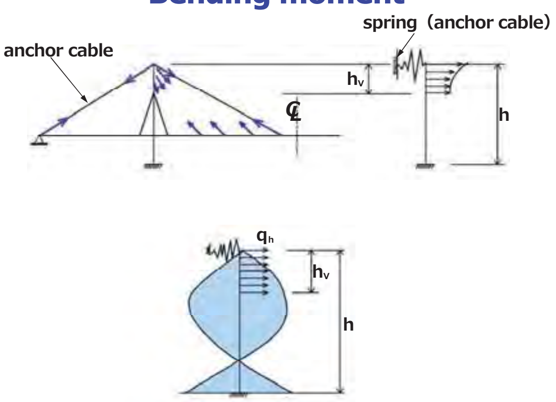


$$M_{max.} = \frac{R_T^2}{2q_h}$$

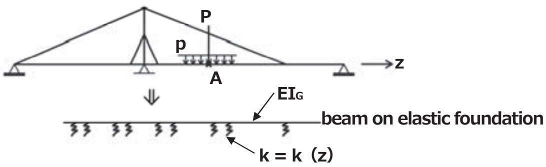
$$R_T = \frac{q_h h}{8} \xi (8 - 6\xi + \xi^3)$$

$$q_h = \frac{pL_C^2}{8h_0} / h_V \quad , \quad \xi = \frac{h_V}{h}$$

Bending moment



Prediction of bending stress change due to web depth (box section) change



Under uniformly distributed loading (p) ,
design bending moment (at A) in given .

$$M_u = \frac{p\pi}{16\beta^2}$$

$$\beta = \sqrt[4]{\frac{k}{4EI_G}}$$

Change of stress (σ)

$$\frac{\sigma_2}{\sigma_1} = \frac{M_2}{M_1}\cdot\frac{W_1}{W_2} = \sqrt{\frac{I_{G,2}}{I_{G,1}}}\cdot\frac{W_1}{W_2}$$

$$= \sqrt{(1 + iA_{w,1} / 6A_f) \;/\; (1 + iA_{w,2} / 6A_f)}$$

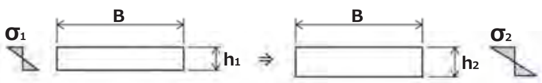
$$\fallingdotseq 1.0$$ (← not affected by web depth)

In case of concentrated load (P)

$$M_C = \frac{P}{4\beta}$$

$$\frac{\sigma_2}{\sigma_1} \fallingdotseq \sqrt{\frac{h_1}{h_2}}$$

box girder



A_f : cross sectional area of upper and lower flanges
A_w : cross sectional area of webs (i : number of web plate)

Under assumption that k is constant ,
M changes due to change of web depth (h)

$$\frac{M_2}{M_1} = \sqrt{\frac{I_{G,2}}{I_{G,1}}}$$

$$I_{G,1} = \frac{h_1^2}{2}\left(A_f + \frac{iA_{w,1}}{6}\right), \quad I_{G,2} = \frac{h_2^2}{2}\left(A_f + \frac{iA_{w,2}}{6}\right),$$

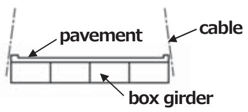
$$W_1 = I_{G,1} / (h_1 / 2), \quad W_2 = I_{G,2} / (h_2 / 2)$$

Prediction of stress (σ_n & σ_b) depending on span length

σ_n : normal stress due to axial force
σ_b : bending stress

Prediction of girder stress depending on span (L)

(multi-cable type , girder with box section)



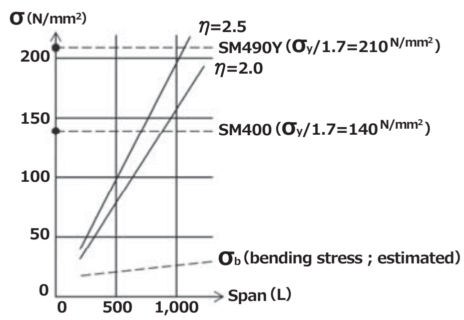
Assumption of dead load (W_D)

$W_D = 1.4\,\gamma_s A_s + W_{DS} = \eta\,\gamma_s A_s$ ($\eta = 2.0\sim2.5$)

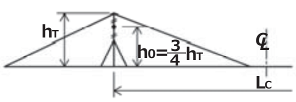
γ_s : weight per unit volume (77kN/m³)
A_s : cross-sectional area of girder resisting axial force
1.4 : increment coefficient (← I assumed)
to take into account of cross beam etc.
not resisting axial force
W_{DS} : superimposed dead load (pavement, curb etc.)

$\sigma_{max.}$ = (155～194)L_C (kN/m²)

L_C : span (m) (η = 2.0～2.5)



$$N_{max.} = \frac{qL_C^2}{8h_0} = \frac{q}{8}\cdot\left(\frac{L_C}{h_0}\right)\cdot L_C = \frac{q}{8}n_h\cdot L_C \quad (n_h = (L_C/h_0))$$



$L_C / h_T \fallingdotseq 5.0$
$L_C / h_0 \fallingdotseq 6.7$

$q = W_D + p = W_D(1 + \omega)$ p : live load

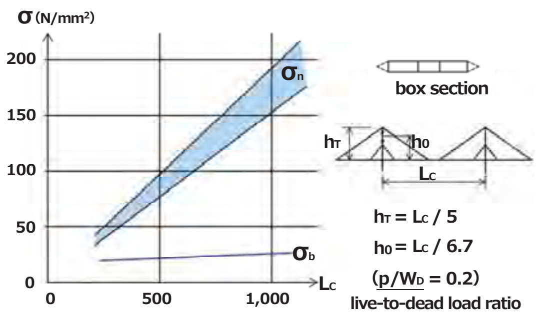
$$N_{max.} = \frac{W_D(1+\omega)}{8}\cdot n_h L_C$$

$$= \frac{\eta\gamma_s A_s}{8}(1+\omega)\,n_h L_C$$

$$\sigma_{max.} = \frac{N_{max.}}{A_s} = \eta\frac{\gamma_s}{8}(1+\omega)\,n_h L_C$$

Assumption is made.
$n_h = 6.7$, $\omega = 0.2$

Predicted stress(σ)-span(L_c) relationship



σ_n : normal stress due to axial force
σ_b : bending stress

[12-3-3]

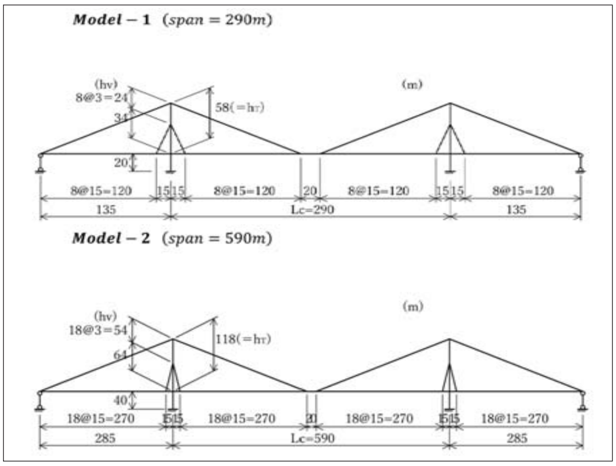
Exercises

– Estimation of stress resultants -

Multi-cable type 3-span continuous bridge

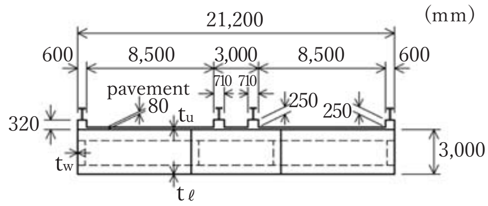
span = 290m, 590m

[1] Axial force and stress in the girder
[2] Tension in cables and required cable area
[3] Up-lift force
[4] Bending moment and stress in the girder
[5] Axial force in the tower
[6] Max. bending moment in the tower



【Model – 1】

Dead load (W_d)



$t_u = t_\ell = t_w = 20mm$ *(assumed)*

(including longitudinal ribs)

Curb	2×0.6×0.32×24.5 kN/m³	=	9.4 kN/m
Median Strip	2×0.71×0.32×24.5 kN/m³	=	11.1 kN/m
Asphalt pavement	2×8.5×0.08×22.5 kN/m³	=	30.6 kN/m
Rail	4× 0.5 kN/m³	=	2.0 kN/m
Steel girder	1.4*×1.088**×77.5 kN/m³	=	118.0 kN/m
	W_d	=	171.1 kN/m

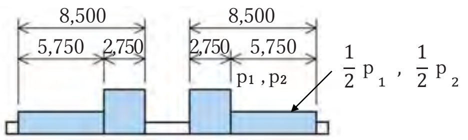
* 1.4 : *take into account steel volume not resisting axial force such as cross beams , diaphragms etc.*

** $A_S = 2 \times 21.2 \times 0.02 + 4 \times 3 \times 0.02 = 1.088\ m^2$

$$\left(I_S = 2 \times 21.2 \times 0.02 \times 1.5^2 + 4 \times \frac{0.02 \times 3^2}{12} = 2.088\ m^4 \right)$$

【Model – 1】

Live load $(\beta - live\ load)$



$\frac{1}{2}P_1$, $\frac{1}{2}P_2$

$$\text{p} = \left(5.5m \times 10\,kN/m^2 + \frac{1}{2} \times 11.5m \times 10\,kN/m^2\right) \times \underline{10m} = \underline{1{,}125\ kN}$$

deal with as concentrated load (assumption)

$$\text{P} = 5.5m \times 3.0\,kN/m^2 + \frac{1}{2} \times 11.5m \times 3.0\,kN/m^2 = 33.75\ kN/m$$

uniform load $\quad$ P = 33.75 kN/m

concentrated load $\quad$ p = 1,125 kN

【Model – 1】

$\{AA\}$ ***Axial force and stress in the girder***

$$N_{max.} = \frac{(W_d + \text{p})}{8h_0} L_C^{\,2} \quad (at\ tower)$$

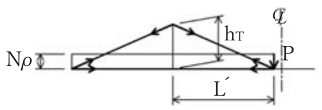
$L_C = 290m$, $h_0 = 34 + 24/2 = 46\ m$

$W_d = 171.1\ kN/m$, $\text{p} = 33.75\ kN/m$

$$N_{max.} = \frac{(171.1 + 33.75)}{8 \times 46} \times 290^2 = 46{,}815\ kN$$

$$\sigma_{max.} = \frac{N_{max.}}{A_S} = \frac{46{,}815}{1.088} = 43{,}028\ kN/m^2 \quad \underline{(= 43.0\ N/mm^2)}$$

Axial force due to concentrated load (P)

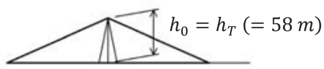


$$N_p = P\frac{L'(\cong L_C/2)}{h_T} = 1{,}125 \times \frac{145}{58} = 2{,}813\ kN$$

$$\sigma_{NP} = \frac{N_p}{A_s} = \frac{2{,}813}{1.088} = 2{,}585\ kN/m^2 \quad (= 2.6\ N/mm^2)$$

$$\overline{\sigma_{max.}} = \sigma_{max.} + \sigma_{NP} = \underline{45.6\ N/mm^2}$$

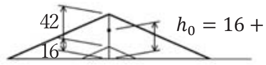
$\{AA-1\}$ ***in case of Radial – type***



$$\sigma_{max.} = \frac{(171.1 + 33.75)}{8 \times 58} \times 290^2 \Big/ 1.088 = \underline{34.1\ N/mm^2}$$

$$\overline{\sigma_{max.}} = 34.1 + 2.6 = 35.7\ N/mm^2$$

$\{\boldsymbol{AA-2}\}$ ***in case of Harp − type***



$h_0 = 16 + 42/2 = 37\ m$

$$\sigma_{max.} = \frac{(171.1 + 33.75)}{8\times37}\times290^2/1.088 = 53.5\ N/mm^2$$

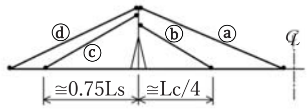
$$\sigma_{max.} = \sigma_{max.} + \sigma_{NP} = 53.5 + 2.6 = 56.1\ N/mm^2$$

Maximum stress at the tower point

N/mm²

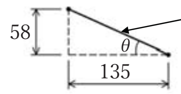
	Fan	Radial	Harp
σ n	45.6	35.7	56.1

$\{\boldsymbol{BB}\}$ ***Cable tension force***



(1) *Cable* ⓐ

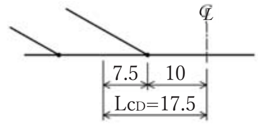
(m)



$L_{cable} = 146.9$

$\sin\theta = 0.395$

[*dead load*]



$T_d \sin\theta = W_d L_{CD} = 171.1\times17.5 = 2{,}994.3\ kN$

$T_d = 7{,}580\ kN$

[*live load (impact is not included)*]

$\underline{first\ ,\quad assume\quad \beta = 0.0150}$

$$T_L \sin\theta = \left(1.2\rho + \frac{P\beta}{2}\right)L_{CD}$$

$$= \left(1.2\times33.75 + \frac{1{,}125\times0.015}{2}\right)\times17.5$$

$= 856.4\ kN$

$T_L = 2{,}168\ kN$

$T = T_d + T_L = 9{,}746\ kN$

Allowable stress of cable is assumed

$\sigma_a = 640\ N/mm^2 \quad \left(\sigma_a = \underline{\sigma_B}/2.5\right)$

↑ *breaking*

$$A_C > \frac{9{,}746\times10^3}{640}\underline{\times1.1} = 16{,}751\ mm^2$$

↑ *margin*

$\phi7\ (A = 38.47\ mm^2)$

$\underline{No.\ of\ wire\quad > \quad \frac{16{,}751}{38.47} = \underline{435.4}\ (436)}$

↑ *check Catalog of cables*

$$K = \frac{EA_C}{L_{cable}}\sin^2\theta/L_{CD}$$

$$= \frac{2\times10^8\times0.0168}{146.9}\times0.395^2/17.5 = 203.9\ \ kN/m^2$$

$$\beta = \sqrt[4]{\frac{K}{4EI}} = \sqrt[4]{\frac{203.9}{4\times2.0\times10^8\times2.088}} = \underline{0.0187}\quad (\neq 0.0150)$$

Set $\beta = 0.0187$, *and repeat.*

$$T_L \sin\theta = \left(1.2\times33.75 + \frac{1{,}125\times0.0187}{2}\right)\times17.5$$

$= 892.8\ kN$

$T_L = 2{,}260\ kN$

$T_D + T_L = 9{,}840\ kN$

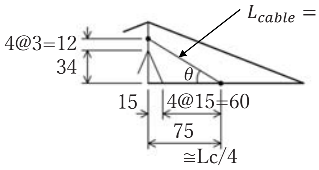
$$A_C > \frac{9{,}840\times10^3}{640}\times1.1 = 16{,}913\ mm^2\ \ (0.0169\ m^4)$$

$$K = \frac{2\times10^8\times0.0169}{146.9}\times0.395^2/17.5 = 205.1\ \ kN/m^2$$

$$\beta = \sqrt[4]{\frac{205.1}{4\times2.0\times10^8\times2.088}} = \underline{0.0187}\quad converged\ !!$$

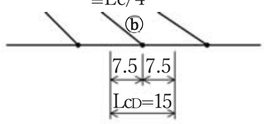
(2) *Cable* ⓑ

(m)



$L_{cable} = 88.0$

$\sin\theta = 0.523$



$T_d \sin\theta = 171.1\times15 = 2{,}567\ kN$

$T_d = 4{,}907\ kN$

$\underline{assume\quad \beta = 0.023}$

$$T_L \sin\theta = \left(1.2\times33.75 + \frac{1{,}125\times0.023}{2}\right)\times15$$

$= \ 801\ kN$

$T_L = 1{,}533\ kN$

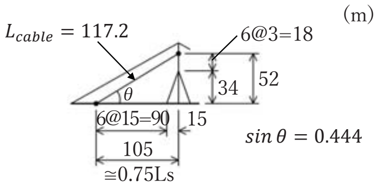
$T_d + T_L = 6{,}440\ kN$

$$A_C > \frac{6{,}440\times10^3}{640}\times1.1 = 11{,}069\ mm^2\ \ (0.0111\ m^4)$$

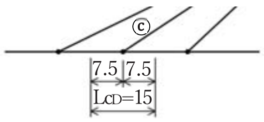
$$K = \frac{2\times10^8\times0.011}{88}\times0.523^2/15 = 455.9\ \ kN/m^2$$

$$\beta = \sqrt[4]{\frac{455.9}{4\times2.0\times10^8\times2.088}} = \underline{0.0230}\quad (\boldsymbol{OK}\ !!)$$

(3) *Cable* ⓒ



$sin\,\theta = 0.444$



$T_d \sin\theta = 171.1\times 15 = 2{,}567\ kN$

$T_d = 5{,}782\ kN$

$\underline{assume \quad \beta = 0.020}$

$T_L \sin\theta = \left(1.2\times 33.75 + \dfrac{1{,}125\times 0.02}{2}\right)\times 15$

$= \ 777\ kN$

$T_L = 1{,}750\ kN$

$T_D + T_L = 7{,}532\ kN$

$A_C > \dfrac{7{,}532\times 10^3}{640}\times 1.1 = 12{,}946\ mm^2 \ \ (0.0129\ m^4)$

$K = \dfrac{2\times 10^8\times 0.013}{117.2}\times 0.444^2/15 = 291.6\ \ kN/m^2$

$\beta = \sqrt[4]{\dfrac{291.6}{4\times 2.0\times 10^8\times 2.088}} = \underline{0.020} \quad (\boldsymbol{OK}\ !!)$

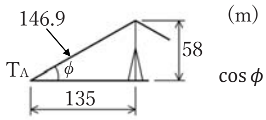
(4) *Cable* ⓓ ← *Anchor cable*

$\Delta N = \dfrac{W_d {L_C}^2}{8h_T}\left\{1 - 4\left(\dfrac{L_S}{L_C}\right)^2\right\} + \dfrac{p{L_C}^2}{8h_T}\ (+N_\rho)$

$= \dfrac{171.1\times 290^2}{8\times 58}\cdot\left\{1 - 4\left(\dfrac{134}{290}\right)^2\right\} + \dfrac{33.75\times 290^2}{8\times 58} + 2{,}813$

$= 4{,}125 + 6{,}117 + 2{,}813$

$= 13{,}055\ kN$ (*)



$\cos\phi = 0.919$

$T_A \cos\phi = \Delta N$

$T_A = \dfrac{\Delta N}{\cos\phi} = 14{,}206\ kN$

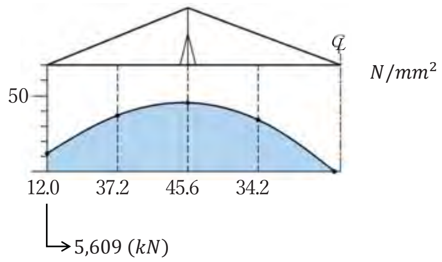
$A_C > \dfrac{14{,}206\times 10^3}{640}\times 1.1 = 24{,}417\ mm^2 \ \ (0.0244\ m^4)$

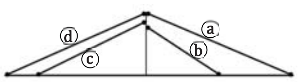
$\{CC\}$ ***UP − lift force***

$R_u = \Delta N \tan\phi = 13{,}055\times\dfrac{58}{135} = 5{,}609\ kN$

((*) $\sigma_n/end = \dfrac{13{,}055}{1.088} = 11{,}999\ \ kN/m^2 = 12.0\ \ N/mm^2$)

$\{\boldsymbol{\sigma_n}\}$



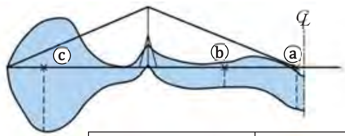


	Ac (mm²)	No. of ϕ7 wire
ⓐ	16,913	440
ⓑ	11,069	286
ⓒ	12,946	338
ⓓ	24,417	634

※ per bridge

$\phi 7\ (A \cong 38.5\ mm^2)$

$\{DD\}$ ***Bending moment and stress in the girder***



	position *	β
ⓐ	≅ Lc / 2	0.0187
ⓑ	≅ Lc / 4	0.0230
ⓒ	≅ 3Ls / 4	0.0200

* from tower point

(1) *at* ⓐ

$M = \dfrac{\rho\pi}{16\beta^2} + \dfrac{P}{4\beta}$

$= \dfrac{33.75\times\pi}{16\times 0.0187^2} + \dfrac{1{,}125}{4\times 0.0187}$

$= 18{,}941 + 15{,}040$

$= 33{,}981\ kN\cdot m$

$\sigma_b = \dfrac{33{,}981}{2.088}\times\underline{1.5} = \underline{24.4\ \ N/mm^2}$

↑ *web depth* (= 3*m*)/2

(2) *at* ⓑ

$M = \dfrac{33.75\times\pi}{16\times 0.0230^2} + \dfrac{1{,}125}{4\times 0.0230}$

$= 12{,}521 + 12{,}228$

$= 24{,}749\ kN\cdot m$

$\sigma_b = \dfrac{24{,}749}{2.088}\times 1.5 = 17{,}779\ \ kN/m^2 = \underline{17.8\ N/mm^2}$

(3) *at* ⓒ

$$M = \frac{\rho L_S}{8\beta} + \frac{P}{4\beta}$$

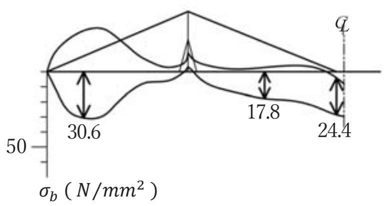
$$= \frac{33.75 \times 135}{8 \times 0.02} + \frac{1,125}{4 \times 0.02}$$

$$= 28,477 + 14,063$$

$$= 42,540\ kN \cdot m$$

$$\sigma_b = \frac{42,540}{2.088} \times 1.5 = \underline{30.6\ N/mm^2}$$

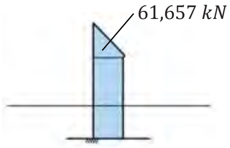
$\{\sigma_b\}$



{EE} *Axial force in the tower*

$$N_T = (W_d + \rho)\, L_C + 2P$$

$$= (171.1 + 33.75) \times 290 + 2 \times 1,125 = 61,657\ kN$$



(+ *self weight*)

{FF} *Max. bending moment in the tower*

$$\xi = \frac{h_V}{h} = \frac{24}{78} = 0.308$$

$$q_h = \frac{p{L_c}^2}{8h_0} / h_V = \frac{33.75 \times 290^2}{8 \times 46} \times \frac{1}{24} = 321\ kN/m$$

$$R_T = \frac{q_h h}{8}\, \xi\, (8 - 6\xi + \xi^3)$$

$$= \frac{321 \times 78}{8} \times 0.308 \times (8 - 6 \times 0.308 + 0.308^3)$$

$$= 5,958\ kN$$

$$M_{max.} = \frac{{R_T}^2}{2 \times R_h} = \frac{5,958^2}{2 \times 321}$$

$$= \underline{55,292\ kN \cdot m}$$

($L_C = 590m$)

{AA} *Axial force and stress in the girder*

$$N_{max.} = \frac{(W_d + p)}{8h_0} {L_C}^2 \quad (at\ tower)$$

$L_C = 590m$, $h_0 = 64 + 54/2 = 91\ m$

$W_d = 171.1\ kN/m$, $p = 33.75\ kN/m$

$$N_{max.} = \frac{(171.1 + 33.75)}{8 \times 91} \times 590^2 = 97,951\ kN$$

$$\sigma_{max.} = \frac{N_{max.}}{A_S} = \frac{97,951}{1.088} = 90,028\ kN/m^2 \quad \underline{(= 90.0\ N/mm^2)}$$

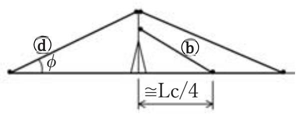
$$N_p \cong P\ \frac{(L_c/2)}{h_T} = 1,125 \times \frac{295}{118} = 2,813\ kN$$

$$\sigma_{NP} = \frac{N_p}{A_s} = \frac{2,813}{1.088} = 2,585\ kN/m^2 \quad (= 2.6\ N/mm^2)$$

$$\overline{\sigma_{max.}} = \sigma_{max.} + \sigma_{NP} = \underline{92.6\ N/mm^2}$$

($L_C = 590m$)

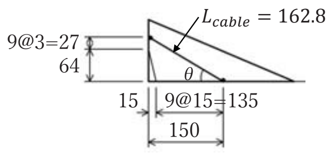
{BB} *Cable tension force*



$$\cos\phi = \frac{285}{308.5} = 0.924$$

(1) *Cable* ⓑ

(m)



$\sin\theta = 0.559$

$L_{CD} = 15$

$$T_d \sin\theta = W_d L_{CD} = 171.1 \times 15.0 = 2,567\ kN$$

$$T_d = 4,591\ kN$$

$\underline{assume \quad \beta = 0.02}$

$$T_L \sin\theta = \left(1.2 \times 33.75 + \frac{1,125 \times 0.02}{2}\right) \times 15$$

$$= 777\ kN$$

$$T_L = 1,390\ kN$$

$$T_d + T_L = 5,981\ kN$$

$$A_C > \frac{5,981 \times 10^3}{640} \times 1.1 = 10,280\ mm^2 \quad (0.0103\ m^4)$$

$$K = \frac{2 \times 10^8 \times 0.0103}{162.8} \times 0.559^2 / 15 = 263.6\ kN/m^2$$

$$\beta = \sqrt[4]{\frac{263.6}{4 \times 2.0 \times 10^8 \times 2.088}} = \underline{0.0199} \quad (\boldsymbol{OK}\ !!)$$

(2) *Cable* ⓓ ← *Anchor cable*

$$\Delta N = \frac{W_d L_C{}^2}{8h_T}\left\{1-4\left(\frac{L_S}{L_C}\right)^2\right\}+\frac{pL_C{}^2}{8h_T}\ (+N_\rho)$$

$$= \frac{171.1\times 590^2}{8\times 118}\cdot\left\{1-4\left(\frac{285}{590}\right)^2\right\}+\frac{33.75\times 590^2}{8\times 118}+2{,}813$$

$$= 4{,}202+12{,}445+2{,}813$$

$$= 19{,}460\ kN^{(*)}$$

$$T_A\cos\phi = \Delta N$$

$$T_A = \frac{\Delta N}{\cos\phi} = 21{,}061\ kN$$

$$A_C > \frac{21{,}061\times 10^3}{640}\times 1.1 = 36{,}199\ mm^2\ \ (0.0362\ m^4)$$

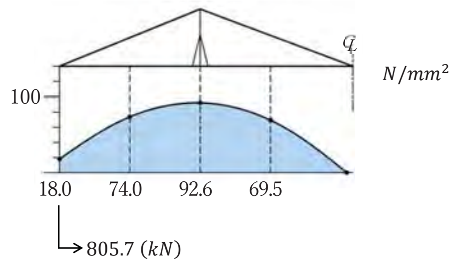
$$No.\ of\ \ \phi 7\ wire\quad > \quad 941$$

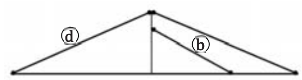
$\{CC\}$ *UP – lift force*

$$R_u = \Delta N\tan\phi = 19{,}460\times\frac{118}{285} = 8{,}057\ kN$$

$$\left(\ ^{(*)}\ \sigma_n/end = \frac{19{,}460}{1.088} = 17{,}886\ kN/m^2 = 17.9\ N/mm^2\ \right)$$

$\{\sigma_n\}$



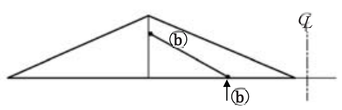


	Ac (mm²)	No. of ϕ7 wire
ⓑ	10,208	268
ⓓ	36,199	941

※ per bridge

$\phi 7\ (A \cong 38.5\ mm^2)$

$\{DD\}$ *Bending moment and stress in the girder*



$$M = \frac{p\pi}{16\beta^2}+\frac{P}{4\beta}$$

$$= \frac{33.75\times\pi}{16\times 0.0199^2}+\frac{1{,}125}{4\times 0.0199}$$

$$= 16{,}725+14{,}133$$

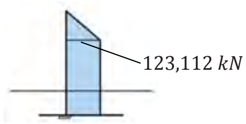
$$= 30{,}858\ kN\cdot m$$

$$\sigma_b = \frac{30{,}858}{2.088}\times 1.5 = 22{,}168\ kN/m^2 = \underline{22.2\ N/mm^2}$$

$\{EE\}$ *Axial force in the tower*

$$N_T = (W_d+p)\,L_C+2P$$

$$= (171.1+33.75)\times 590+2\times 1{,}125 = 123{,}112\ kN$$



(+ *self weight*)

$\{FF\}$ *Max. bending moment in the tower*

$$\xi = \frac{h_V}{h} = \frac{54}{158} = 0.342$$

$$q_h = \frac{pL_c{}^2}{8h_0}/h_V = \frac{33.75\times 590^2}{8\times 91}\times\frac{1}{54} = 299\ \ kN/m$$

$$R_T = \frac{q_h h}{8}\,\xi\,(8-6\xi+\xi^3)$$

$$= \frac{299\times 158}{8}\times 0.342\times(8-6\times 0.342+0.342^3)$$

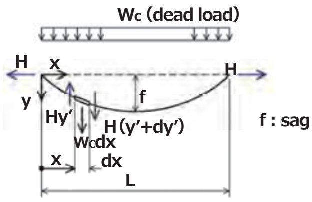
$$= 12{,}093\ kN$$

$$M_{max.} = \frac{R_T{}^2}{2\times R_h} = \frac{12{,}093^2}{2\times 299}$$

$$= \underline{244{,}550\ kN\cdot m}$$

[12-4-1]

Design & Erection of Cables



$$Hy' = W_c dx + H(y'+dy') \qquad (\ )' = \frac{d}{dx}(\ )$$

$$0 = W_c dx + Hdy'$$

$$\Downarrow$$

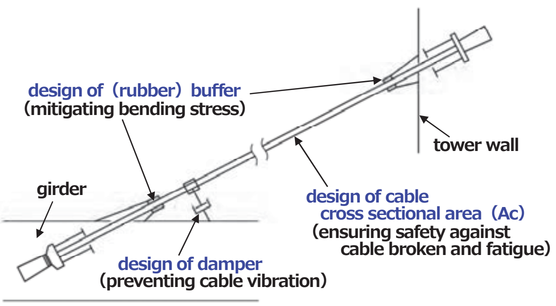
$$Hy'' + W_c = 0$$

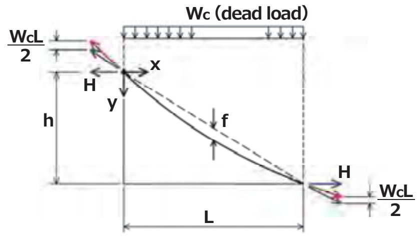
$$\Downarrow \quad (\text{at } x=0,\ L \rightarrow y=0)$$

$$y = \frac{W_c}{2H}x(L-x)$$

$$f = \frac{W_c L^2}{8H}, \quad H = \frac{W_c L^2}{8f}$$

Design of cables

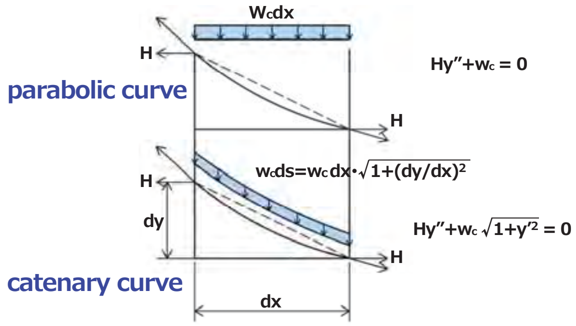


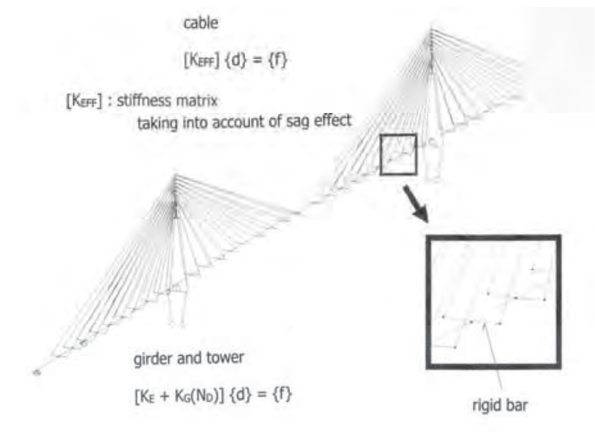


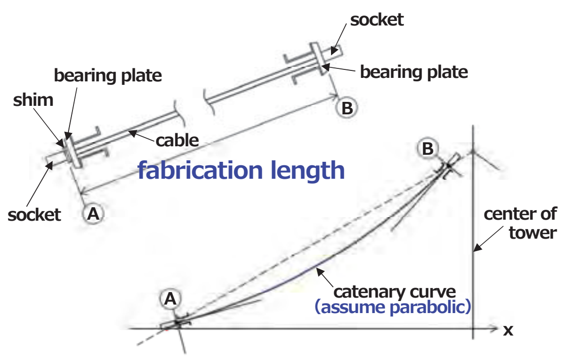
$$y = -\frac{W_c}{2H}x^2 + \left(\frac{h}{L} + \frac{W_c}{2H}L\right)x$$

$$f = \frac{W_c L^2}{8H}$$

Cable curve







$$E_{EFF} = \frac{E_0}{1 + \frac{\gamma^2 L^2 E_0}{12\sigma^3}}$$

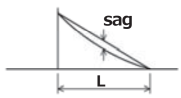
E_{EFF} : elastic modulus of cable with sag (equivalent Young's modulus)

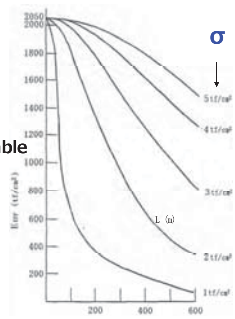
E_0 : Young's modulus of straight cable

γ : weight per unit length

L : horizontal projection of cable

σ : tensile stress in cable





$$E_{EFF} = \frac{E_0}{1+\dfrac{\gamma^2 L^2}{12\sigma_m^3}\cdot\dfrac{(1+\mu)^4}{16\mu^2}\cdot E_0}$$

$$\sigma_m = \frac{\sigma_0+\sigma_u}{2}\ ,\ \mu = \frac{\sigma_0}{\sigma_u}$$

σ_0 : max. stress
σ_u : min. stress

$$E_{EFF} = \frac{E_0}{1+\dfrac{(\gamma L)^2\,(T_i+T_f)A_c E_0}{24\,T_i^2 T_f^2}} \qquad \text{(by ASCE)}$$

A_c : cross-sectional area of cable
T_i : min. tension
T_f : max. tension

---

by JHBS

1) Strength check（against breaking）

$\sigma_D + \sigma_{L+i} < \sigma_a \quad (\sigma_a = \sigma_B\ /\ (\gamma=2.5))$

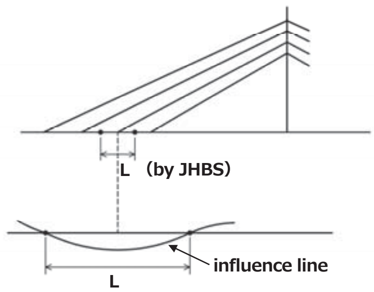
σ_D ($=T_D/A_C$) : stress due to dead load
σ_{L+i} ($=T_{L+i}/A_C$) : stress due to live load (including impact)
T_D, T_{L+i} : tension in cables
σ_B : breaking stress of cables

2) Fatigue strength check

γ (safety factor) is set to be
equal to or greater than 2.5,
safety against fatigue is ensured (by JHBS)

Safety check and cross-sectional area（A_c）

Impact $i = \dfrac{20}{50+L}$



In many cases in Japan, i = 0.2 has been used

---

by Honshu-Shikoku Bridge Authority

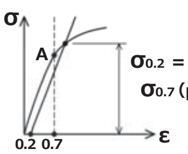
1) Strength check (against breaking)

$\sigma_D + \sigma_{L+i} < \sigma_a \quad (\sigma_a = \sigma_B / 2.5)$

2) Strength check (against yielding)

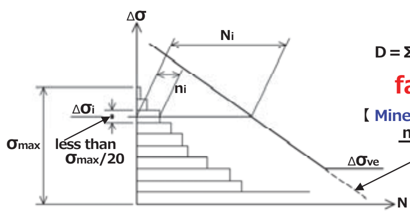
$\sigma_D + \sigma_{L+i} < \sigma_a \quad (\sigma_a = \sigma_{cy} / 2.0)$

σ_{cy} : yield stress



$\sigma_{0.2} = 1.07\sigma_{0.7}$ ($=\sigma_{cy}$)
$\sigma_{0.7}$ (pt. A) has been used as σ_{cy} in Japan

Linear cumulative damage rule



$D = \Sigma\left(\frac{n_i}{N_i}\right) = 1.0$

fatigue damage

【Miner's law (or) modified Miner's law】

n_i : (cycle of $\Delta\sigma_i$) in obtained by rainflow counting method

---

3) Stress check including secondary stress

$\sigma_D + \sigma_{L+i} + \sigma_b < \sigma_a \quad (\sigma_a = \sigma_B / 2.0)$

σ_b : secondary (bending) stress

4) Fatigue strength check

Check stress（$\Delta\sigma$*）is by rainflow method

Check method is Minor's law

$$\Sigma\frac{n_i}{N_i} < 1.0$$

* $\Delta\sigma$ includes bending stress

Fatigue check by DIN1073 (1974)

$\sigma_{fa} = 628$ (MPa) $\qquad K_f \geqq 0.681$

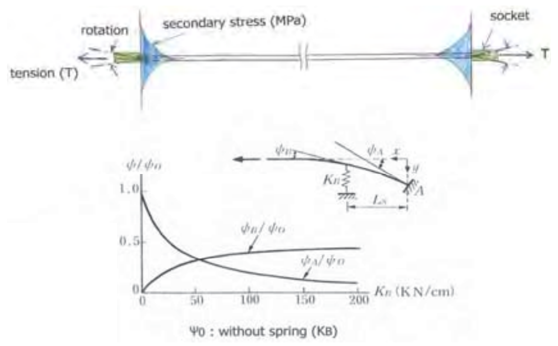
$= \dfrac{245.2}{1-0.895K_f} \qquad K_f < 0.681$

$$K_f = \frac{\sigma_D+0.5\sigma_{L,min}}{\sigma_D+0.5\sigma_{L,max}}$$

$$\sigma_f = \sigma_D+0.5\sigma_{L,max}$$

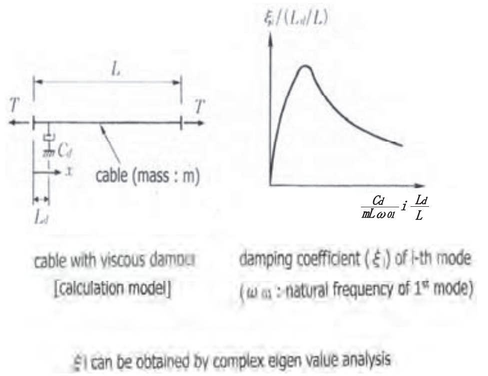
σ_{fa} : allowable stress
σ_f : stress to be checked
σ_D : stress due to dead load
$\sigma_{L,min.}$: min. stress due to live load
$\sigma_{L,max.}$: max. stress due to live load

Secondary stress (bending stress)



Type of cable

Suppression of cable vibration by damper



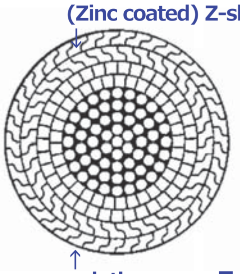
[Top priority]

Protecting water penetration

High fatigue strength at anchor system (socket)

Locked coil rope

(Zinc coated) Z-shape wire

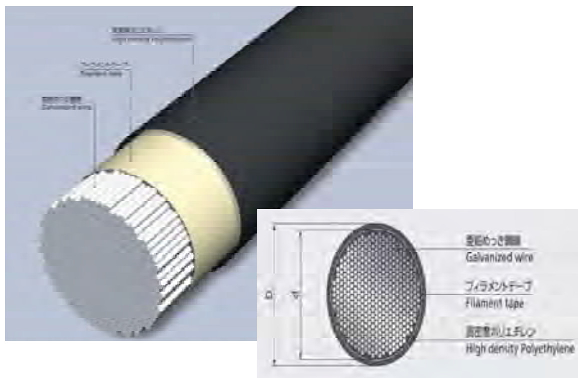


painting

$E_{LCR} < E_{parallel}$

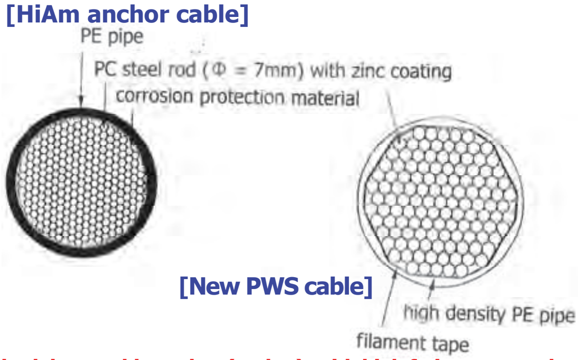
$\sigma_{B,\,LCR} < \sigma_{B,\,parallel}$

Cable section [New PWS]



Cables (parallel wire)

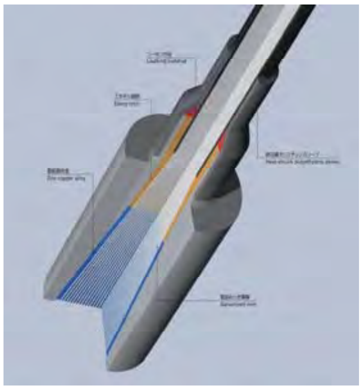
[HiAm anchor cable]



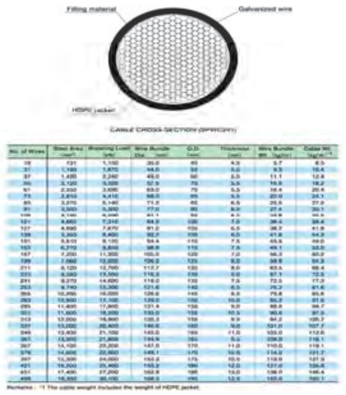
[New PWS cable]

both have cable anchor (socket) with high fatigue strength

Cable socket (cable anchor)



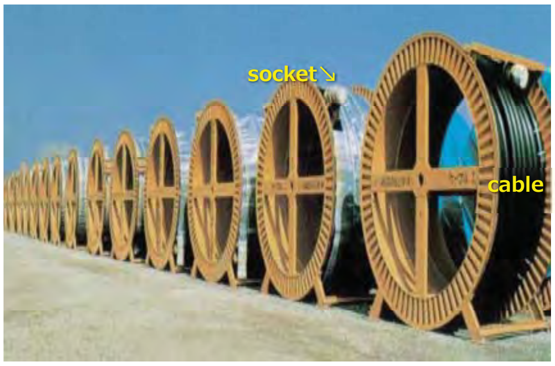
Physical property [HiAm anchor]



Erection of cables

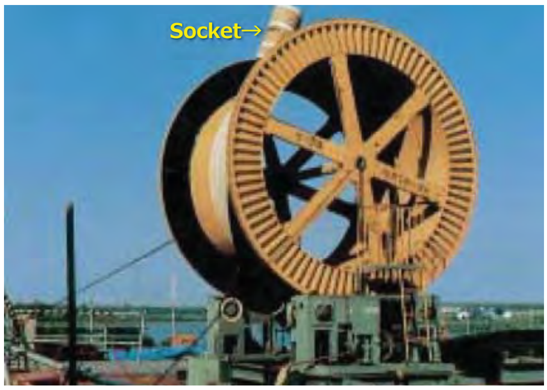
Strand and section (SEE cable)

Reeled cable



Reeled cable (at factory)

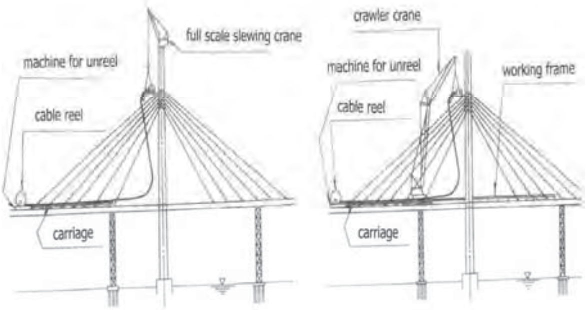
Set to unreeler

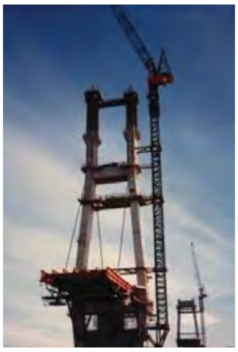


(transport to site)

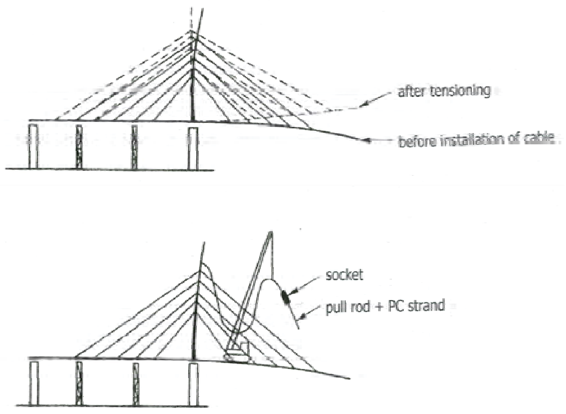
Reeled cable set to unreeler machine

Cable expansion

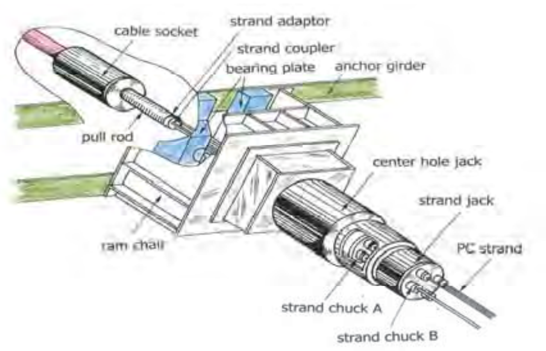

full scale slewing crane
machine for unreel
cable reel
carriage
crawler crane
machine for unreel
working frame
cable reel
carriage



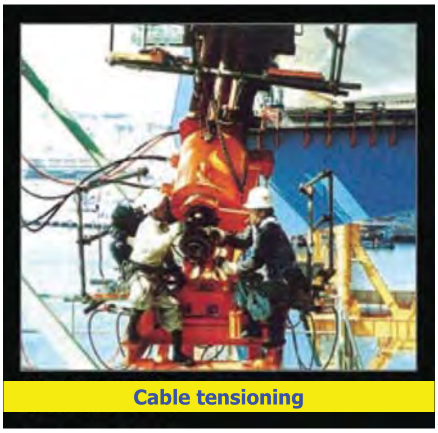

crane
Lift up of cable socket to tower anchorage



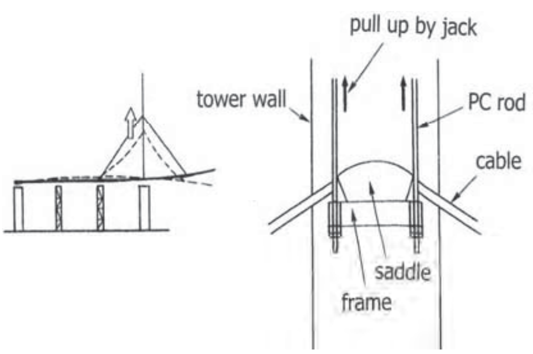
Tension introduction by center hole jack

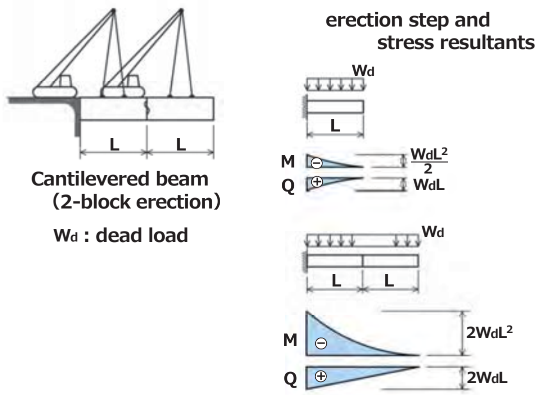


Cable led to girder anchor



Tension introduction





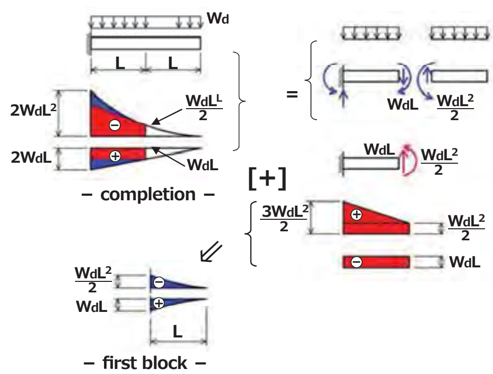
Safety check at each erection stage and (center-hole) jack volume (size) for cable tensioning

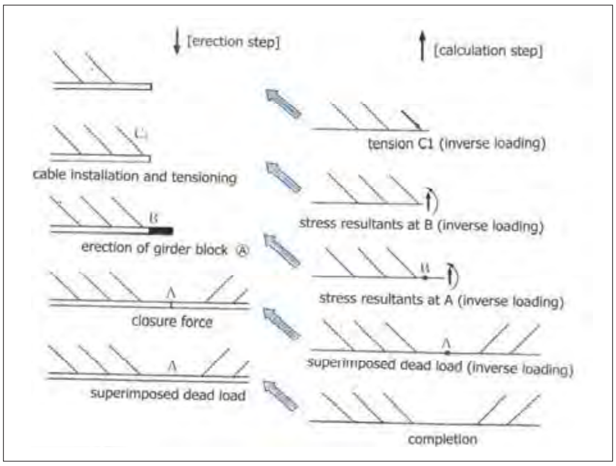
⇓

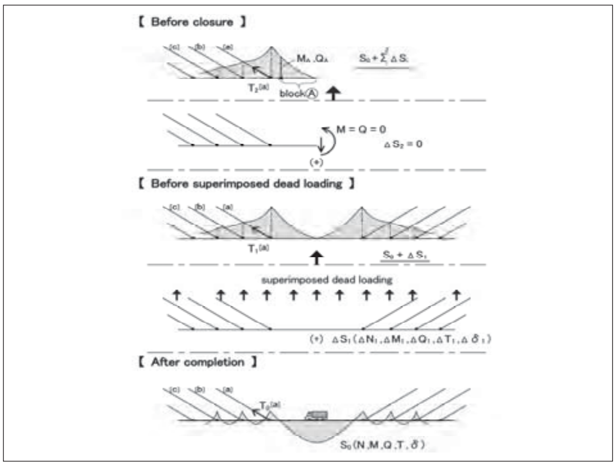
inverse analysis (explain at next PPT)

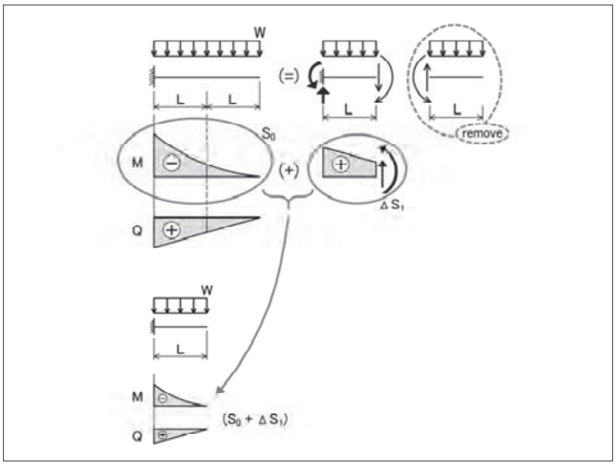
using geometrical non-linear analysis

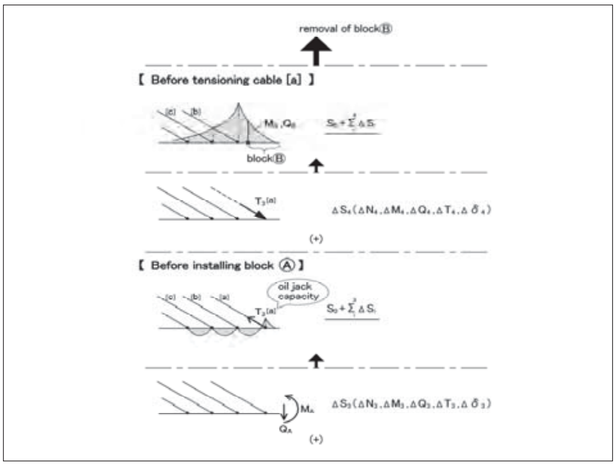
(since displacement under erection is large)











$\boxed{S_0 + \sum_i^n \Delta S_i}$

design (computed) value at each erection stage

compare and check, and adjust ※

measured value

(girder level and tension in cables)
[by vibration method and sometimes, load cell]

※mainly, change (adjust) the tension using shim plates
[insert (or) remove]

S_0 ($N_0, M_0, Q_0, T_0, \delta_0$) : values at completion
ΔS_i ($\Delta N_i, \Delta M_i, \Delta Q_i, \Delta T_i, \Delta \delta_i$) : changed values

N : axial force , M : bending moment
Q : shear force , T : tension in cables
δ : deflection

Tension in cable by vibration method

$\Gamma < 3$ (use 2nd mode)
sag is large

$$\Gamma = \frac{w}{g}(f_2 L)^2 \left(1.02 - 6.26\frac{C}{f_2}\right) \quad (\xi \geqq 10)$$

$\Gamma \geqq 3$ (use 1st mode)
sag is small

$$\Gamma = \frac{4w}{g}(f_1 L)^2 \left\{0.857 - 10.89\left(\frac{C}{f_1}\right)^2\right\} \quad (3 \leqq \xi \leqq 17)$$

$$\Gamma = \frac{4w}{g}(f_1 L)^2 \left\{1 - 2.2\frac{C}{f_1} - 2\left(\frac{C}{f_1}\right)^2\right\} \quad (17 \leqq \xi)$$

$$\left[\Gamma = \frac{4w}{g}(f_1 L)^2 \quad (100 < \xi)\right]$$

f_1, f_2 : measured 1st and 2nd frequency

At each stage,

- configuration
  (girder level and tower inclination)
- tension in cables

are measured and checked

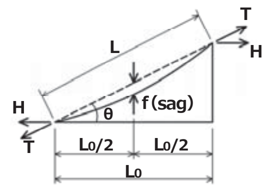
$$C = \sqrt{\frac{EIg}{wL^4}}$$

$$\xi = \sqrt{\frac{T}{EI}} \cdot L$$

$$\Gamma = \sqrt{\frac{wL}{128EA(\delta)^3 \cos^5\theta}} \cdot \left(\frac{0.31\xi + 0.5}{0.31\xi - 0.5}\right)$$

$$\delta = f/L_0$$

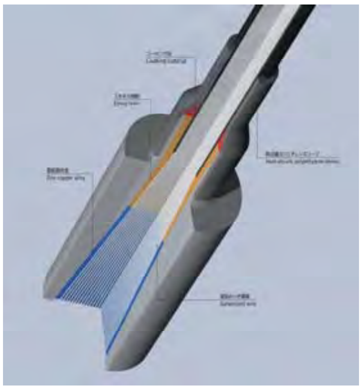
EA : axial rigidity
EI : flexural rigidity
w : cable unit weight
g (= 9.8 m/sec²)



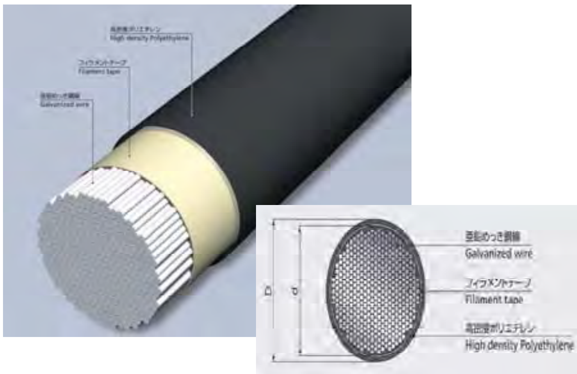
Appendix - 1

Reference from
NEW-PWS cable brochure

Cable socket (cable anchor)



Cable section



Reeled cable (at factory)

(transport to site)

Reeled cable set to unreeler machine

Lift up of cable socket to tower anchorage

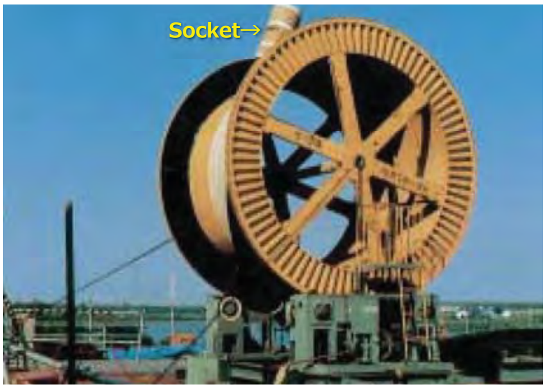
Cable expansion

Cable tensioning

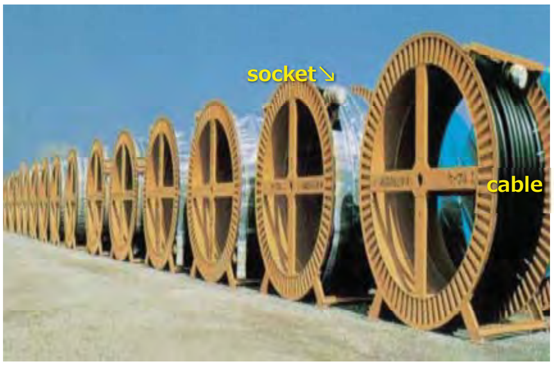
Appendix - 2

Reference from
HiAm & DINA cable brochure

Set to unreeler

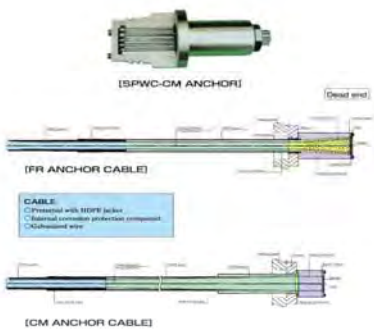


Reeled cable

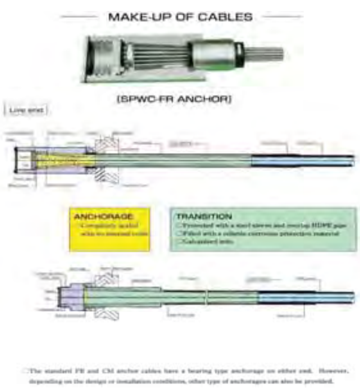


Cable led to girder anchor

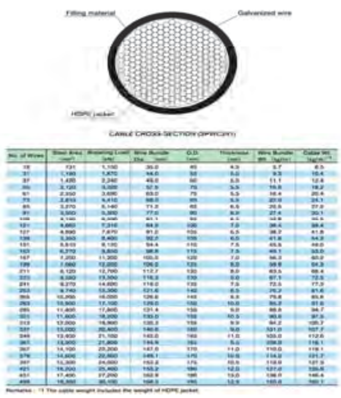
Cable anchor



Cable anchor



Physical property

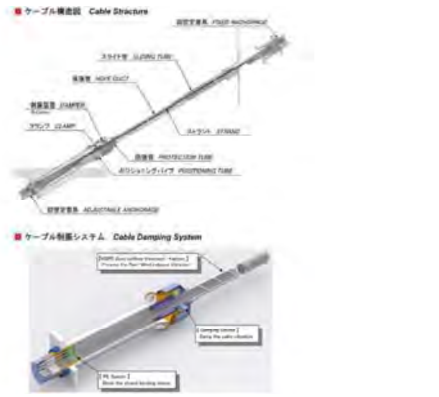


Appendix - 3

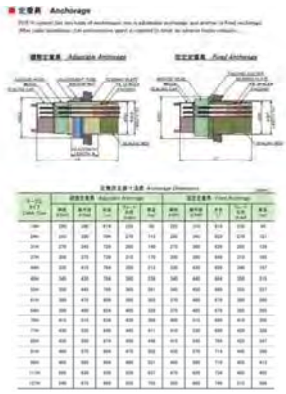
Reference from
SEEE cable brochure

Strand and section

Cable structure and damping system



Anchorage

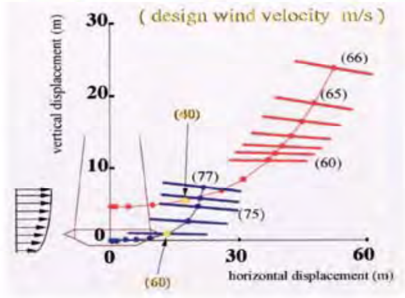


[12-4-2]

Design of Girder

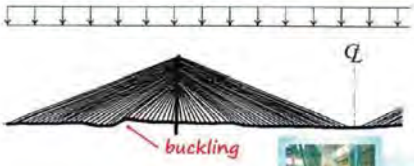
Static instability -(2)-

Divergence under displacement-dependent wind loading



Static instability –(1)-

・Elasto-plastic buckling of main girder



Side view of buckling of the girder (Tatara Br.)

Dynamic instability

flutter

Identification by analysis (based on non-linear 3D FEA)

-buckling-

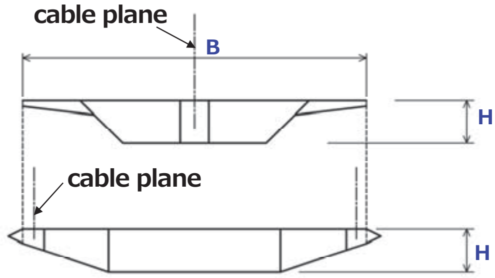
Elasto-plastic finite displacement analysis

-divergence-

Non-linear elastic analysis under displacement-dependent wind load

-flutter-

Complex eigenvalue analysis using modal coordinate



B(width) : from traffic (volume, flow)

H(height) : from maintainability, fabrication and preventing in-plane global buckling

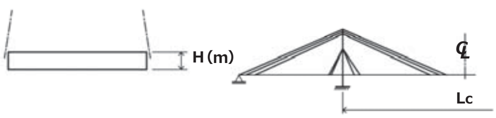
Priority

At the design (after basic design), first step to do,

Check performance (safety) of the girder and tower under wind load

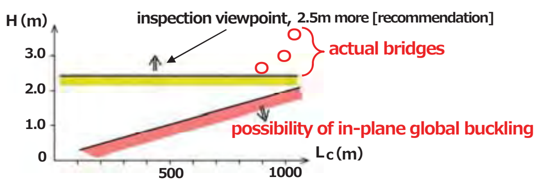
Check performance under (huge) earthquake

in case of box-girder and multi-cable type



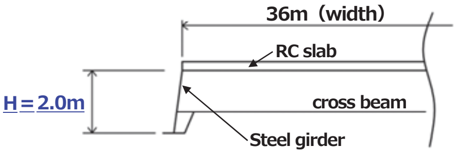
$H > L_c / (500 \sim 600)$ (← for 4-lane bridge)

↑no possibility of in-plane buckling



in case of open-section

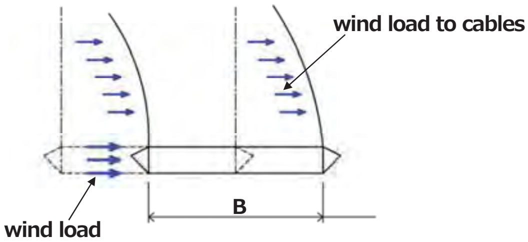
ex. Alex Fraser bridge (span=485m)



- Composite 2-I-girder -

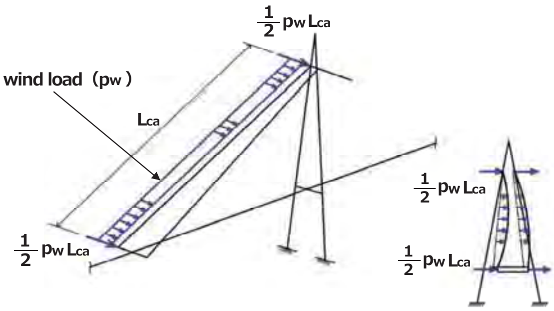
see practice and judge!!

under lateral (wind) loading



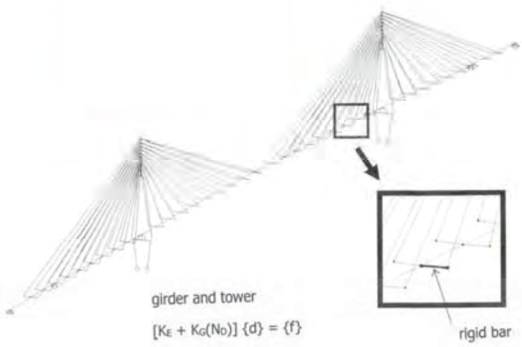
$L_C / B < 40$ has been recommended

Wind Load on Cables



L_{ca} = cable length

Example of frame model



$$p = \frac{1}{2}\rho V_d^2 C_d G$$

p (N/m^2) : wind load per unit area

ρ ($= 1.23\ kg/m^3$) : air density

V_d (m/sec) : design wind velocity

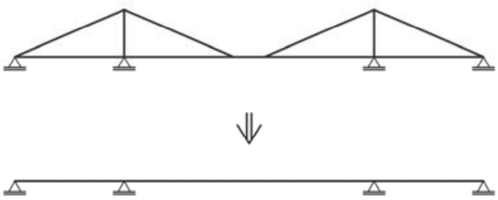
C_d: drag coefficient

G : gust factor

$p_w (N/m) = pA_n$

A_n: projection area (m^2/m)

Impact $i = \dfrac{20}{50 + L}$

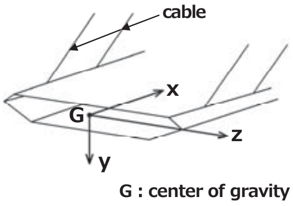


the same definition of (L) as continuous beam

Stress (σ, τ)

$$\sigma = \frac{N}{A} + \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

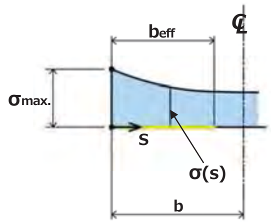
$$= \sigma_n + \sigma_{bz} + \sigma_{by}$$



$$\tau = \left(\frac{Q}{A_w}\right)^{*} + \tau_s$$

$= \tau_b + \tau_s$ (shear stress due to torsion)

∗ shear flow theory is applied for more exact evaluation



$$\sigma_{max.}\, b_{eff} = \int_0^b \sigma(s)\, ds$$

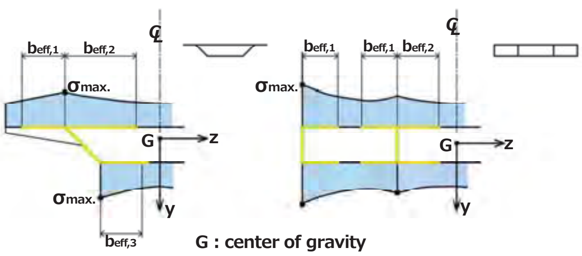
$$\sigma_{max.} = \frac{\int_0^b \sigma(s)\, ds}{b_{eff}}$$

by JHBS, b_{eff} is defined by

$b_{eff} = f\,(b, L_{eq.})$

L_{eq} : equivalent span length

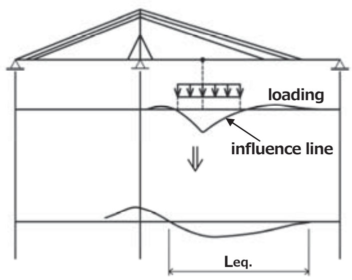
shear lag



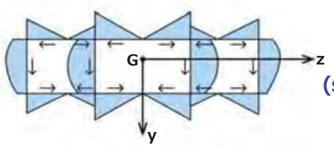
(I_z and position of centroid G) are calculated using effective width (b_{eff}) to evaluate ($\sigma_{max.}$)

Equivalent span length ($L_{eq.}$)

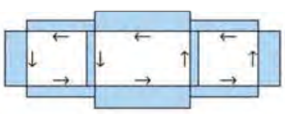
$b_{eff} = f\,(b, L_{eq.})$ by JHBS



example of (τ_b, τ_s) distribution



τ_b
(shear due to flexure by shear flow theory)



τ_s
(shear due to pure torsion)

[1] Against global buckling

$$\frac{\sigma_c}{\sigma_{caz}} + \frac{\sigma_{bz}}{\sigma_{bagz}} + \frac{\sigma_{by}}{\sigma_{bao}} < 1.0$$

σ_c : axial compressive stress

σ_{caz} : allowable column buckling stress

$\sigma_{bagz}, \sigma_{bao}$ $(= \sigma_y / 1.7)$: allowable bending stress

↑ since no lateral torsional buckling will produce

[2] Against local (plate) buckling

$\sigma_c + \sigma_{bz} + \sigma_{by} < \sigma_{cal}$

σ_{cal} : allowable plate buckling stress

($\sigma_c, \sigma_{bz}, \sigma_{by}$) are calculated based on linearized finite displacement analysis

Safety check of the girder

[1] Global buckling strength

Side view of buckling of the girder
(Tatara Br.)

〔 case－2 〕 E_f-method(inelastic eigenvalue analysis)

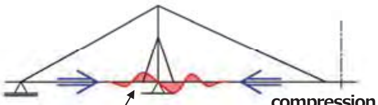
1) elastic eigenvalue analysis

$$\left| K_E\,(E_i\,,I_i) + \kappa K_G\;(N_i) \right| = 0$$

$$L_{e\,,\,i} = \pi \sqrt{E_i\,I_i\,/\,(\kappa N_i)}$$

[K_E] : elastic stiffness matrix
[K_G] : geometric matrix
$L_{e,i}$: buckling length of elasticity of element (i)
E_i : young 's modulus of elasticity of element (i)
I_i : geometrical moment of inertia of element (i)
κ : min. eigenvalue
N_i : compressive axial force of element (i)

calculation of (σ_{caz})



〔 case－1 〕

$\sigma_{cr} = \sigma_y \longrightarrow \sigma_{caz} = \sigma_y\,/\,1.7$

↑ no buckling is assumed

the following check has to be made

a) elastic geometrical non-linear analysis

(or) b) elasto-plastic & geometrical non-linear analysis

{↑Ultimate strength analysis}

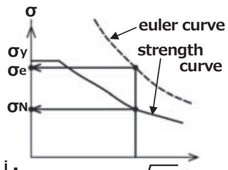
2) modify $E_i \longrightarrow E_{f\,i}$

$$E_{f,i} = \frac{\sigma_{N\,,\,i}}{\sigma_{e\,,\,i}}\,E_i$$

$\sigma_{e,i}$: buckling stress of element (i)
$\sigma_{N,i}$: strength of element (i)

$$\left| K_E\,(E_{f,i}\,,I_i) + \kappa K_G\;(N_i) \right| = 0$$

$$L_{e\,,\,i} = \pi \sqrt{E_{f,i}\,I_i\,/\,(\kappa N_i)}$$



$\lambda = \frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}}\cdot\frac{L_e}{r}$

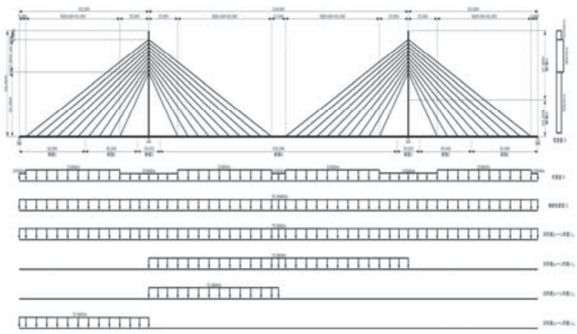
$(r = \sqrt{I\,/A}\,)$

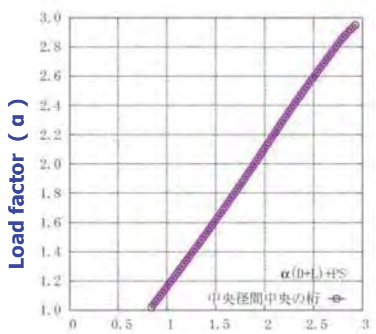
3) until converged value of $L_{e,i}$.
calculation is continued

4) $\sigma_{caz} = \sigma_{cr}(L_{e,i})/1.7$

Load cases of Ultimate Strength Analysis

Live Load Cases
L1: Loading on the all spans
L2: Loading on the center span
L3: Loading on the left half of center span
L4: loading on the left side span





Load – deflection at the center of the girder

Analysis Case L1 [α(D+L)+Ps]

Girder yielding 1.79(D+L)+PS

Cable yielding 2.61(D+L)+PS

Tower yielding 2.63(D+L)+PS

Maximum Load 2.95(D+L)+PS

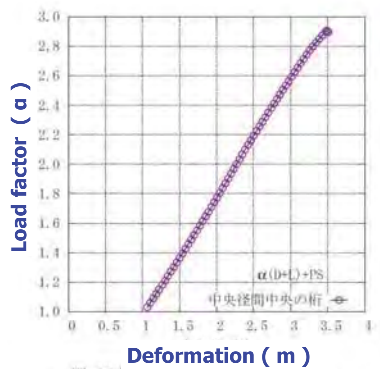
Analysis Case L2 [α(D+L)+Ps]

Girder yielding 2.03(D+L)+PS

Tower yielding 2.38(D+L)+PS

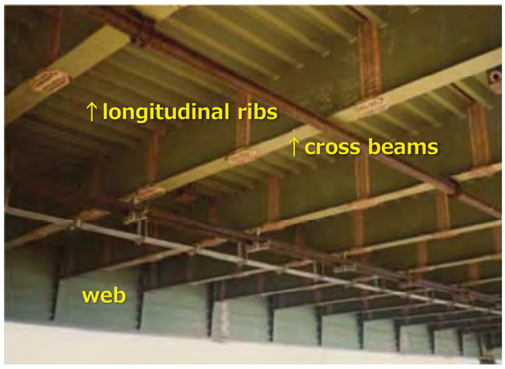
Cable yielding 2.68(D+L)+PS

Maximum Load 2.91(D+L)+PS



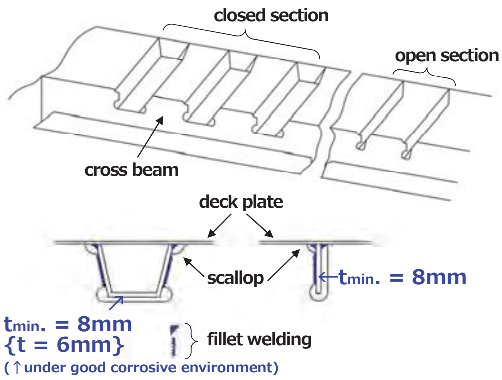
Load – Deflection at the center of the girder

Steel deck



[2] Local buckling strength

Steel deck plate

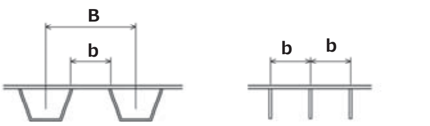


Deck plate thickness and rib arrangement

t = 0.037b* mm (B-live load)

t = 0.035b* mm (A-live load)

(t ≧ 12 mm)

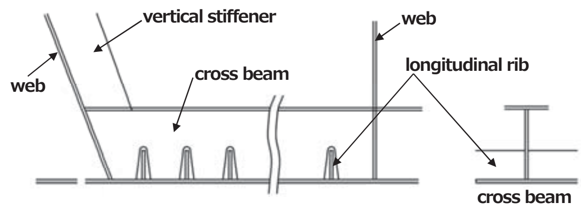


B = 620 ～ 660 mm
b = 300 ～ 340 mm

b = 300 ～ 340 mm

*defined from viewpoint of no damage to pavement
[mostly no possibility of local buckling]

Design of lower flange



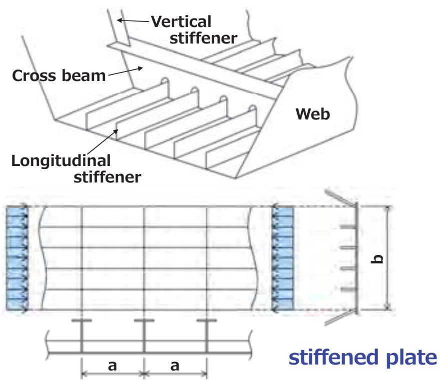
Recent topics (due to fatigue problem)

Mostly , 12 mm thickness has been used so far.

Due to severe fatigue damage,

16 mm thickness is recommended

calculation of (σ_{cal})



stiffened plate

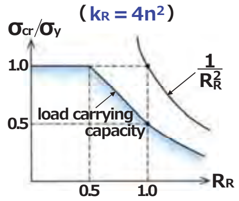
Global buckling of stiffened plates | **Buckling of longitudinal ribs**

Local buckling of plates

Ultimate strength of stiffened plate by JHBS

$$R_R = \sqrt{\frac{\sigma_y}{\sigma_E}} = \sqrt{\frac{\sigma_y}{E}\cdot\frac{12(1-\nu^2)}{\pi^2 k_R}}\cdot\left(\frac{b}{t}\right)$$

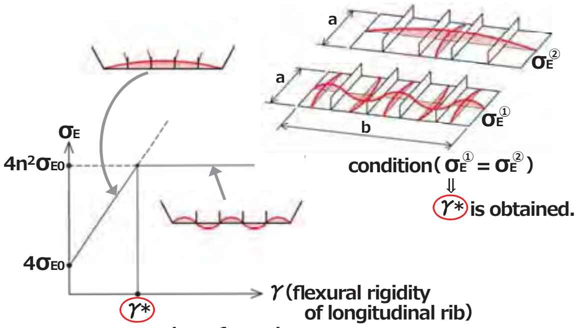
$(k_R = 4n^2)$



$\frac{\sigma_{cr}}{\sigma_y} = 1.0$	$R_R \leqq 0.5$
$\frac{\sigma_{cr}}{\sigma_y} = 1.5 - R_R$	$0.5 < R_R \leqq 1.0$
$\frac{\sigma_{cr}}{\sigma_y} = \frac{0.5}{R_R^2}$	$1.0 < R_R$

$\sigma_{cal} = \sigma_{cr}/1.7$

{Local elastic buckling stress} = {Global elastic buckling stress}

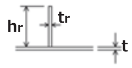


n : number of panel
n-1 : number of longitudinal ribs

Design of longitudinal ribs (I_ℓ)

$$\gamma_\ell = \frac{I_\ell}{\frac{bt^3}{12(1-\nu^2)}} \cong \frac{I_\ell}{\frac{bt^3}{11}} \longrightarrow I_\ell \geqq \frac{bt^3}{11}\cdot\gamma_{\ell,req.}$$

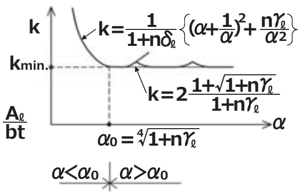
$$A_\ell = \frac{bt}{10n} \qquad A_\ell = h_r t_r \qquad I_\ell = \frac{h_r{}^3 t_r}{3}$$



From condition,
$\sigma_E^{①}$ (buckling stress of plate between ribs) = $\sigma_E^{②}$ (buckling stress of stiffened plate)

$k = 4n^2$

$\delta_\ell = \frac{A_\ell}{bt}$



1) $\alpha \leqq \alpha_0$ (&) $I_t \geqq \frac{bt^3}{11}\cdot\frac{1+n\gamma_{\ell,req.}}{4\alpha^3}$

$$\gamma_{\ell,req.} = 4\alpha^2 n\left(\frac{t_0}{t}\right)^2(1+n\delta_\ell) - \frac{(\alpha^2+1)^2}{n} \qquad (t \geqq t_0) \quad (R_R < 0.5)$$

$$= 4\alpha^2 n(1+n\delta_\ell) - \frac{(\alpha^2+1)}{n} \qquad (t < t_0) \quad (R_R > 0.5)$$

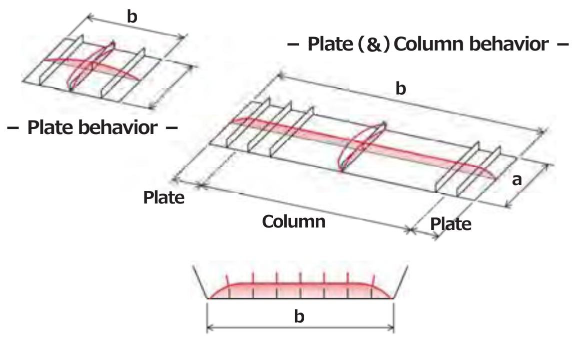
(t_0 is the thickness when $R_R = 0.5$)

2) the others $[(\alpha > \alpha_0), (\alpha \leqq \alpha_0$ & $I_t < \frac{bt^3}{11}\cdot\frac{1+n\gamma_{\ell,reg.}}{4\alpha^3})]$

$$\gamma_{\ell,reg.} = \frac{1}{n}\left[\left\{2n^2\left(\frac{t_0}{t}\right)(1+n\delta_\ell)-1\right\}^2-1\right] \qquad (t \geqq t_0) \quad (R_R < 0.5)$$

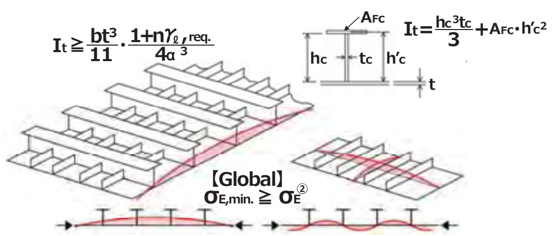
$$= \frac{1}{n}\left[\left\{2n^2(1+n\delta_\ell)-1\right\}^2-1\right] \qquad (t > t_0) \quad (R_R > 0.5)$$

Buckling strength of stiffened plate with small aspect ratio ($\alpha = a/b$)



Design of cross beam (I_t)

$$I_t \geqq \frac{bt^3}{11}\cdot\frac{1+n\gamma_{\ell,req.}}{4\alpha^3} \qquad I_t = \frac{h_c{}^3 t_c}{3} + A_{Fc}\cdot h'_c{}^2$$

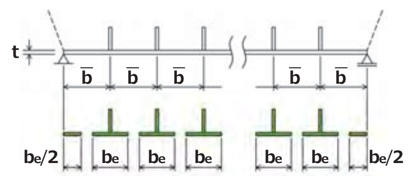


$$\alpha \leqq \alpha_0 \qquad 2\frac{1+\sqrt{(1+n\gamma_\ell)(1+\gamma_t/\alpha)}}{1+n\gamma_\ell} \geqq \frac{1}{1+n\gamma_\ell}\left\{\left(\alpha+\frac{1}{\alpha}\right)^2+\frac{n\gamma_\ell}{\alpha^2}\right\}$$

$$\gamma_{t,req.} = \frac{1+n\gamma_{\ell,req.}}{4\alpha^3} - \underline{\frac{\alpha}{2} + \frac{5}{4(1+n\delta_\ell)}}$$ neglected

$$\alpha > \alpha_0 \qquad 2\frac{1+\sqrt{(1+n\gamma_\ell)(1+\gamma_t/\alpha)}}{1+n\gamma_\ell} \geqq 2\frac{1+\sqrt{(1+n\delta_\ell)}}{1+n\delta_\ell} \Rightarrow \gamma_t = 0$$

Column approach



$$N_{cr} = \left\{\left(\frac{\sigma_{cr}}{\sigma_y}\right)_c\cdot n\cdot A_T + \left(\frac{\sigma_{cr}}{\sigma_y}\right)_p\cdot b_e\cdot t\right\} \qquad (3)$$

N_{cr} : load carrying capacity of stiffened plate
$(\sigma_{cr})_c$: load carrying capacity of column
n : number of rib
A_T : cross-sectional area of column with T-section
$(\sigma_{cr})_p$: load carrying capacity of plate

Evaluation of strength

$\underline{(\sigma_{cr})_c}$

$$\left(\frac{\sigma_{cr}}{\sigma_y}\right)_c = 1.0 \qquad (\bar{\lambda} \le 0.2)$$
$$= 1.109 - 0.545\bar{\lambda} \qquad (0.2 < \bar{\lambda} \le 1.0)$$
$$= 1.0/(0.773 + \bar{\lambda}^2) \qquad (1.0 < \bar{\lambda} \qquad)$$

$$\bar{\lambda} = \frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}}\left(\frac{a}{r}\right) \qquad r = \sqrt{\frac{I_T}{A_T}}$$

I_T : geometrical moment of inertia of T-section

a : distance of cross beams

$$\frac{b_e}{b} = 0.702R_e^3 - 1.640R_e^2 + 0.654R_e + 0.926$$

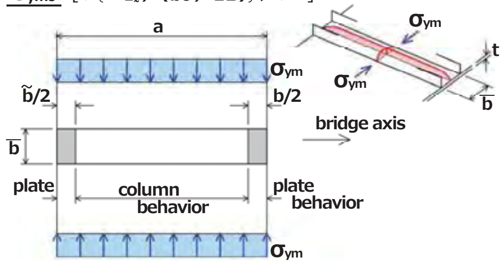
$$R_e = 0.526\frac{b}{t}\sqrt{\frac{\sigma_{cr}}{E}}$$

First, σ_{cr} is assumed and repeat calculation until converged σ_{cr} is obtained

$\underline{(\sigma_{cr})_P}$: load carrying capacity of plate with width (b_e), and simply supported at 4-side.

Strength of plate under σ_{ym} only (σ_{ymo})

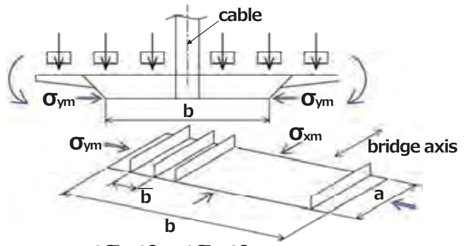
$\underline{\sigma_{ymo}}$ $[\gamma\{=I_\ell/(bt^3/11)\} > \gamma^*]$



$$\sigma_{ymo} = \frac{\sigma_{ym} + 0.9\sigma_{ym}(\alpha - 1)}{\alpha}$$

σ_{ymo} : strength under σ_{ym} only

Check of biaxial compression



$$\left(\frac{\sigma_{xm}}{\sigma_{xmo}}\right)^2 + \left(\frac{\sigma_{ym}}{\sigma_{ymo}}\right)^2 < 1.0$$

↑proposed by Kitada (1988)

σ_{xmo} (= strength under σ_{xm} only) is estimated by eq. (3)

$$\frac{\sigma_{ymc}}{\sigma_y} = 1.0 \qquad (\bar{\lambda} \le 0.2)$$
$$= 1.109 - 0.545\bar{\lambda} \qquad (0.2 < \bar{\lambda} \le 1.0)$$
$$= 1.0/(0.773 + \bar{\lambda}^2) \qquad (1.0 < \bar{\lambda} \qquad)$$

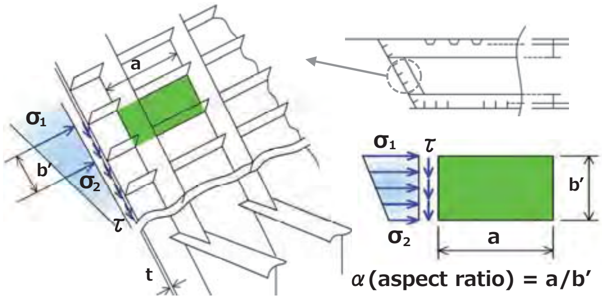
$$\bar{\lambda} = \frac{\sqrt{12}}{\pi}\frac{b}{t}\sqrt{\frac{\sigma_y}{E}}$$

$$\frac{\sigma_{ym}}{\sigma_y} = 0.542R^3 - 1.249R^2 + 0.412R + 0.968 \quad (0.3 \le R \le 1.3)$$

↑proposed by Komatsu (1978)

$$R = \frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}\cdot\frac{12(1-\nu^2)}{\pi^2 k}}\cdot\left(\frac{b}{t}\right) \qquad (k = 4.0)$$

Design of web



$$\Psi = \sigma_2/\sigma_1$$
$$\eta = \tau/\sigma_1$$

$$\sigma_{vKi} = \frac{\sqrt{\sigma_1^2 + 3\tau^2}}{\frac{1+\Psi}{4}\cdot\frac{\sigma_1}{\sigma_{ocr}} + \sqrt{\left(\frac{3-\Psi}{4}\cdot\frac{\sigma_1}{\sigma_{ocr}}\right)^2 + \left(\frac{\tau}{\tau_{ocr}}\right)^2}}$$

$$\sigma_{ocr} = K\,\sigma_E$$
$$\tau_{ocr} = K_\tau \cdot \sigma_E$$
$$\sigma_E = \frac{\pi^2 E}{12(1-\nu^2)}\cdot\left(\frac{t}{b'}\right)^2$$

K, K_τ : buckling coefficient

Safety check against buckling

$$\sqrt{\sigma_1^2 + 3\tau^2} < \frac{\sigma_{vK}}{\nu_{B,req.}}$$

$$\nu_{B,req.} = 1.25 + (0.3 + 0.15\Psi)^{-4.3\eta}$$
$$= 1.55 + 0.15\Psi \qquad (\eta = 0)$$

σ_{vK}/σ_y	$= 1.0$	$R < 0.5$
σ_{vK}/σ_y	$= 1.5 - R$	$0.5 \le R < 1.0$
σ_{vK}/σ_y	$= 0.5/R^2$	$1.0 \le R$

$$R = \sqrt{\frac{\sigma_y}{\sigma_{vKi}}}$$

σ_y : yield stress

[3] Global & local coupled buckling strength

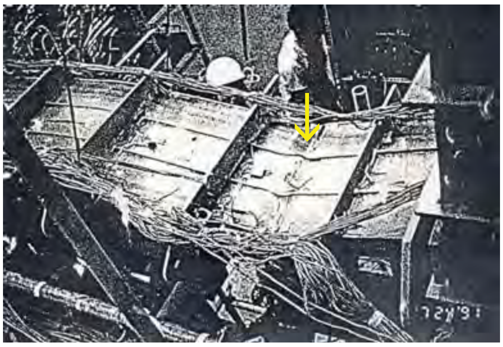
$$\sigma_a = \overset{*}{\sigma}_{caz} \times (\overset{**}{\sigma}_{cal} / [\sigma_y / 1.7])$$

σ_{caz} (in[1])→ σ_a (allowable coupled buckling stress)
[reduced]

* σ_{caz} : allowable global buckling stress

** σ_{cal} : allowable local buckling stress

global-local coupled buckling



Design of anchor girder

[4] check of combined stresses (σ, τ)

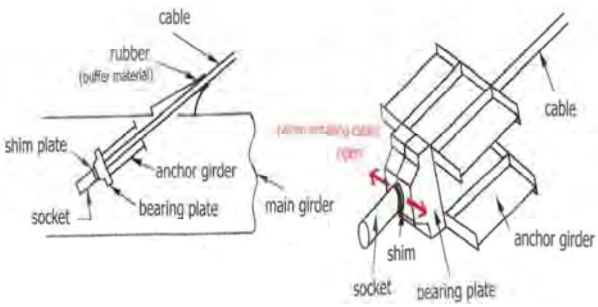
$$\sigma_e = \sqrt{\sigma^2 + 3\tau^2} < 1.1\sigma_a$$

$$\sigma_e = \sqrt{\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau^2} < 1.1\sigma_a$$

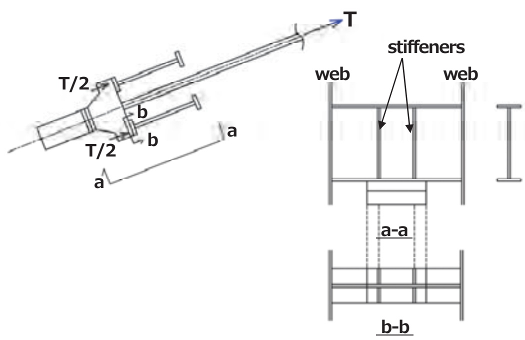
σ_e : equivalent stress

$\sigma_a = \sigma_y / (\gamma = 1.7)$

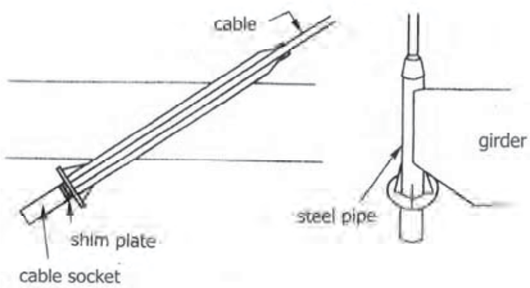
Anchor girder

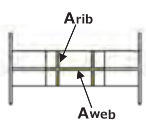


Design of anchor girder



Pipe anchor





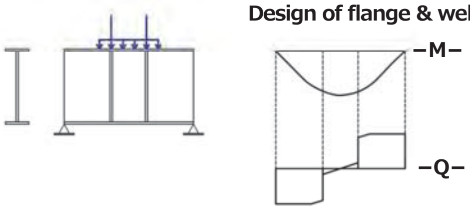
Design of stiffeners

$$\sigma = \frac{T/2}{A} < \sigma_a$$

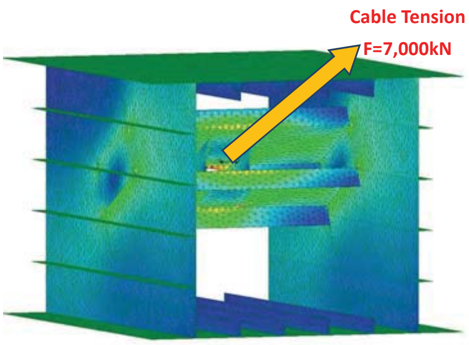
$(\sigma_a = \sigma_y / 1.7)$

$A = 4 \cdot A_{rib} + A_{web}$

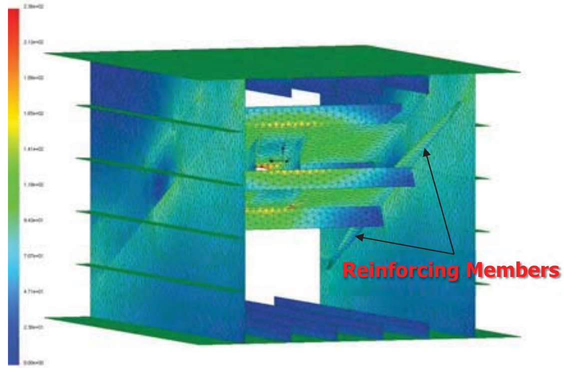
Design of flange & web



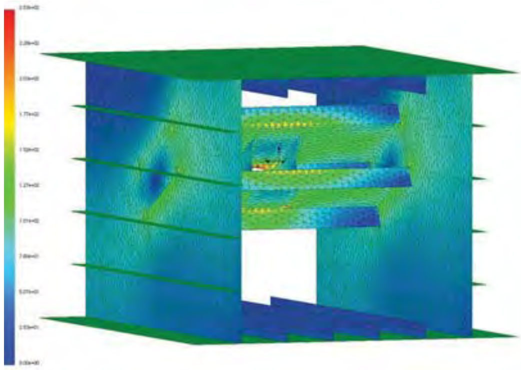
FE Analysis of the Cable Anchoring Section



The Stress State of the Anchoring Section
The Case of adding a Reinforcing Members



The Stress State of the Anchoring Section
Primary Structure



[12-4-3]

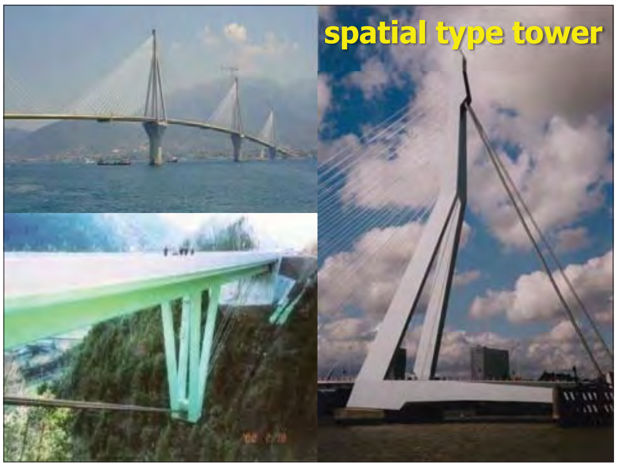
Design of Tower

"A" or "Inverted-Y" shaped Tower

"H" shaped tower

Single tower

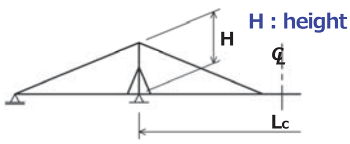
spatial type tower



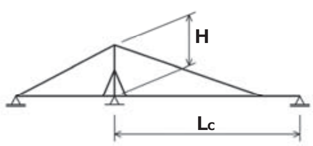
Two towers

Height of tower

H : height from deck level

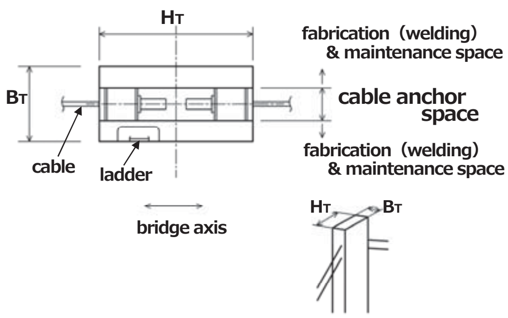


$L_c / H ≒ 5.0$

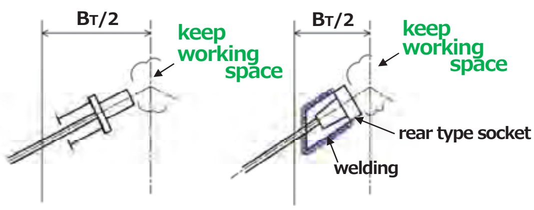


$(1.8L_c) / H ≒ 5.0$

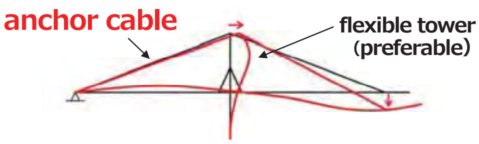
Basic dimension of tower



Determination of B_T



anchor girder type   anchor plate type

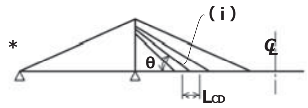


in-plane flexural stiffness
depends on anchor cable,
not on tower stiffness.

flexible tower is preferable.

First step

Cable size* & anchor system (type and erection) are determined.



Cable (i) –tension (T_i) and area ($A_{c,i}$)

$$T_i = \left(W_d + 1.2p + \frac{P\beta}{2} \right) L_{CD} / \sin\theta$$

$$A_{c,i} = \frac{T_i}{\sigma_a} \underline{(\times 1.1)}$$

margin

p : distributed loading
P : concentrated loading

↓
selection of cable type
↓
design of anchor system
↓ ← fabrication view point
maintenance view point
Assumption of (B_T and H_T)
↓
Structural analysis using frame model

Shear lag is also taken into account

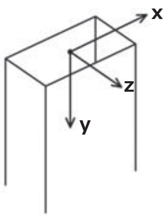
Effective width
$b_{eff.} = b_{eff.}(\text{width}, L_{eq.})$

Equivalent length ($L_{eq.}$)
is obtained depending on
moment distribution pattern

[parabolic] or [straight]

Safety check

Stress (σ, τ)

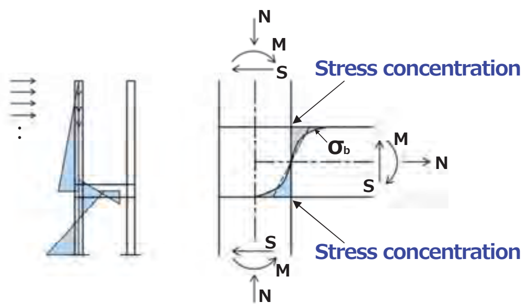


$$\sigma = \frac{N}{A} + \frac{M_z}{I_z}x + \frac{M_y}{I_x}z$$
$$= \sigma_c + \sigma_{bz} + \sigma_{bx}$$
$$\tau = \tau_b + \tau_s$$

(similar to main girder)

Procedure of safety check
is similar to main girder

Design of rigid frame corner part



[1] Against global buckling

$$\frac{\sigma_c}{\sigma_{caz}} + \frac{\sigma_{bz}}{\sigma_{bagz}} + \frac{\sigma_{by}}{\sigma_{bao}} < 1.0$$

σ_c : axial compressive stress

σ_{caz} : allowable column buckling stress

$\sigma_{bagz}, \sigma_{bao}$ $(=\sigma_y/1.7)$: allowable bending stress

↑ since no lateral torsional buckling will produce

[2] Against local (plate) buckling

$$\sigma_c + \sigma_{bz} + \sigma_{by} < \sigma_{cal}$$

σ_{cal} : allowable plate buckling stress

$(\sigma, \sigma_{bz}, \sigma_{by})$ are calculated based on linearized finite displacement analysis

2) modify $E_i \longrightarrow E_{fi}$

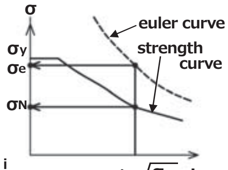
$$E_{f,i} = \frac{\sigma_{N,i}}{\sigma_{e,i}} E_i$$

$\sigma_{e,i}$: buckling stress of element (i)

$\sigma_{N,i}$: strength of element (i)

$$\left| K_E\left(E_{f,i}, I_i\right) + \kappa K_G\left(N_i\right) \right| = 0$$

$$L_{e,i} = \pi\sqrt{E_{f,i} I_i / (\kappa N_i)}$$



$$\Lambda = \frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}} \cdot \frac{L_e}{r}$$

$$\left(r = \sqrt{I/A}\right)$$

3) until converged value of $L_{e,i}$ calculation is continued

4) $\sigma_{caz} = \sigma_{cr}(L_{e,i})/1.7$

E_f-method (inelastic eigenvalue analysis)

1) elastic eigenvalue analysis

$$\left| K_E\left(E_i, I_i\right) + \kappa K_G\left(N_i\right) \right| = 0$$

$$L_{e,i} = \pi\sqrt{E_i I_i / (\kappa N_i)}$$

$[K_E]$: elastic stiffness matrix

$[K_G]$: geometric matrix

$L_{e,i}$: buckling length of elasticity of element (i)

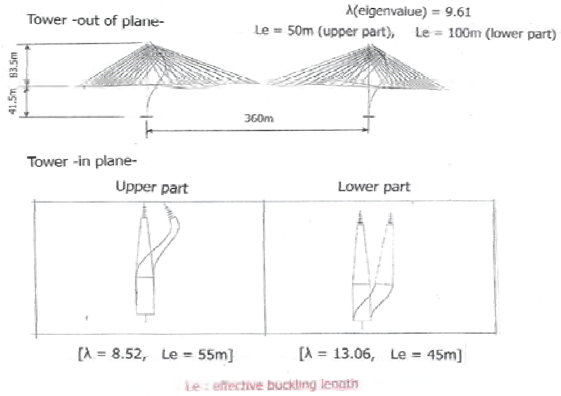
E_i : young 's modulus of elasticity of element (i)

I_i : geometrical moment of inertia of element (i)

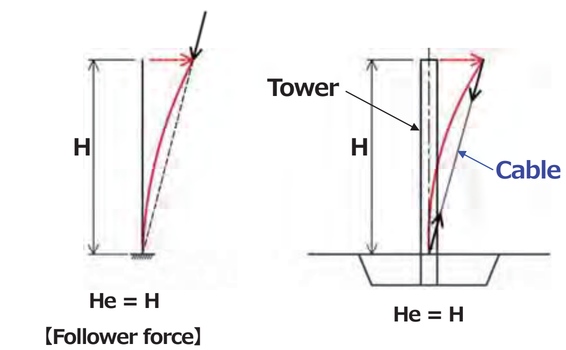
κ : min. eigenvalue

N_i : compressive axial force of element (i)

Example of application of E_f method



Effective buckling length (H_e) of single tower



Load cases of Ultimate Strength Analysis

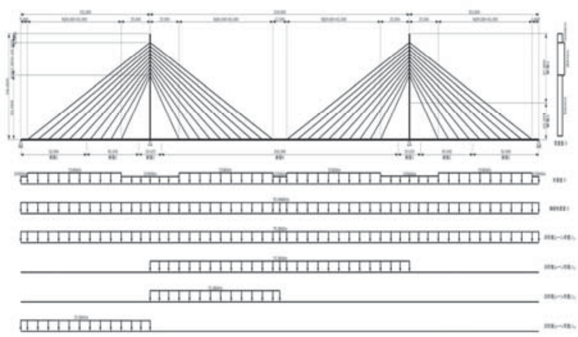
Live Load Cases

L1 : Loading on the all spans

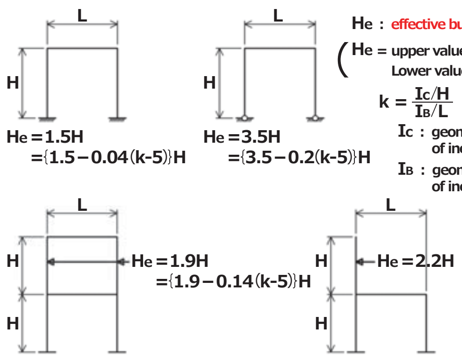
L2 : Loading on the center span

L3 : Loading on the left half of center span

L4 : loading on the left side span



Rigid frame (Rahmen) structures



H_e : effective buckling length

(H_e = upper value ($k \leqq 5$) Lower value ($5 < k \leqq 10$))

$$k = \frac{I_C/H}{I_B/L}$$

I_C : geometrical moment of inertia of column

I_B : geometrical moment of inertia of beam

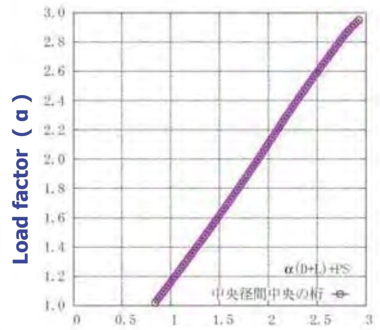
Analysis Case L1 [α(D+L)+Ps]

Girder yielding 1.79(D+L)+PS

Cable yielding 2.61(D+L)+PS

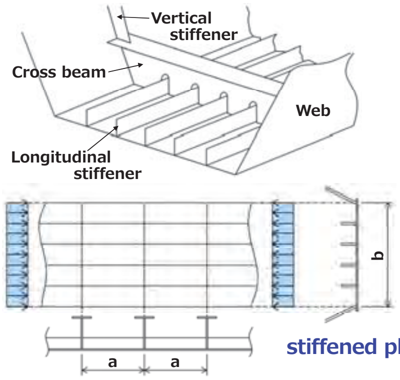
Tower yielding 2.63(D+L)+PS

Maximum Load 2.95(D+L)+PS



Load – deflection at the center of the girder

calculation of (σ_{cal})



Design of stiffened plate

Explained at [Design of the girder]

[3] Global & local coupled buckling strength

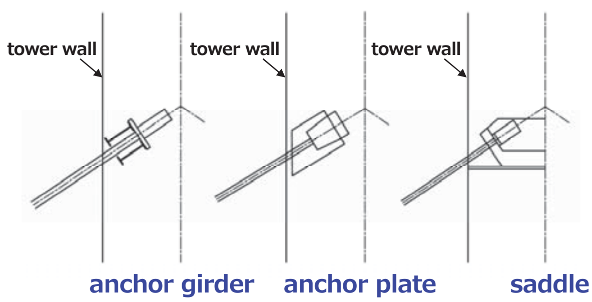
$$\sigma_a = \sigma^{*}_{caz} \times (\sigma^{**}_{cal} / [\sigma_y / 1.7])$$

σ_{caz} (in[1]) → σ_a (allowable coupled buckling stress) [reduced]

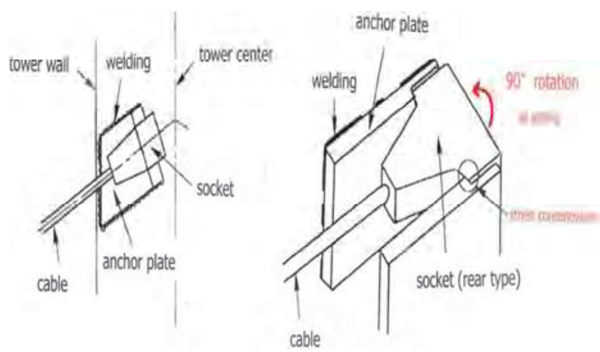
$*$ σ_{caz} : allowable global buckling stress

$**$ σ_{cal} : allowable local buckling stress

Cable anchor in the tower

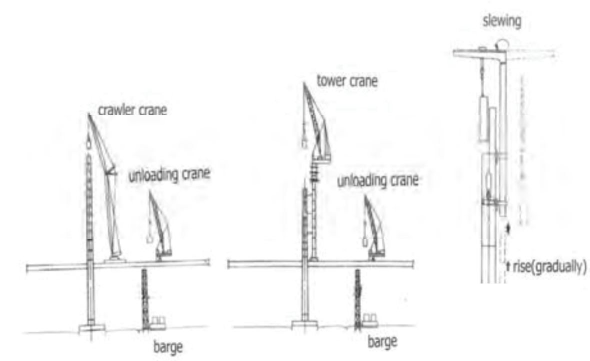


Anchor plate



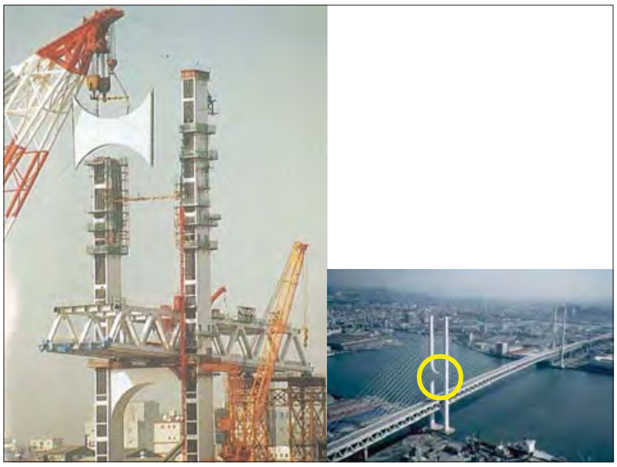
[12-5-1]

Erection of Girder & Tower



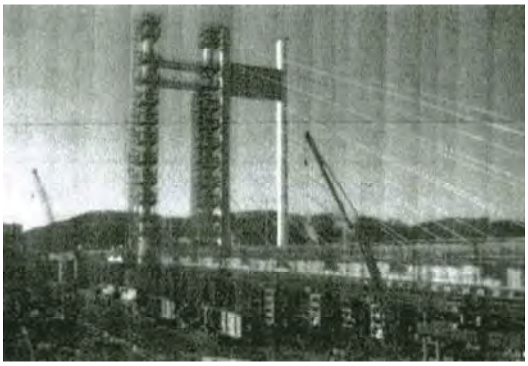
[Tower erection]

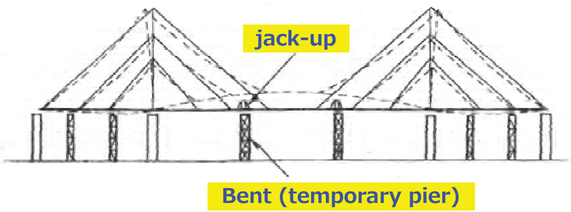
Large block erection by floating crane



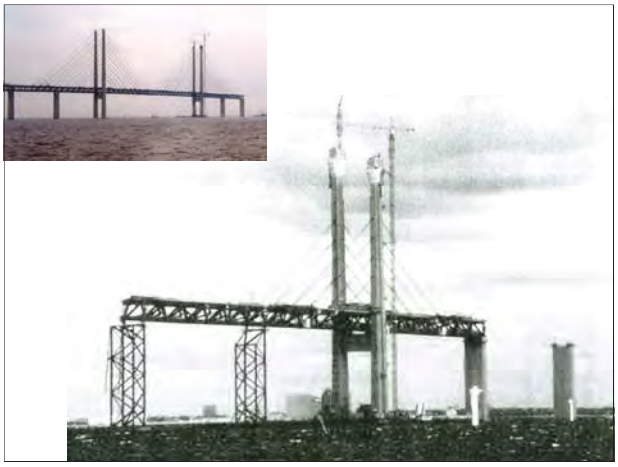
[Girder erection]

[1] All bent (staging) erection



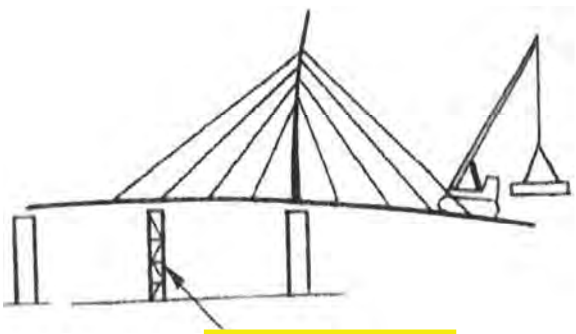


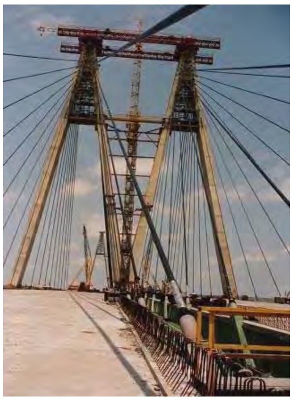
[2] Girder erection
(large block by floating crane)

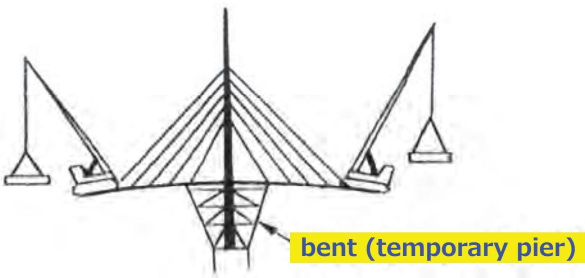


[3] Cantilevered erection

Side span (erection by temporary piers)
+ Center span (by cantilevered erection)





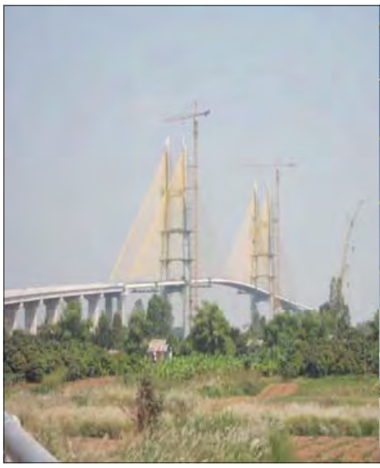
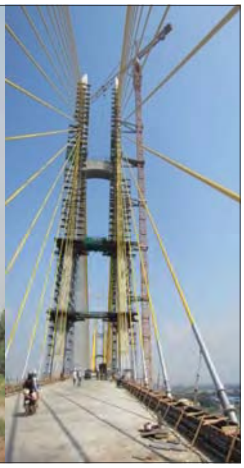


[4] Balanced cantilevered erection

[5] Push-out erection method

Tower cranes

(not for erection of RC tower,
for erection of cables and
for lifting materials)

Crawler crane and tower crane



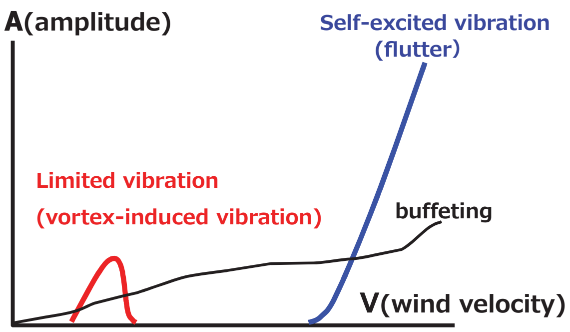

erection

(DVD)

[12-5-2]

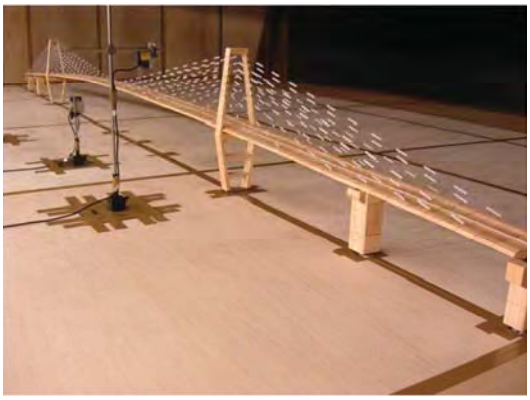
Wind-resistant Design

Dynamic response of structure due to wind



Wind tunnel test (section model test)

Wind tunnel test (full model test)



Countermeasures

(Oscillation suppression method)

1) Mechanical means

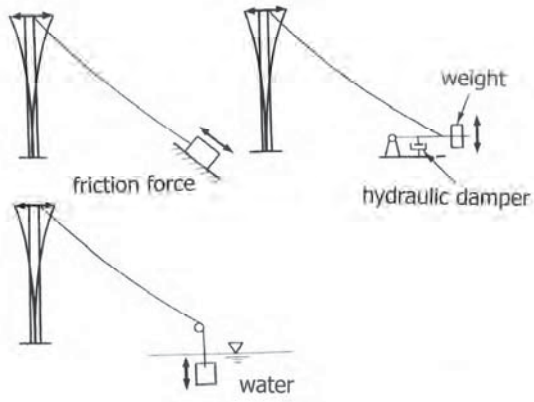
a) Passive-type

・add damping

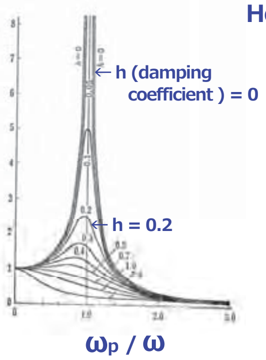
(untuned type)[←cable]

・TMD(tuned type)

b) Active-type



L



How to avoid resonance

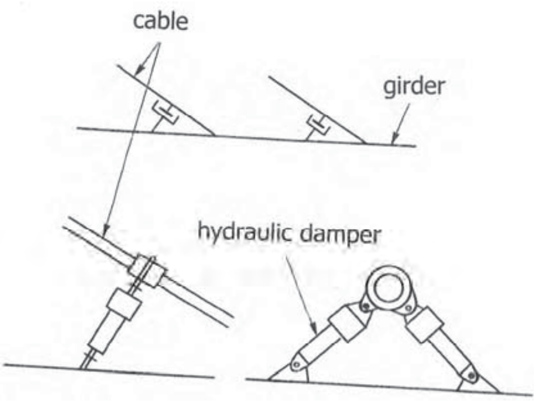
(1) Avoid ($\omega_p / \omega = 1.0$)

ω_p : frequency of force

ω : frequency of structures

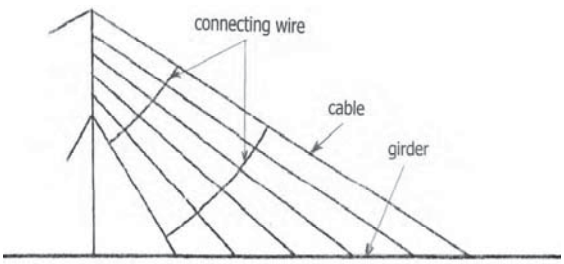
change $\omega \left(= \sqrt{\frac{K}{m}}\right)$

(2) Add(increase) damping

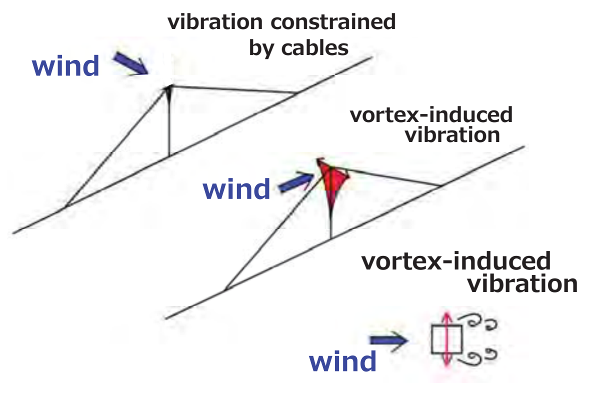


Tuned mass damper

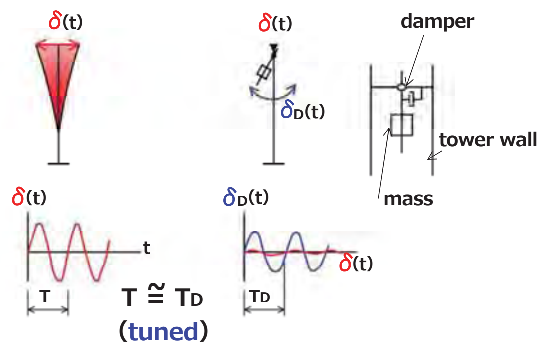
Connecting cables by wires



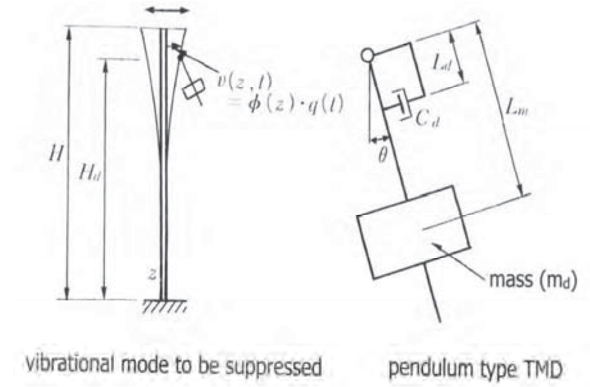
(1990)



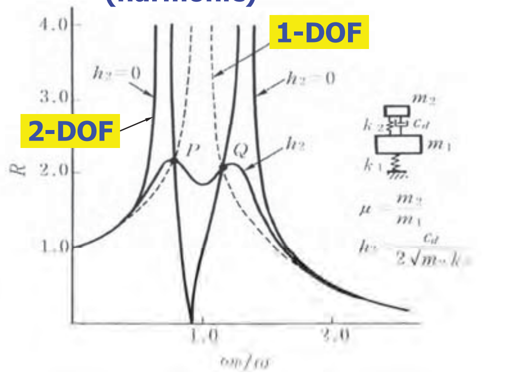
TMD (Tuned Mass Damper)



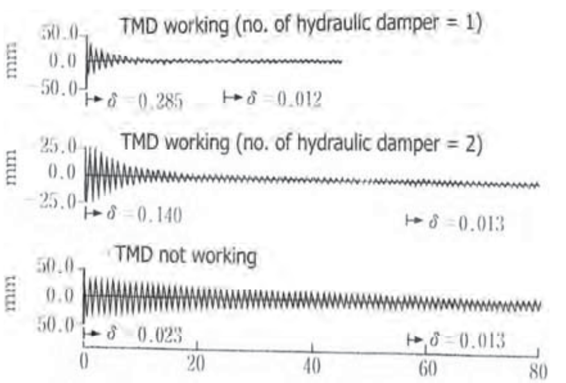
Pendulum type TMD

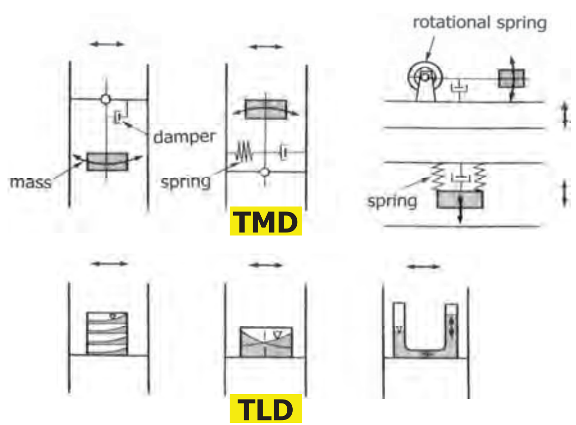


2-DOF forced vibration with damping (harmonic)

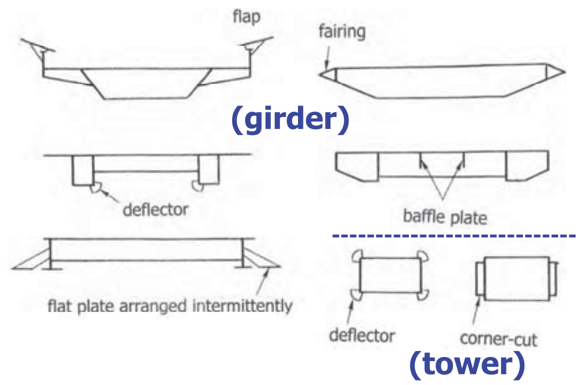


Effectiveness of TMD

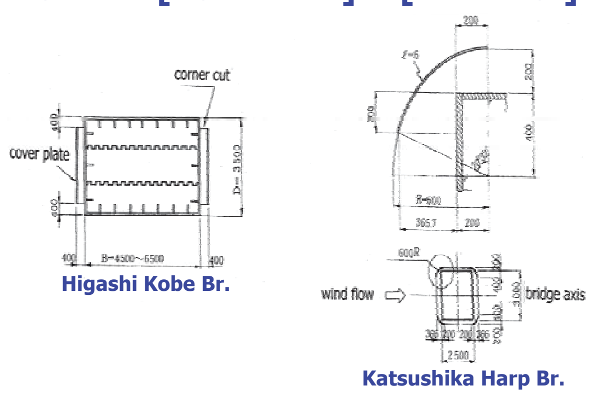




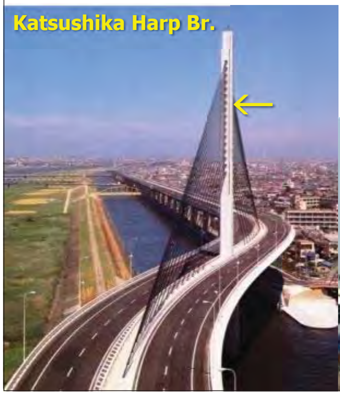
2) Aerodynamic means



Tower – [Corner cut] & [Deflector]



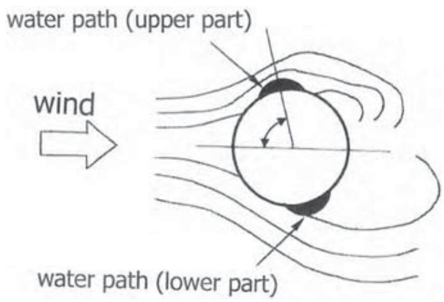
Deflector was employed



Cables

Corner cut was employed

Rain vibration

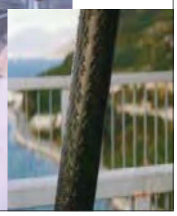


Rain vibration of cables

[conditions of occurrence]

[rainy day (not heavy rain)]
+
[wind speed : from 10 to 15m/s]
+
[wind direction : nearly parallel
to bridge axis]

Cable with indented surface



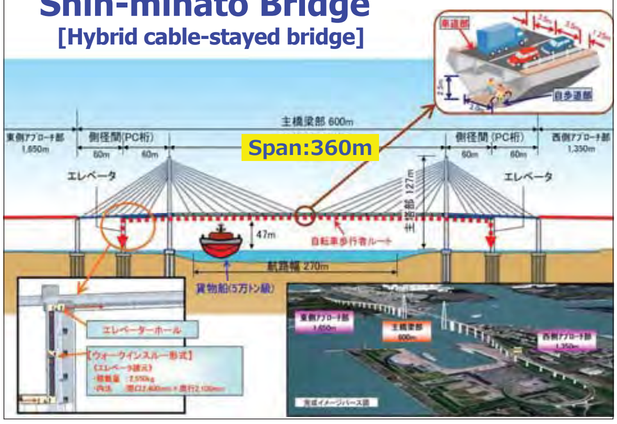
Suppressing cable vibration

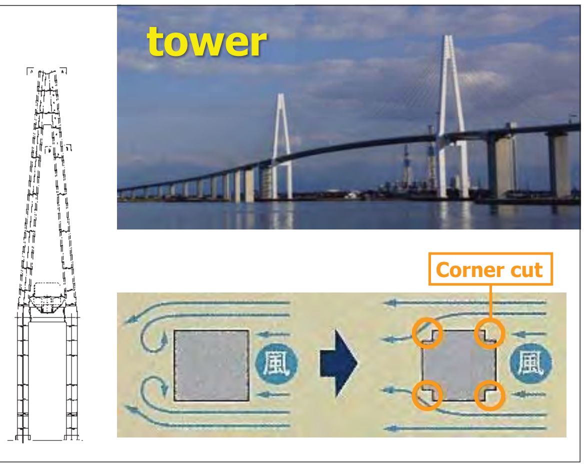
Indented surface cable was employed

Gear-type surface

Cross section (Main girder)

Shin-minato Bridge
[Hybrid cable-stayed bridge]

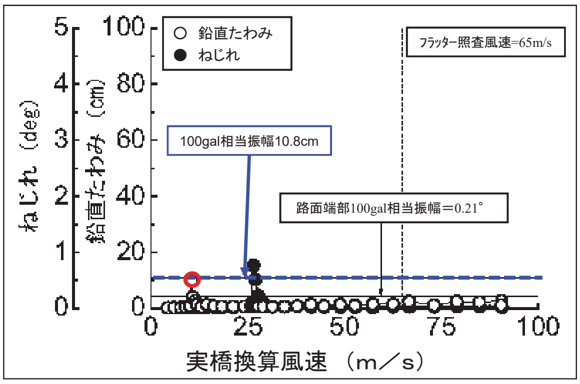




Rain vibration of cables

(DVD)

Wind tunnel test(2002)



[Upper part of cables (long cables)]

↓ changed to

Vortex-induced vibration of the girder

(DVD)

Opening related ceremony

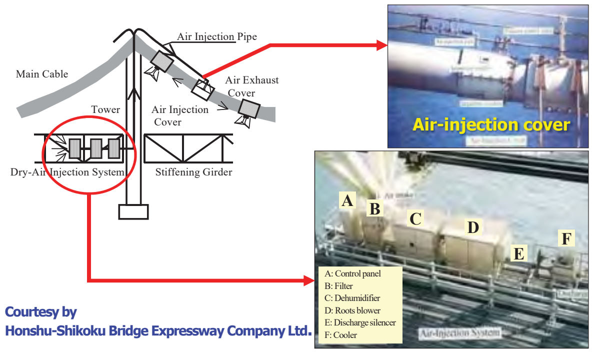
Opening of walkway

Flap (Countermeasure)

[12-5-3]

Span Limitation of Self-anchored Cable-stayed System

Dry-air Injection System of Akashi Kaikyo Bridge



Courtesy by
Honshu-Shikoku Bridge Expressway Company Ltd.

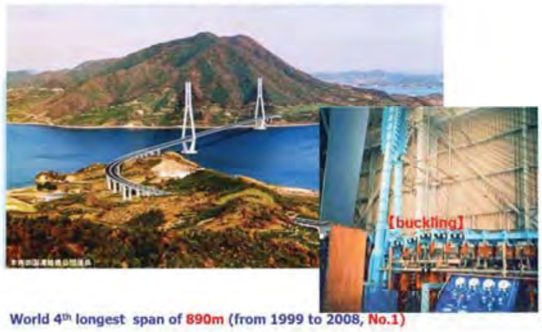
Akashi Kaikyo Bridge

(Honshu-Shikoku Bridge Expressway Company Ltd.)

World longest span of 1,991m

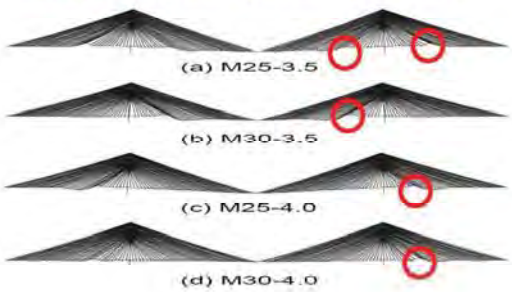
Tatara Bridge

(Honshu-Shikoku Bridge Expressway Company Ltd.)



World 4th longest span of 890m (from 1999 to 2008, No.1)

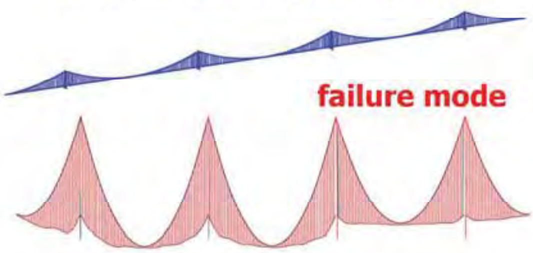
Buckling (failure mode) of 1400-m model by nonlinear FEA



Span limitation of cable-stayed bridges

Chinese magazine "Bridge"

5-span continuous suspension bridge (3000-m)



Span limitation

1) cable-stayed bridges

from 1,200 to 1,400 m

2) Suspension bridges

around 3,000 m

will be possible!!

topics

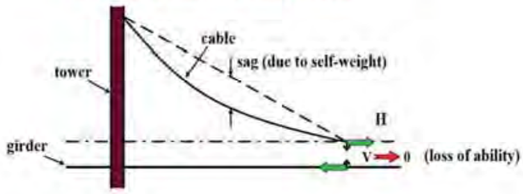
1) Span limitation of self-anchored steel cable-stayed bridges

2) Possibility of further span extension
   a) spatial net system
   b) partially earth-anchored system

3) Span limitation of self-anchored composite and PC cable-stayed br.

AA : From mechanical viewpoint!!

Controlled by suspending ability of cables



·3,000m will be possible by current material !!

·4,000-5,000m will be possible by new material !!
(light-weight & high strength)

- 1st Topic -

We have to take into account two aspects

AA : Mechanical viewpoint

BB : Economical viewpoint

Another critical issue
Need technology mitigating

1)long-cable vibration

Under construction, mitigation of

2) Vortex-induced vibration
of the girder with a cantilevered length from 600 to 700 meters

Possible vib. depends on the site condition
Solution by wind tunnel test

What is the key point (subject) for fair comparison ???

⇒ design main girder with minimum size

(ensuring safety against static and dynamic instabilities)

Weight of girder controls size of cables, towers, substructures and foundations

BB : From economical viewpoint!!

Competition (fair!!)

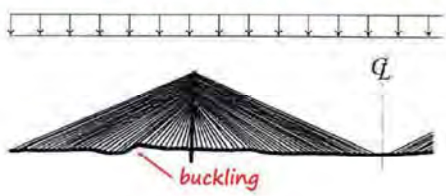
Cable-stayed bridges

vs.

Suspension bridges

Static instability –(1)-

·*Elasto-plastic buckling* of main girder



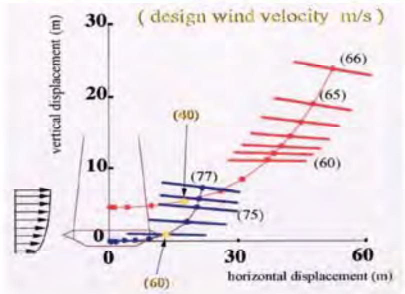
Side view of buckling of the girder
(Tatara Br.)

Dynamic instability

flutter

Static instability -(2)-

Divergence under
displacement-dependent wind loading



Identification by analysis (based on non-linear 3D FEA)

-buckling-

Elasto-plastic finite displacement analysis

-divergence-

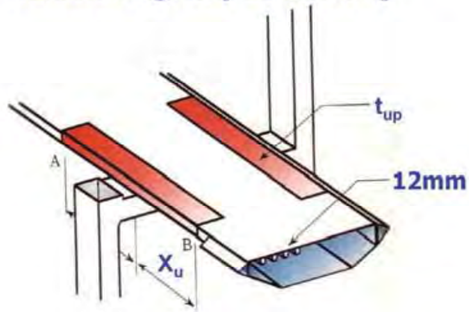
Non-linear elastic analysis
under displacement-dependent wind load

-flutter-

Complex eigenvalue analysis
using modal coordinate

Summary (new findings) for basic planning

Increase out-of-plane flexural rigidity efficiently



Lateral instability under wind load

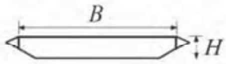
$$V_{(divergence*)} < V_{(flutter)}$$

*under disp.-dependent wind loading

Span-to-girder width ratio (lateral instability)

$L_c/B < 40 \Rightarrow L_c/B < (50\sim55)$

(popular view) (my recommendation)



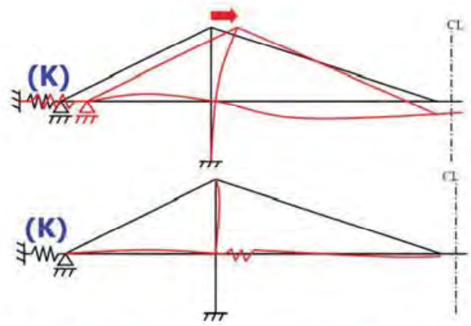
In-plane instability under gravity loading

If the width is around 10 meters (for 2-lane bridge),

and span exceeds 400 meters more,

Lateral stability has to be carefully checked.

If large (L/B) ratio, a suspension bridge is recommended.

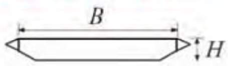


Setting spring constant of (K) is important

Span-to-girder depth ratio (in-plane global buckling instability)

$L_c/H < (600\sim700)$

From a viewpoint of maintenance,
2.5-m more depth is preferable,
⇒ 1500-m span bridge, no check ok!!



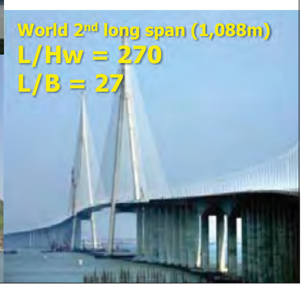
Ultimate strength of cable-stayed bridge system

$f_{ult.} = \min.\{f_{cy}, f_{cal}\}$

$f_{ult.}$: ultimate strength (collapse)
f_{cy} : yield stress of cable
f_{cal} : local (plate) buckling strength

Under condition that
$L_c/H \leqq (600\sim700)$: [global buckling criterion]
$L_c/B \leqq (50\sim55)$: [lateral instability]

(L/Hw) & (L/B) of long span cable-stayed bridges



Global buckling itself can be prevented by $L_c/H < (600\sim700)$

Yield of cable leads to global buckling

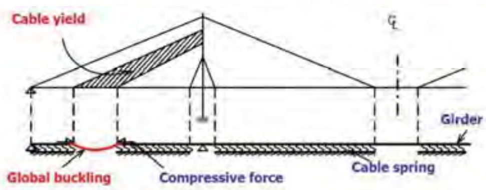
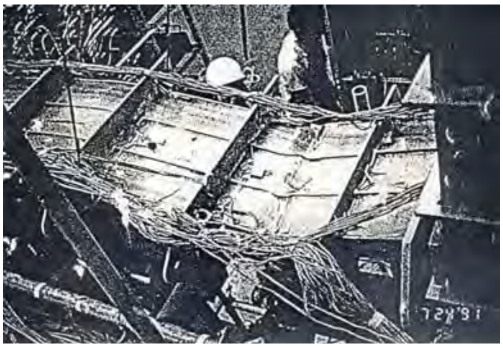


Plate local buckling leads to global buckling

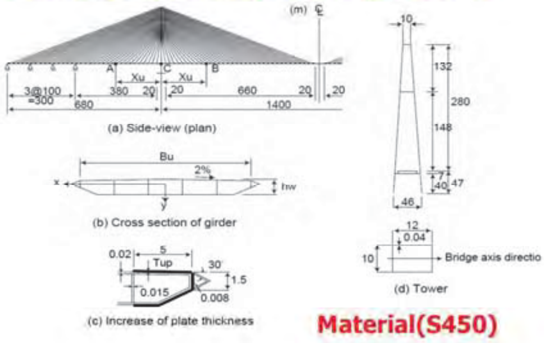
(coupled local/global buckling)

1400-m cable-stayed bridge model



Buckling of the girder near tower (Tatara Br.)

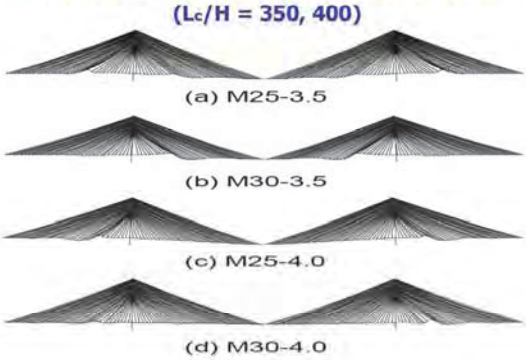
1400-m Cable-stayed Bridge Model



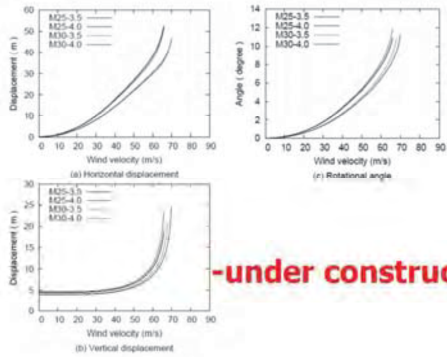
Material(S450)

Buckling (failure) Mode Shape

(L_c/H = 350, 400)

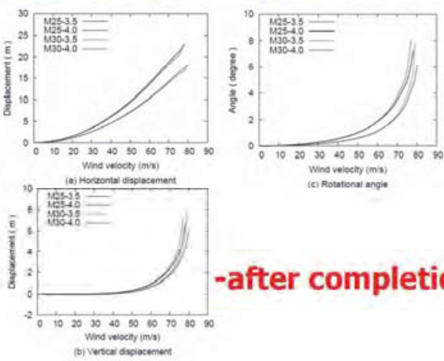


Displacements at the tip of cantilevered girder



-under construction-

Displacements of the girder at the center of span



-after completion-

Flutter Onset Wind Velocity (m/s)

	Completed		Under construction	
Model	[30-mode]	Selberg	[20-mode]	Selberg
M25-3.5	120 (144)	131	100 (151)	94
M25-4.0	127	135	102	100
M30-3.5	120	131	102	97
M30-4.0	126 (151)	136	105 (168)	103

Note: Cable vibration is taken into account for values in parentheses.

Comparison of steel volume
(rough estimation)

Suspension	vs.	cable-stayed
(1.00)		(1.28)
(1.00)		(1.16*)

*cable unit price = 1.75 x (steel unit price)

[+ anchorage]   [+ intermediate piers]

My conclusion!!

From 1,200 to 1,400m cable-stayed bridges

will be possible!!

Depending on the site (soil) condition,
should be included as one of alternatives

Which solution is economical??

Exact identification will be difficult.

It depends on many parameters such as,
- soil condition at site
- unit price of cable & steel

etc. etc.

-2nd Topic-

Further span extension??

- How to -

1) Spatial cable system (by Gimsing)
2) Partially earth-anchored system

1)Spatial cable system

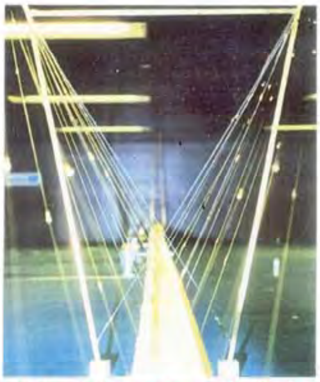
(not promising : my opinion)

the reason why???

Proposal

Bocairente Bridge

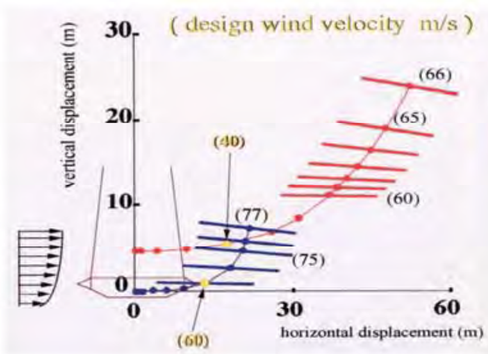
Proposal for a new Ebro Bridge



by Gimsing

In order to increase out-of-plane flexural rigidity

Lateral instability



since $V_{(divergence)} < V_{(flutter)}$

Important parameters to enhance $V_{(divergence)}$ under wind load are

1) In-pane flexural rigidity of the system

2) Torsional rigidity of the system

Effect of increase of out-of-plane flexural rigidity

will be minor

2) Partially earth-anchored system

(promising system or not depends on the erection cost)

Brief summary

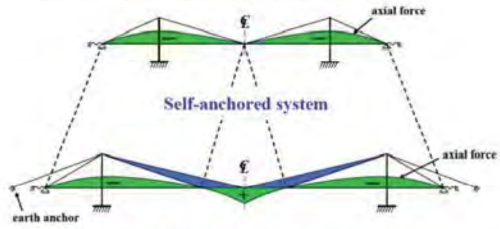
1)Small reduction of out-of-plane bending moment

2)No contribution to enhance the wind velocity at lateral instability

3)Higher cost for erection

4)From aesthetic reason, it may be OK

Cable-stayed System



Partially earth-anchored system

N(+) = N(-)

$\Rightarrow L_c$ (Partially earth-anchored) $= \sqrt{2} L_c$(Self-anchored)

from theoretical consideration,

In the girder,
(max. compressive axial force)
= (max. tensile axial force)

$\sqrt{2}$ times span extension is possible

Realistic (or) Economical??

After completion

a) 10% reduction of displacement and bending moment at the design wind velocity owing to earth-anchored system

b) Critical wind velocity is nearly the same!!

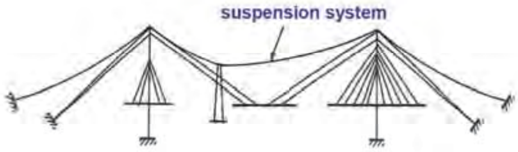
(Span = 1400m, Girder width = 34m, Girder depth = 4.6m : Earth-anchored length in the span is 370m)

What's difference under wind action?

Self-anchored vs. Partially earth-anchored

How to erect ??

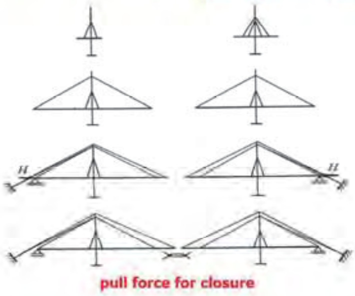
Suspension system for erection only
[extra equipment & cost]



Under (cantilevered) erection

Nearly the same behavior!!
with self-anchored system

Cantilevered erection
Equipment for resisting horizontal force (H) and pull force for closure



I talked,
from 1,200 to 1,400m cable-stayed bridges

will be possible!!

(through economical comparison with suspension bridges)

↓

Partially earth-anchored system

with a span of 1,600m will be possible!!

(taking into account of feasibility of erection)

Pull force for closure of the girder is 10MN

[Cable-stayed system]
Up to 1,200-1,400 meters
(from economical reason)
[Suspension system]
Up to 3,000-3,500 meters
(depending on cable material & aerodynamic stability)

-3rd Topic-

Steel-concrete composite cable-stayed bridges

Spatial & Partially earth-anchored systems

will be hopeless (from economical reason)

Since 1986 of Alex Fraser Bridge at Vancouver,
[its span of 485m was the world record at that time]
Worldwide construction except Japan

The Golden Ear Bridge at Vancouver (span:242m)
PFI (German Company)

Extradosed type PC bridge in Slovakia

2nd Forth roadway bridge (composite continuous bridge)

1) Longest span
2) Structural form (continuous type)

Nhat Tan bridge

Binh bridge
(ship collision)

For worldwide competition

Inevitable alternative to compete PC bridge!!

⇒ **Span Limitation!!**

$f_n < f_{ck}(35\sim50^{MPa}) / 3.5$

↓

300~400^{m}span OK!!

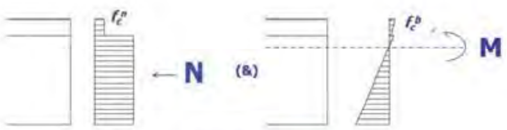
(only axial force, and if no buckling)

Technical issue

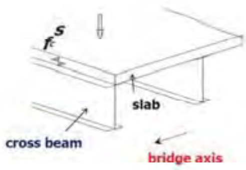
1) Aerodynamic stability [poor]
ensured by wind tunnel test

2) Stress transfer due to creep & shrinkage
↑ 《Precast RC slab》

3) Under construction, since flexible,
shape control



$f_c^n + f_c^b < f_{ck} / 3.5$

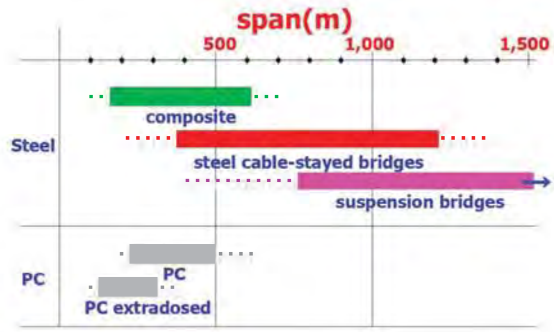


$f_c^n + f_c^b + f_c^s < (f_{ck} / 3.5) \times 1.4$

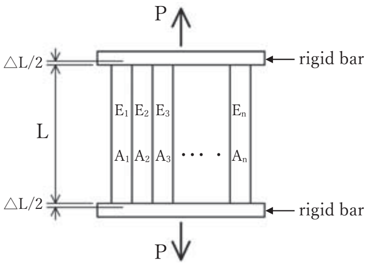
$[\ f_{ck} = 55^{MPa}\ ]$

⇒ **span of 500~600^{m}**

Applicable span



【10-3-2】(1)



E_i : Young's modulus of elasticity

A_i : Cross sectional area

$$\varepsilon = \frac{\Delta L}{L} = \frac{P_1}{E_1A_1} = \frac{P_2}{E_2A_2} = \cdots\cdot = \frac{P_n}{E_nA_n}$$

$$\mathrm{P} = \Sigma P_i = P_1 + \frac{E_2A_2}{E_1A_1}P_1 + \cdots\cdot + \frac{E_nA_n}{E_1A_1}P_1$$

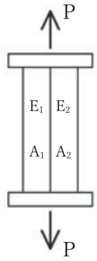
$$= \frac{E_1A_1 + E_2A_2 + \cdots\cdot + E_nA_n}{E_1A_1}P_1$$

$$P_1 = \frac{E_1A_1}{\Sigma E_iA_i}P$$

$$P_2 = \frac{E_2A_2}{\Sigma E_iA_i}P$$

$$\vdots$$

【10-3-2】(2)



$$P_1 = \frac{E_1A_1}{E_1A_1 + E_2A_2}P$$

$$P_2 = \frac{E_2A_2}{E_1A_1 + E_2A_2}P$$

(Q1) Find P_1 ,P_2

under conditions

$E_1 = 2 \times 10^5\ N/mm^2$

$A_1 = 100\ mm^2$

$E_2 = 3 \times 10^4\ N/mm^2$

$A_2 = 500\ mm^2$

$P = 1{,}000\ kN$

【10-3-2】(3)

《 A1 》

$$P_1 = \frac{2 \times 10^5 \times 100}{2 \times 10^5 \times 100 + 3 \times 10^4 \times 500} = 0.571P$$

$$= 571\mathrm{kN}$$

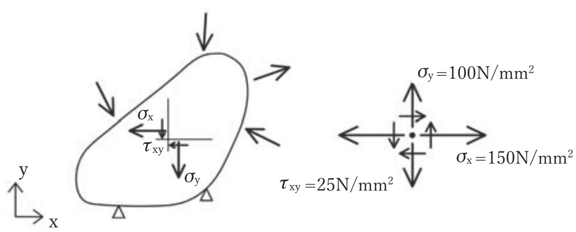
$$P_2 = \frac{3 \times 10^4 \times 500}{2 \times 10^5 \times 100 + 3 \times 10^4 \times 500} = 0.429P$$

$$= 429\mathrm{kN}$$

$$(P_1 + P_2 = 1{,}000\mathrm{kN} = P)$$

【10-3-2】(4)

(Q2) Find σ_1 , σ_2 , θ_0



《 A2 》

$$\sigma_{1,2} = \frac{150+100}{2} \pm \frac{1}{2}\sqrt{(150-100)^2 + 4 \times 25^2}$$

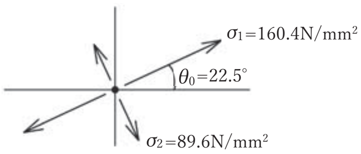
$$= 125 \pm 35.4 \ \ (N/mm^2)$$

$\underline{\sigma_1 = 160.4 \ \ N/mm^2}$, $\underline{\sigma_2 = 89.6 \ \ N/mm^2}$

$$\theta_0 = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = \frac{1}{2}\tan^{-1}\left(\frac{2 \times 25}{150-100}\right) = \frac{1}{2}\tan^{-1}(1)$$

$\tan(2\theta_0) = 1 \ \rightarrow \ \underline{\theta_0 = 22.5°}$

$$\frac{\cos\theta_0}{\sigma_y - \sigma_x} < 0 \qquad (\cos\theta > 0\ ,\ \ \sigma_y - \sigma_x < 0)$$



Safety check (by JHBS)

$\Sigma\sigma < \sigma_a$

$$\sigma_a = min.\ \{\underline{\sigma_y}\ ,\underline{\sigma_{cr}}\}/1.7$$

yield stress *buckling strength*

$\Sigma\tau < \tau_a$

$$\tau_a = \tau_y/1.7 \qquad (\tau_y = \sigma_y/\sqrt{3})$$

$\underline{\sigma_e(\Sigma\sigma\ ,\Sigma\tau) < \ 1.1\sigma_a}$

$$\sigma_a = \sigma_y/1.7$$

(Q3) Calculate σe , and check safety

1) $\sigma_x = 150\ N/mm^2\ , \tau_{xy} = 30\ N/mm^2$

2) $\sigma_x = 180\ N/mm^2\ , \sigma_y = 120\ N/mm^2\ , \quad \tau_{xy} = 50\ N/mm^2$

3) $\sigma_x = 180\ N/mm^2\ , \sigma_y = -120\ N/mm^2\ , \tau_{xy} = 50\ N/mm^2$

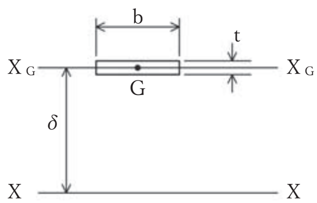
$\sigma_a = 210\ N/mm^2$ (SM490Y)

《 A3 》

1) $\sigma_e = \sqrt{150^2\ + 3 \times 30^2} = 158.7\ N/mm^2 < 1.1\sigma_a$
$(= 231\ N/mm^2)$

2) $\sigma_e = \sqrt{180^2\ - 180 \times 120 + 120^2 + 3 \times 50^2} = 180.8\ N/mm^2$
$< 1.1\sigma_a$

3) $\sigma_e = \sqrt{180^2\ + 180 \times 120 + 120^2 + 3 \times 50^2} = 275.5\ N/mm^2$
$> 1.1\sigma_a$



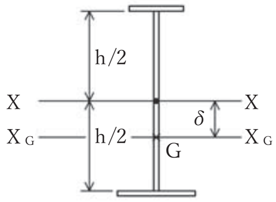
$A = bt$

$$I_{XG} = \frac{bt^3}{12}$$

$$I_X \ = I_{XG} + A\delta^2$$

$$I_{XG} = I_X - A\delta^2$$

G : center of gravity (centroid)



$$I_X \ = I_{XG} + A\delta^2$$

$$I_{XG} = I_X - A\delta^2$$

A : cross sectional area of section

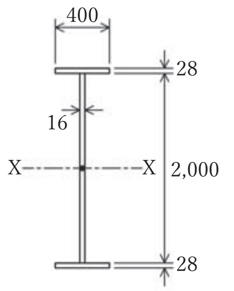
G : center of gravity (centroid)

【10-3-3】(2)

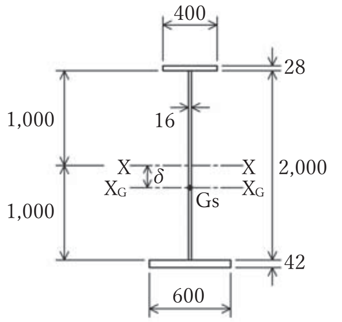
(Q) Find A ,I

(mm)

1)



2)

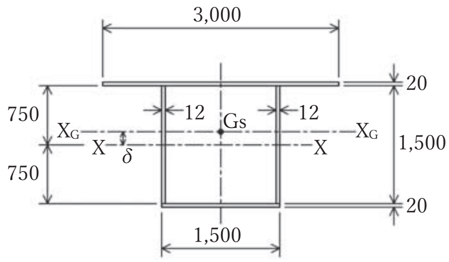


Gs : center of gravity

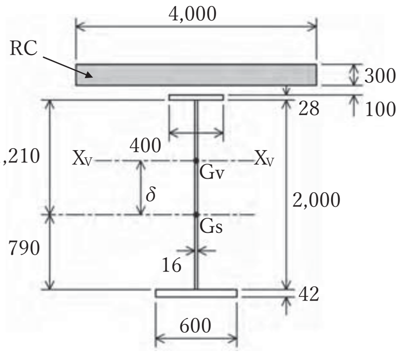
【10-3-3】(3)

(mm)

3)



4)



$$\left(n = \frac{E_{steel}}{E_{concrete}} = 7\right)$$

【10-3-3】(4)

《 A 》

(cm)

1) $A = 2 \times 40 \times 2.8 + 1.6 \times 200 = 544\ cm^2$

$$I_x = 2 \times 40 \times 2.8 \times 101.4^2 + 2 \times \underbrace{\frac{2.8^3 \times 40}{12}}_{\uparrow 146.3} + \underbrace{\frac{200^3 \times 1.6}{12}}_{\uparrow 1{,}066{,}667}$$

$= 2{,}303{,}159 + 146.3 + 1{,}066{,}667$

$\cong 3{,}369{,}826^{(*)}\ cm^4$

(*) 146.3 *is excluded*

2) (cm)

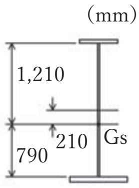
	A	y	Ay	Ay²
1-U PL 400×28	112	101.4	11,357	1,151,580 73*
1-W PL 2,000×16	320	–	–	1,066,667
1-L PL 600×42	252	−102.1	−25,729	2,626,951 370*
Σ	684		−14,372	4,845,198

$$\delta = \frac{-14{,}372}{684} = -21.0 \quad (= -210\text{mm})$$

$-301{,}644^{**}$

$I_{XG} = 4{,}543{,}554\text{cm}^4$

(mm)



(*) *is excluded to calculate* (I_{XG})

(**) $\delta^2 A = 21 \times 21 \times 684 = 301{,}644$

【10-3-3】(5)

3) (cm)

	A	y	Ay	Ay²
1-U PL 3,000×20	600	76	45,600	3,465,600 200*
2-W PL 1,500×12	360	–	–	675,000
1-L PL 1,500×20	300	−76	−22,800	1,732,800 100*
Σ	1,260		22,800	5,873,400

$$\delta = \frac{22{,}800}{1{,}260} = 18.1 \quad (= 181\text{mm})$$

$-412{,}789^{**}$

$I_{XG} = 5{,}460{,}611\text{cm}^4$

(*) *is excluded to calculate* (I_{XG})

(**) $\delta^2 A = 18.1 \times 18.1 \times 1{,}260 = 412{,}789$

4) (cm)

	A	y	Ay	Ay²
1-D PL 4,000×300	1,714.3*	148.8**	255,088	37,957,071 128,571***
1-ST Girder****	684	–	–	4,543,554
Σ	2,398.3		255,088	42,629,196

$$\delta = \frac{255{,}088}{2{,}398.3} = 106.4 \quad (= 1{,}064\text{mm})$$

$-27{,}151{,}058^{*****}$

$I_{XG} = 15{,}478{,}138\text{cm}^4$

* $1{,}714.3 = 400 \times 30/7$ ($n = 7$)

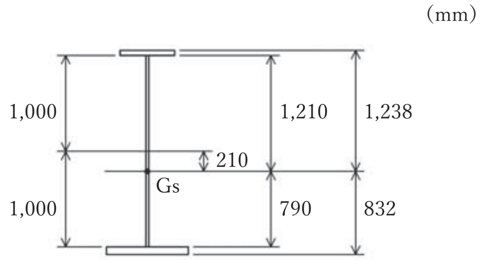
** $148.8 = 121.0 + 2.8 + 10 + 15$

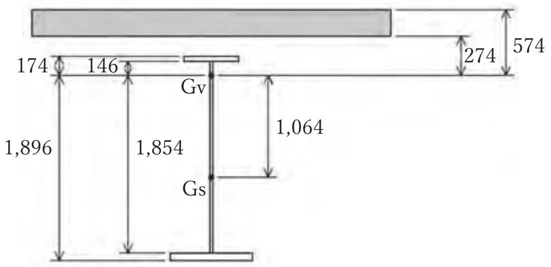
*** $128{,}571 = \dfrac{30^3 \times 400}{12}/7$

**** see 《A》, 2)

***** $\delta^2 A = 106.4 \times 106.4 \times 2{,}398.3 = 27{,}151{,}058$

【10-3-3】(6)

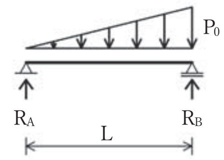




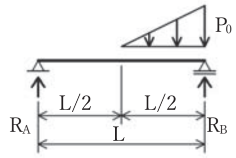
【10-3-4】(1)

(Q) Find Reactions (R_A , R_B , H_A)

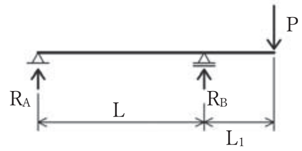
1)



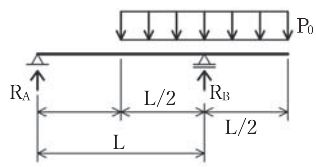
2)



3)

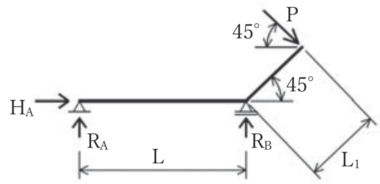


4)

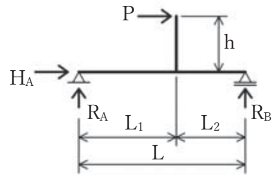


【10-3-4】(2)

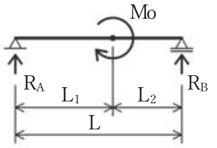
5)



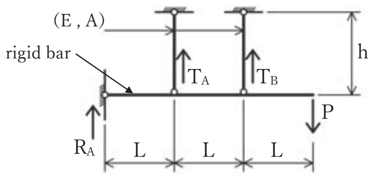
6)



7)

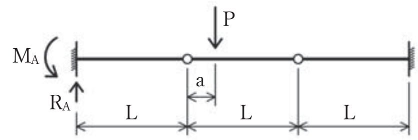


8) *Find* (R_A , T_A , T_B)

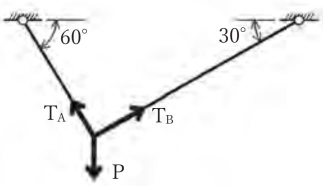


【10-3-4】(3)

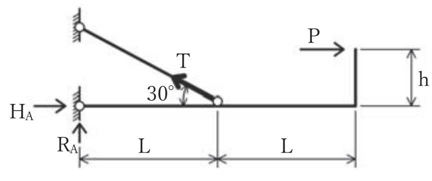
9) *Find* (M_A , R_A)



10) *Find* (T_A , T_B)

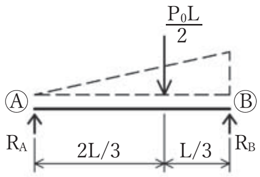


11) *Find* (H_A , R_A , T)



《 A 》

1)

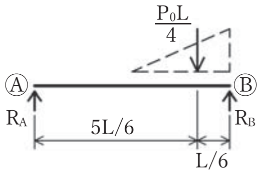


at Ⓐ $R_B L = \frac{P_0 L}{2} \times \frac{2}{3} \mathrm{L} = \frac{P_0 L^2}{3}$

$\underline{R_B = \frac{P_0 L}{3}}$

$R_A = \frac{P_0 L}{2} - R_B = \underline{\frac{P_0 L}{6}}$

2)



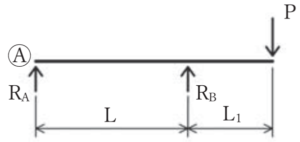
at Ⓐ $R_B L = \frac{P_0 L}{4} \times \frac{5L}{6} = \frac{5}{24} P_0 L^2$

$\underline{R_B = \frac{5}{24} P_0 L}$

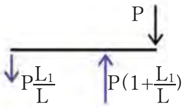
$R_A = \frac{P_0 L}{4} - \frac{5}{24} P_0 L = \underline{\frac{1}{24} P_0 L}$

《 A 》

3)



at Ⓐ $R_B L = P(\mathrm{L} + L_1)$

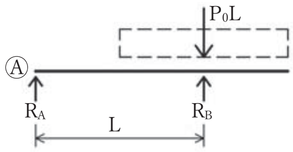


$\underline{R_B = P\left(1 + \frac{L_1}{\mathrm{L}}\right)}$

$R_A = P - R_B = P - P - P\frac{L_1}{\mathrm{L}}$

$\underline{R_A = -P\frac{L_1}{\mathrm{L}}}$

4)



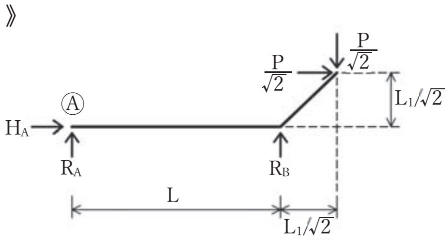
at Ⓐ $R_B L = P_0 L \times L = P_0 L^2$

$\underline{R_B = P_0 L}$

$R_A = P_0 L - R_B = \underline{0}$

《 A 》

5)



$\underline{H_A = -\frac{P}{\sqrt{2}}}$

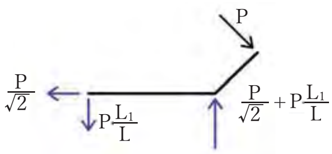
at Ⓐ $R_B L = \frac{P}{\sqrt{2}} \cdot \frac{L_1}{\sqrt{2}} + \frac{P}{\sqrt{2}}\left(\mathrm{L} + \frac{L_1}{\sqrt{2}}\right)$

$= \frac{PL_1}{2} + \frac{\mathrm{PL}}{\sqrt{2}} + \frac{PL_1}{2}$

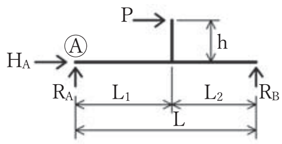
$= \frac{\mathrm{PL}}{\sqrt{2}} + PL_1$

$\underline{R_B = \frac{\mathrm{P}}{\sqrt{2}} + P\frac{L_1}{L}}$

$R_A = \frac{\mathrm{P}}{\sqrt{2}} - R_B = \underline{-P\frac{L_1}{\mathrm{L}}}$

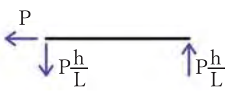


6)



$\underline{H_A = -P}$

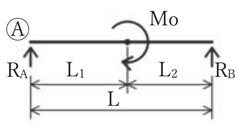
at Ⓐ $R_B L = Ph$



$\underline{R_B = P\frac{h}{\mathrm{L}}}$

$R_A = -R_A = \underline{-P\frac{h}{\mathrm{L}}}$

7)



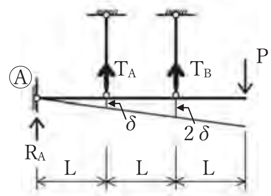
at Ⓐ $R_B L = Mo$

$\underline{R_B = \frac{Mo}{\mathrm{L}}}$

$R_A = -R_B = \underline{-\frac{Mo}{\mathrm{L}}}$

【10-3-4】(8)

8)



$$\delta = \frac{T_A h}{EA} \qquad 2\delta = \frac{T_B h}{EA}$$

$$\rightarrow 2T_A = T_B$$

at Ⓐ $\quad \overset{\frown}{T_A} L + \overset{\frown}{T_B}(2L) = \overset{\frown}{P}(3L)$

$$T_A + 2T_B = 3P$$

$$5T_A = 3P \quad (T_B = 2T_A)$$

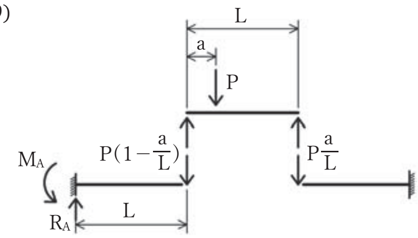
$$\underline{T_A = \frac{3}{5}P}$$

$$\underline{T_B = \frac{6}{5}P}$$

$$R_A = P - (T_A + T_B) = \underline{-\frac{4}{5}P}$$

【10-3-4】(9)

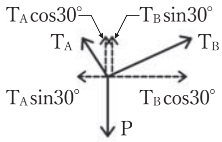
9)



$$\underline{R_A = P\left(1 - \frac{a}{L}\right)}$$

$$\underline{M_A = P(L - a)}$$

10)



$$T_A\frac{\sqrt{3}}{2} + T_B\frac{1}{2} = P \quad \rightarrow (\sqrt{3}T_A + T_B = 2P)$$

$$T_A\frac{1}{2} = T_B\frac{\sqrt{3}}{2} \quad \rightarrow (T_A = \sqrt{3}T_B)$$

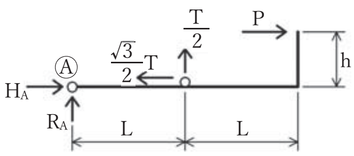
$$3T_B + T_B = 4T_B = 2P$$

$$\underline{T_B = \frac{P}{2}}$$

$$\underline{T_A = \frac{\sqrt{3}}{2}P}$$

【10-3-4】(10)

11)



$$H_A - \frac{\sqrt{3}}{2}T + P = 0$$

at Ⓐ $\quad \overset{\frown}{\frac{T}{2}} L = \overset{\frown}{P}h \quad \rightarrow \quad \underline{T = 2P\frac{h}{L}}$

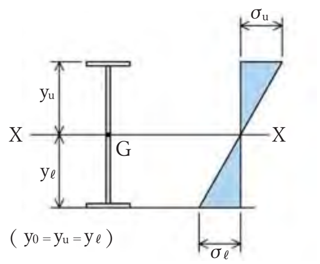
$$R_A + \frac{T}{2} = 0 \quad \rightarrow \quad \underline{R_A = -P\frac{h}{L}}$$

$$H_A = \frac{\sqrt{3}}{2}T - P = \frac{\sqrt{3}}{2}\cdot 2P\frac{h}{L} - P$$

$$= \sqrt{3}P\frac{h}{L} - P$$

$$= \underline{P\left(\frac{\sqrt{3}h}{L} - 1\right)}$$

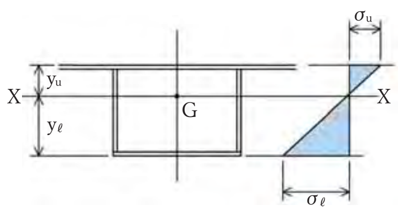
【10-4-4】(1)



$$\sigma_u = \frac{M}{I}y_u = \frac{M}{w_u}$$

$$\sigma_\ell = \frac{M}{I}y_\ell = \frac{M}{w_\ell}$$

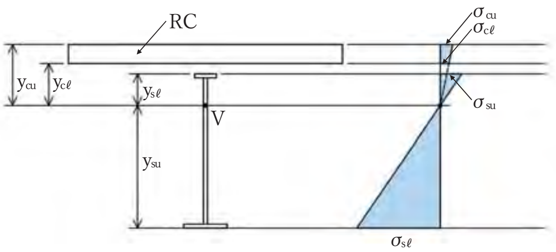
$$(\sigma_u = \sigma_\ell)$$



$$\sigma_u = \frac{M}{I}y_u = \frac{M}{w_u}$$

$$\sigma_\ell = \frac{M}{I}y_\ell = \frac{M}{w_\ell}$$

G : centroid



$$\sigma_{su} = \frac{M}{I_v} y_{su}$$

$$\sigma_{s\ell} = \frac{M}{I_v} y_{s\ell}$$

$$\sigma_{cu} = \frac{M}{I_v} y_{cu} / n$$

$$\sigma_{c\ell} = \frac{M}{I_v} y_{c\ell} / n$$

n : Es / Ec (Young's modulus ratio)

(Q)

1) See【10-3-3】(2),
when M=5 (MN・m), calculate
(σ_u, σ_ℓ)

2) See【10-3-3】(4),
when M=10 (MN・m), calculate
(σ_{cu}, $\sigma_{c\ell}$), (σ_{su}, $\sigma_{s\ell}$)

《 A 》

1)

$$\frac{\sigma_u}{\sigma_\ell} = \frac{5 \times 10^9}{4{,}543{,}554 \times 10^4} \times \frac{1{,}238}{832} = \frac{136.2}{91.6}\ N/mm^2$$

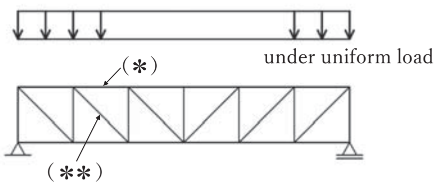
2)

$$\frac{\sigma_{cu}}{\sigma_{c\ell}} = \frac{10 \times 10^9}{15{,}478{,}138 \times 10^4} \times \frac{574}{274} / 7 = \frac{5.3}{2.5}\ N/mm^2$$

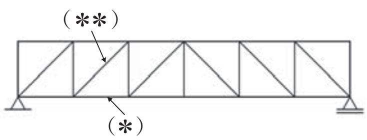
$$\frac{\sigma_{su}}{\sigma_{s\ell}} = \frac{10 \times 10^9}{15{,}478{,}138 \times 10^4} \times \frac{174}{1{,}896} = \frac{11.2}{122.5}\ N/mm^2$$

(Q1) Identity the members (* , **) are subjected to tension or compression.

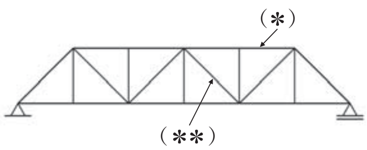
1)



2)

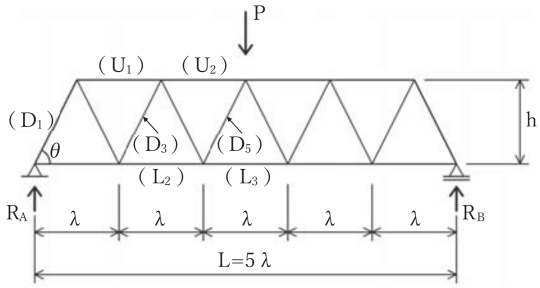


3)



【10-5-3】(2)

(Q2) Find member force from
node and section methods.



diagonal member length $L_D = \sqrt{h^2 + (\lambda/2)^2}$

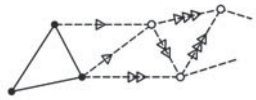
$\sin\theta = h/L_D$, $\cos\theta = (\lambda/2)/L_D$

【10-5-3】(3)

(Q3) Stable and unstable (plane problem)

[*Internal*]

m : *number of members*



$m = 3 + 2(j - 3) = 2j - 3$

j : *number of nodes*

$m \geqq 2j - 3$	→ *stable*
$m = 2j - 3$	→ *stable (determinate)*
$(n_i = m + 3 - 2j)$	→ *degree of redundancy*
$m < 2j - 3$	→ *unstable*

[*External*]

$r \geqq 3$	→ *stable*
$r = 3$	→ *stable (determinate)*
$(n_r = r - 3)$	→ *degree of redundancy*
$r < 3$	→ *unstable*

r : *number of reactions*

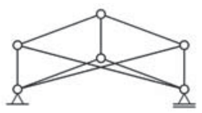
[*Total system*]

$m + r \geqq 2j - 3 + 3 = 2j$	→ *stable*
$m + r = 2j$	→ *stable (determinate)*
$(n_t = m + r - 2j)$	→ *degree of redundancy*
$m + r < 2j$	→ *unstable*

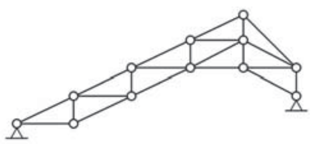
【10-5-3】(4)

Judge the following truss structures
stable , unstable.

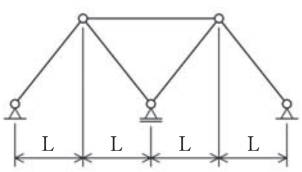
1)



2)



3)



【10-5-3】(5)

《 A1 》

1)

(*) *compression* (−)

(**) *tension* (+)

2)

(*) *tension* (+)

(**) *compression* (−)

3)

(*) *compression* (−)

(**) *compression* (−)

[*note*]

Have a deformed image under uniform load !!

【10-5-3】(6)

《 A2 》

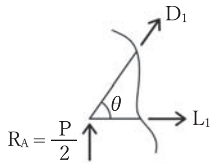
(*Reactions*)

$$R_B \cdot L = P(2.5\lambda)$$

$$R_B = 2.5P\lambda \,/\, (L = 5\lambda) = \frac{P}{2}$$

$$R_A = P - \frac{P}{2} = \frac{P}{2}$$

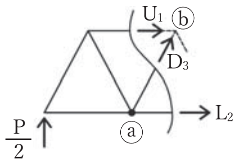
(D_1) [*put* $\sin\theta = s$, $\cos\theta = c$]



$$D_1 s + \frac{P}{2} = 0$$

$$D_1 = -\frac{P}{2} \cdot \frac{1}{s}$$

(U_1 , D_3 , L_2)

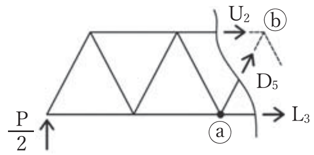


at ⓐ $U_1 h + \frac{P}{2}\lambda = 0$ $\qquad U_1 = -\frac{P}{2} \cdot \frac{\lambda}{h}$

at ⓑ $L_2 h = \frac{P}{2} \cdot \frac{3\lambda}{2}$ $\qquad L_2 = \frac{3}{4}P \cdot \frac{\lambda}{h}$

$D_3 s + \frac{P}{2} = 0$ $\qquad D_3 = -\frac{P}{2} \cdot \frac{1}{s}$

【10-5-3】(7)



at ⓐ $U_2 h + \frac{P}{2}(2\lambda) = 0$ $\qquad U_2 = -P\frac{\lambda}{\mathrm{h}}$

at ⓑ $L_3 h = \frac{P}{2} \cdot \frac{5}{2}\lambda$ $\qquad L_3 = \frac{5}{4}P\frac{\lambda}{\mathrm{h}}$

$D_5 \cdot S + \frac{P}{2} = 0$ $\qquad D_5 = -\frac{P}{2} \cdot \frac{1}{S}$

【10-5-3】(8)

《 A3 》

1) $m = 9$
$j = 6$
$r = 3$

[*I*] $(m = 9) = (2j = 12) - 3 = 9$ → *stable , determinate*

[*E*] $r = 3$ → *stable , determinate*

[*T*] $(m + r = 12) = 2 \times 6 = 12$ → *stable , determinate*

2) $m = 22$
$j = 12$
$r = 4$

[*I*] $(m = 22) > (2j = 24) - 3 = 21$ → *stable* ($n_i = 1$)

[*E*] $r = 4 > 3$ → *stable* ($n_r = 1$)

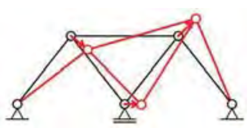
[*T*] $(m + r = 26) > (2j = 24)$ → *stable* ($n_t = 2$)

3) $m = 5$
$j = 5$
$r = 5$

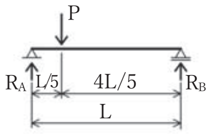
[*I*] $m = 5 < (2j = 10) - 3 = 7$ → *unstable*

[*E*] $r = 5 > 3$

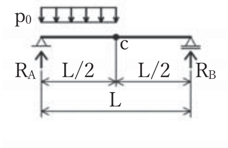
[*T*] $(m + r = 10) = (2j = 10)$



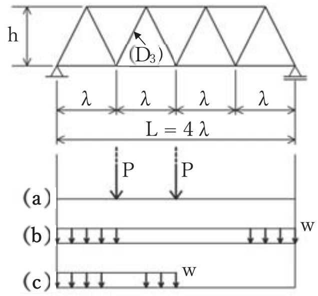
(Q1) Reaction (R_A) is (4P/5). Using influence line, conform it.



(Q2) Reaction (R_B) is ($p_0L/8$) and moment (M_C) is ($p_0L^2/16$). Using influence line, conform them.



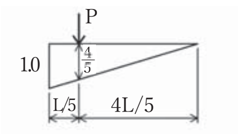
(Q3) The lower deck type truss is subjected to three types of loading. Find axial force in the diagonal member (D_3) using influence line.



$$S = \sin\theta$$
$$= h/\sqrt{h^2 + (\lambda/2)^2}$$

《 A1 》

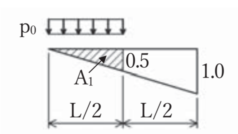
Influence line of R_A is as follows.



$$R_A = P\eta = \frac{4}{5}P$$

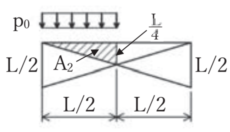
《 A2 》

Influence line of R_B , M_C are as follows.



$$R_B = p_0 A_1 = \frac{P_0 L}{8}$$

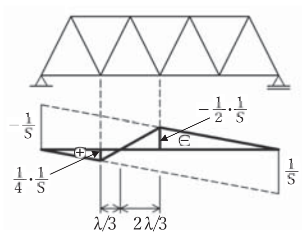
$$A_1 = \frac{1}{2} \times 0.5 \times \frac{L}{2} = \frac{L}{8}$$



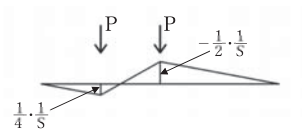
$$M_C = p_0 A_2 = \frac{P_0 L^2}{16}$$

$$A_2 = \frac{1}{2} \times \frac{L}{4} \times \frac{L}{2} = \frac{L^2}{16}$$

《 A3 》

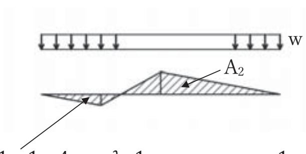


(*a*)



$$D_3 = P\left(\frac{1}{4}\cdot\frac{1}{S} - \frac{1}{2}\cdot\frac{1}{S}\right) = \underline{-\frac{P}{4}\cdot\frac{1}{S}}$$

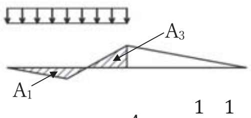
(*b*)



$$A_1 = \frac{1}{2}\cdot\frac{1}{4S}\cdot\frac{4}{3}\lambda = \frac{\lambda}{6}\cdot\frac{1}{S} \qquad A_2 = -\frac{1}{2}\cdot\frac{1}{2S}\cdot\frac{8}{3}\lambda = \underline{-\frac{2\lambda}{3}\cdot\frac{1}{S}}$$

$$D_3 = w(A_1 + A_2) = w\left(\frac{\lambda}{6}\cdot\frac{1}{S} - \frac{2\lambda}{3}\cdot\frac{1}{S}\right) = -\frac{w\lambda}{2}\cdot\frac{1}{S}$$

(*c*)

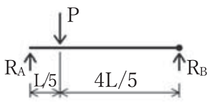


$$A_3 = -\frac{1}{2}\cdot\frac{1}{2S}\cdot\frac{2}{3}\lambda = -\frac{\lambda}{6}\cdot\frac{1}{S}$$

$$D_3 = \mathrm{w}(A_1 + A_3) = w\left(\frac{\lambda}{6}\cdot\frac{1}{S} - \frac{\lambda}{6}\cdot\frac{1}{S}\right) = \underline{0}$$

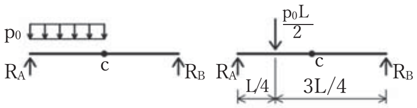
[***note***]

(Q1)



at Ⓑ $\quad R_A L = P \times \left(\frac{4}{5}L\right) = \frac{4}{5}PL \quad \rightarrow R_A = \frac{4}{5}P$

(Q2)

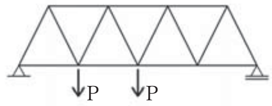


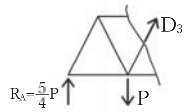
at Ⓐ $\quad R_B L = \frac{p_0 L}{2}\cdot\frac{L}{4} = \frac{p_0 L^2}{8} \quad \rightarrow R_B = \frac{p_0 L}{8}$

$$M_C = R_B\frac{L}{2} = \frac{p_0 L^2}{16}$$

(Q3)

(*a*)

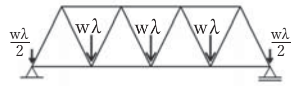


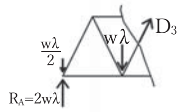


$$D_3 S + \frac{5}{4}\mathrm{P} = P$$

$$\rightarrow D_3 = -\frac{P}{4}\cdot\frac{1}{S}$$

(*b*)

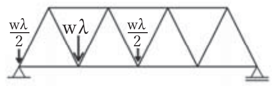


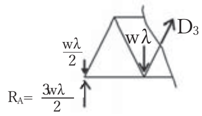


$$D_3 S + 2w\lambda = \frac{w\lambda}{2} + w\lambda$$

$$\rightarrow D_3 = -\frac{w\lambda}{2}\cdot\frac{1}{S}$$

(*c*)





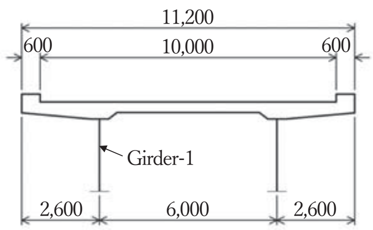
$$D_3 S + \frac{3w\lambda}{2} = \frac{w\lambda}{2} + \mathrm{w}\lambda$$

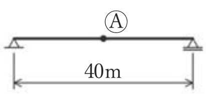
$$\rightarrow D_S = 0$$

(Q1) The member is subjected to tension.

1,000 kN (under dead load)
1,400 kN (under live load)
200 kN (under temperature change)
the cross-sectional area of the member is 180 cm², and the material grade is SM400 (σ_y=235 N/mm², σ_a=140 N/mm²).
Check the safety

(Q2)





(***a***) *Find live load to Girder-1*

(***b***) *Find design bending moment and shear force* at Ⓐ *(Girder-1)*

《 A1 》

$$\sigma_D = \frac{1{,}000 \times 10^3}{180 \times 10^2} = 55.6\ N/mm^2 \quad (dead\ load)$$

$$\sigma_L = \frac{1{,}400 \times 10^3}{180 \times 10^2} = 77.8\ N/mm^2 \quad (live\ load)$$

$$\sigma_T = \frac{200 \times 10^3}{180 \times 10^2} = 11.1\ N/mm^2 \quad (temperature\ change)$$

$$\sigma_D + \sigma_L = 133.4\ N/mm^2 < \sigma_a = 140\ N/mm^2$$

$$\sigma_D + \sigma_L + \sigma_T = 144.5\ N/mm^2 < 140 \times 1.15 = 161\ N/mm^2$$

(in case of check using stress resultants)

$N_D = 1{,}000\ kN$

$N_L = 1{,}400\ kN$

$N_T = 200\ kN$

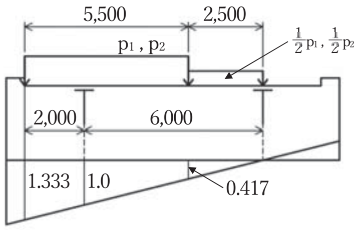
$N_{ult.} = \sigma_y A = 4{,}230\ kN$

$N_a = N_{ul}t./1.7 = 2{,}488\ kN$

$N_D + N_L = 2{,}400\ kN < N_a$

$N_D + N_L + N_T = 2{,}600\ kN < N_a \times 1.15 = 2{,}861\ kN$

《 A2 》



(*a*)

1] Distributed load (p1)

1-1 For bending moment (p1=10 kN/m²)

$$\frac{1.333+0.417}{2}\times 5.5m\times 10kN/m^2 = 48.13\ kN/m$$

$$\frac{0.417}{2}\times 2.5m\times\frac{10}{2}kN/m^2 = \ 2.61\ kN/m$$

$$\Sigma \quad 50.74\ kN/m$$

1-2 For shear force (p1=12 kN/m²)

$$\Sigma \quad 50.74\times\frac{12}{10} = 60.89\ kN/m$$

2] Distributed load (p2=3.5 kN/m²) (L<80m)

$$\Sigma \quad 50.74\times\frac{3.5}{10} = 17.76\ kN/m$$

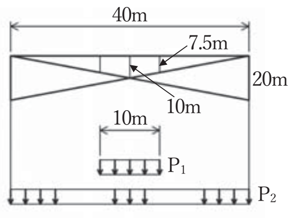
(*b*)

1] impact

$$i = \frac{20}{50+L} = \frac{20}{90} = 0.222 \quad (L = 40m)$$

2] Influence line and loading

2-1 Design bending moment by live load



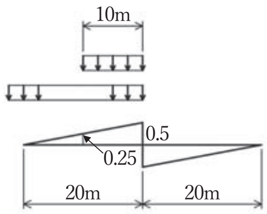
$$M = \frac{1}{2}\times 10m\times 40m\times 17.76\ kN/m\times(1+0.222)$$

$$+2\times\left(\frac{10+7.5}{2}\right)m\ \times 5m\times 50.74\ kN/m\times(1+0.222)$$

$$= 4{,}340.5 + 5{,}425.4$$

$$= 9{,}765.9\ kN\cdot m$$

2-2 Design shear force by live load



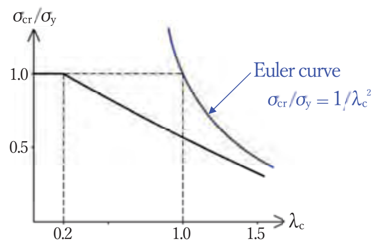
$$Q = \frac{1}{2}\times 0.5\times 20m\times 17.76\ kN/m\times(1+0.222)$$

$$+\frac{0.5+0.25}{2}\times 10m\times 60.89\ kN/m\times(1+0.222)$$

$$= 108.5 + 279.0$$

$$= 387.5\ kN$$

Strength (σ_{cr}) of columns (by JHBS)



$$\sigma_{cr}/\sigma_y = 1.0 \qquad (\lambda_c \leqq 0.2)$$

$$\sigma_{cr}/\sigma_y = 1.109 - 0.547\lambda_c \qquad (0.2 < \lambda_c \leqq 1.0)$$

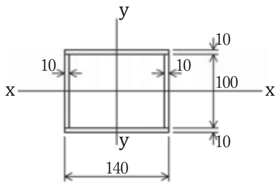
$$\sigma_{cr}/\sigma_y = \frac{1.0}{0.773+{\lambda_c}^2} \qquad (1.0 < \lambda_c)$$

$$\lambda_c = \sqrt{\frac{\sigma_y}{\sigma_E}}$$

$$\sigma_a = \sigma_{cr}/\gamma \quad (\gamma \cong 1.7)$$

【11-3-1】(2)

(Q1) Find elastic buckling stress (σ_E) and strength (σ_{cr}) of columns with a height of 5,000 mm, and with support (boundary) conditions { (a):PIN-PIN , (b):FIX-FIX }. The material grade is SM400(σ_y=235 N/mm²).



【11-3-1】(3)

《 A1 》

$$A = 2\times 14\times 1 + 2\times 10\times 1 = 48\ cm^2$$

$$I_x = 2\times 14\times 1\times 5.5^2 + 2\times\frac{10^3\times 1}{12} = 1{,}013.7\ cm^4$$

$$I_y = 2\times 10\times 1\times 6.5^2 + 2\times\frac{14^3\times 1}{12} = 1{,}302.3\ cm^4 > I_x$$

($\boldsymbol{a}$) $PIN-PIN\ support\ (\ L_e = 5{,}000\ mm\ )$

$$P_E = \frac{\pi^2}{L^2}EI = \frac{\pi^2}{(5{,}000)^2}\times 2.0\times 10^5\times 1{,}013.7\times 10^4$$

$$= 7.996\times 10^5\ (N)$$

$$\sigma_E = \frac{P_E}{A} = \frac{7.996\times 10^5}{48\times 10^2} = 166.6\ (N/mm^2)$$

$$\lambda_c = \sqrt{\frac{\sigma_y}{\sigma_E}} = \sqrt{\frac{235}{166.6}} = 1.190\ (>1.0)$$

$$\sigma_{cr}/\sigma_y = \frac{1.0}{0.773+{\lambda_c}^2} = 0.457$$

$$\sigma_{cr} = 0.457\ \sigma_y = 107.4\ (N/mm^2)$$

$$\sigma_a = \sigma_{cr}/1.7 = \underline{\underline{63.2\ (N/mm^2)}}$$

JHBS

$$\gamma = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{1{,}013.7}{48}} = 4.596\ (\ cm\ )$$

$$L_e/\gamma = \frac{500}{4.596} = 108.8$$

$$\sigma_a = \frac{1{,}200{,}000}{6{,}700+(L_e/\gamma)^2} = \underline{\underline{64.7\ (N/mm^2)}}$$

【11-3-1】(4)

($\boldsymbol{b}$) $FIX-FIX\ support\ (\ \underline{L_e = 2{,}500\ mm}\ )$

$$P_E = 4P_{E(PIN-PIN)} = 4\times 7.996\times 10^5 = 3.198\times 10^6\ (N)$$

$$\sigma_E = 4\sigma_{E(PIN-PIN)} = 4\times 166.6 = 666.4\ (N/mm^2)$$

$$\lambda_c = \sqrt{\frac{\sigma_y}{\sigma_E}} = \sqrt{\frac{235}{666.4}} = 0.594$$

$since$, $(\ 0.2 < \lambda_c \le 1.0\ )$

$$\sigma_{cr}/\sigma_y = 1.109 - 0.547\lambda_c = 0.784$$

$$\sigma_{cr} = 0.784\ \sigma_y = 184.3\ (N/mm^2)$$

$$\sigma_a = \sigma_{cr}/1.7 = \underline{\underline{108.4\ (N/mm^2)}}$$

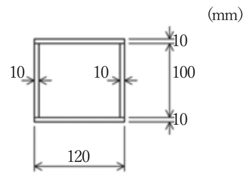
JHBS

$$L_e/\gamma = \frac{250}{4.596} = 54.7$$

$$\sigma_a = 140 - 0.82\left(\frac{L_e}{\gamma} - 18\right) = \underline{\underline{109.9\ (N/mm^2)}}$$

【11-3-1】(5)

※ Check of plate strength



$$b = 100\ mm\ ,\ \ t = 10\ mm$$

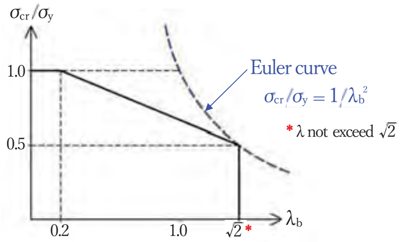
$$\sigma_E = 4.0\times\frac{\pi^2 E}{12(1-V^2)}\cdot\left(\frac{t}{b}\right)^2 = 722{,}315\left(\frac{t}{b}\right)^2$$

$$= 7223.2\ (N/mm^2)$$

$$R = \sqrt{\frac{\sigma_y}{\sigma_E}} = \sqrt{\frac{235}{7223.2}} = 0.180 < 0.5$$

$$\rightarrow \sigma_{cr} = \sigma_y$$

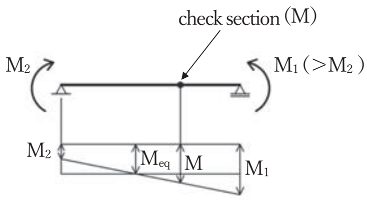
Strength (σ_{cr}) of beams (by JHBS)



$\sigma_{cr}/\sigma_y = 1.0 \quad (\lambda_b \leqq 0.2)$

$\sigma_{cr}/\sigma_y = 1.0 - 0.412(\lambda_b - 0.2) \quad (0.2 < \lambda_b \leqq \sqrt{2})$

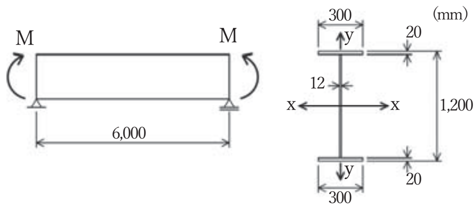
In case that M varies between fix point,



$M_{eq.} = max.\{(0.6M_1 + 0.4M_2), (0.4M_1)\}$

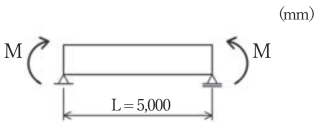
σ_a *can be increased to* $\{(M/M_{eq})\,\sigma_a\}$

(Q1) The simply supported beam with laterally constrained at supports is subjected to bending moment(M). Find elastic buckling moment(M_E) and ultimate moment(M_{cr}).
The material grade is SM400 (σ_y=235 N/mm²).

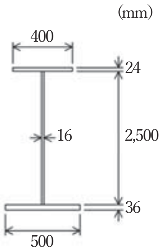


(Q2) Find increment coefficient of allowable stress. The beam is given in (Q1) , and subjected to the following moment.
The satisfy check section is Ⓑ

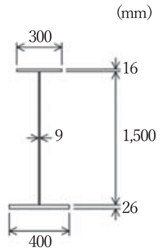
(Q3) The following beam is subjected to bending moment and laterally constrained at supports. Find allowable stress based on JHBS. The material grade is SM490Y.



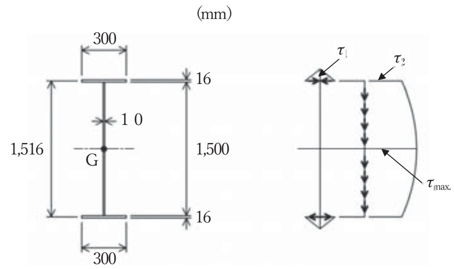
(a)



(b)



(Q4) Obtain shear stress under shear (Q).



《 A1 》

$$I_x = 2 \times 30 \times 2 \times 59^2 + \frac{1.2 \times 116^3}{12} = 573{,}810\ cm^4$$

$$I_y = 2 \times \frac{30^3 \times 2}{12} + \frac{1.2^3 \times 120}{12} \cong 9{,}000\ cm^4$$

$$I_w \cong I_y \left(\frac{h}{2}\right)^2 = 9{,}000 \times \left(\frac{120}{2}\right)^2 = 32{,}400{,}000\ cm^6$$

(I_w : *warping constant*)

$$M_E = \frac{\pi}{L}\sqrt{EI_yGJ\left(1 + \frac{\pi^2 \times EI_w}{L^2GJ}\right)}$$

$$\cong \left(\frac{\pi}{L}\right)^2 E\sqrt{I_yI_w}$$

$$= \left(\frac{\pi}{6{,}000}\right)^2 \times 2 \times 10^5 \times \sqrt{9 \times 10^7 \times 3.24 \times 10^{13}}$$

$$= \underline{2.958 \times 10^9\ N \cdot mm}$$

$$since\ ,\ W_x = \frac{5.738 \times 10^9}{600} = 9.56 \times 10^6$$

$$M_y = 235 \times W_x = 235 \times 9.56 \times 10^6 = 2.247 \times 10^9\ N \cdot mm$$

$$\lambda_b = \sqrt{\frac{M_y}{M_E}} = \sqrt{\frac{2.247 \times 10^9}{2.958 \times 10^9}} = 0.872\ (> 0.2)$$

$$\sigma_{cr}/\sigma_y = 1.0 - 0.412(\lambda_b - 0.2) = 0.723$$

$$\sigma_{cr} = 0.723\ \sigma_y = 169.9\ N/mm^2$$

$$M_{cr} = \sigma_{cr}\ W_x = 169.9 \times 9.56 \times 10^6 = \underline{1.624\ (MN \cdot m)}$$

$$\sigma_{ba} = \sigma_{cr}/1.7 = \underline{\underline{99.9\ N/mm^2}}$$

by JHBS

$$A_c = 30 \times 2.0 = 60\ cm^2\ ,\ A_w = 120 \times 1.2 = 144\ cm^2$$

$$A_w/A_c = 144/60 = 2.4 > 2.0$$

$$K = \sqrt{3 + A_w/(2A_c)} = 2.05$$

$$\frac{9}{\mathrm{K}}(= 4.5) < \frac{L}{b}(= 20) < 30$$

$$\sigma_{ba} = 140 - 1.2\left(K\frac{L}{b} - 9\right) = 101.6\ N/mm^2$$

《 A2 》

$$M_{eq} = 0.6M_1 + 0.4M_2 = 0.8M$$

$$M_{eq} = 0.4M_1 = 0.4M$$

$$\downarrow$$

$$M_{eq} = 0.8M$$

$$M/M_{eq} = 1.25$$

$\underline{\sigma_{ba}\ can\ be\ increased\ \ 1.25}\ \sigma_{ba}$

《 A3 》

(***a***) $L/b = 5{,}000/400 = 12.5$

$$A_C = 40 \times 2.4 = 96\ cm^2\ , \quad A_w = 250 \times 1.6 = 400\ cm^2$$

$$A_w/A_C = 400/96 = 4.17 \geq 2$$

$$K = \sqrt{3 + \frac{A_w}{2A_C}} = 2.25$$

$$\frac{7}{k}(= 3.1) < \frac{L}{b}(= 12.5) < 27$$

$$\underline{\sigma_{ba}} = 210 - 2.3\left(K\frac{L}{b} - 7\right) = \underline{161.4\ (N/mm^2)}$$

(***b***) $L/b = 5{,}000/300 = 16.7$

$$A_C = 30 \times 1.6 = 48\ cm^2\ , \quad A_w = 150 \times 0.9 = 135\ cm^2$$

$$A_w/A_C = 135/48 = 2.81 \geq 2$$

$$K = \sqrt{3 + \frac{A_w}{2A_C}} = 2.10$$

$$\frac{7}{k}(= 3.33) < \frac{L}{b}(= 16.7) < 27$$

$$\underline{\sigma_{ba}} = 210 - 2.3\left(K\frac{L}{b} - 7\right) = \underline{145.4\ (N/mm^2)}$$

《 A4 》

$$I = 2 \times 30 \times 1.6 \times 75.8^2 + \frac{150^3 \times 1}{12}$$

$$= 832{,}831.4\ (cm^4)\ \ (\rightarrow 8.328 \times 10^9\ mm^4)$$

$$\tau_1 = \frac{Q}{I} \times \frac{300 \times 1{,}516}{4} = 1.365 \times 10^{-5}Q$$

$$\tau_2 = \frac{Q}{I} \times \frac{300 \times 1{,}516}{2} \times \frac{16}{10} = 4.369 \times 10^{-5}Q$$

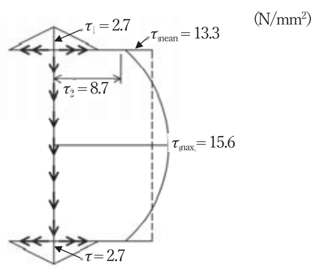
$$\tau_{max.} = \frac{Q}{I} \times \left[\frac{1{,}516^2}{8} + \frac{300 \times 1{,}516}{2} \times \frac{16}{10}\right] = 7.818 \times 10^{-5}Q$$

$$\tau_{mean} = \frac{Q}{A_w} = \frac{Q}{1{,}500 \times 10} = 6.667 \times 10^{-5}Q$$

when $Q = 200\ kN\ \ (2 \times 10^5\ N)$,

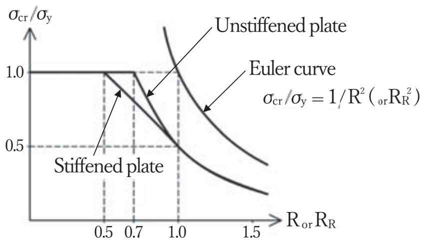
$\tau_1 = 2.7\ N/mm^2$, $\tau_2 = 8.7\ N/mm^2$, $\tau_{max.} = 15.6\ N/mm^2$

$\tau_{mean} = 13.3\ N/mm^2$



【11-3-3】(1)

Strength of plate (by JHBS)



Unstiffened plate

$\sigma_{cr}/\sigma_y = 1.0$ ($R \leqq 0.7$)

$\sigma_{cr}/\sigma_y = 0.5/R^2$ ($0.7 < R$)

Stiffened plate

$\sigma_{cr}/\sigma_y = 1.0$ ($R_R \leqq 0.5$)

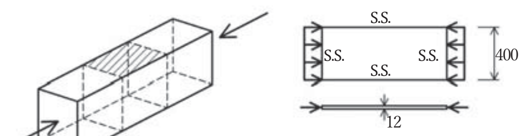
$\sigma_{cr}/\sigma_y = 1.5 - R_R$ ($0.5 < R_R \leqq 1.0$)

$\sigma_{cr}/\sigma_y = 0.5/{R_R}^2$ ($1.0 < R_R$)

【11-3-3】(2)

(Q1) Obtain the ultimate strength(σ_{cr}) of the following plates. The material grade is SM400 (σ_y=235 N/mm^2).

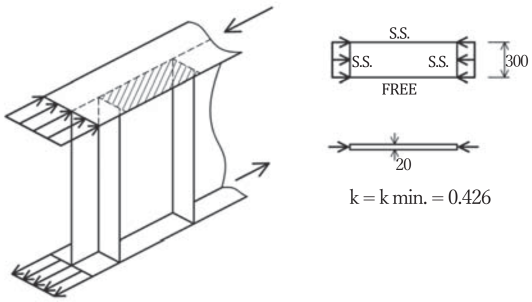
(*a*)



k = k min. = 4.0

S.S. = Simple support

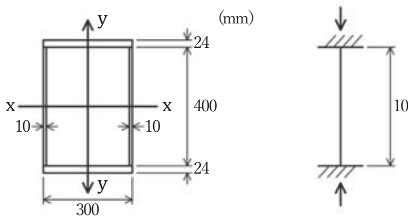
(*b*)



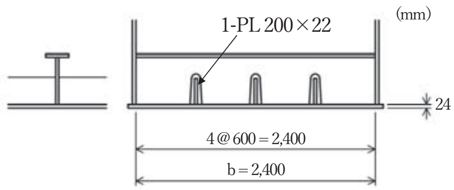
k = k min. = 0.426

【11-3-3】(3)

(Q2) Find the allowable stress ($\sigma_a = \sigma_{cr}/1.7$) of the following column with height of 10m and (FIX-FIX) support. The material grade is SM490Y (σ_y =355 N/mm^2)



(Q3) Check the safety of the following stiffened plate.
The material grade is SM400 (σ_y=235 N/mm^2)

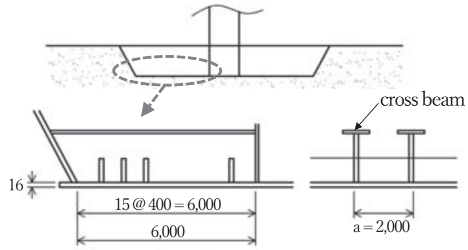


pitch of cross beam $(a) = 2{,}880mm$

$$\alpha = \frac{a}{b} = \frac{2{,}880}{2{,}400} = 1.2$$

【11-3-3】(4)

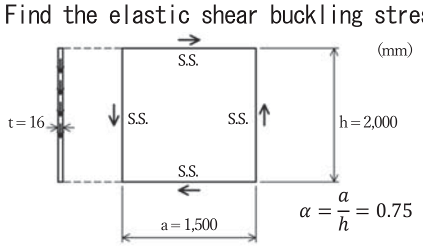
(Q4) Design the following lower flange.



$$\alpha = \frac{2{,}000}{6{,}000} = 0.33$$

(***a***) *Find the allowable stress of the lower flange.*
(***b***) *Design the longitudinal rib.*
(***c***) *Design the cross beam.*

(Q5) Find the elastic shear buckling stress (τ_E)



$$\alpha = \frac{a}{h} = 0.75$$

$$\tau_E = k_\tau \frac{\pi^2 E}{12(1 - V^2)} \left(\frac{t}{h}\right)^2$$

$$k_\tau = 5.34 + 4.00\,(h/a)^2 \qquad a/h > 1.0$$
$$= 4.00 + 5.34\,(h/a)^2 \qquad a/h \leqq 1.0$$

(Q6) Find the ultimate strength($\tau_{ult.}$) of the (Q3) plate using Basler's formula.
The material grade is SM400

$$\left(\tau_y = \frac{\sigma_y}{\sqrt{3}} = 135\ N/mm^2 \right)$$

$$\frac{\tau_{ult.}}{\tau_y} = \frac{\tau_{cr}}{\tau_y} + \underbrace{\frac{\sqrt{3}}{2} \cdot \frac{\left(1 - \frac{\tau_{cr}}{\tau_y}\right)}{\sqrt{1+\alpha^2}}}_{(\,post\ buckling\ strength\,)}$$

$$\tau_{cr} = \tau_E \qquad (\qquad \tau_E \leqq 0.8\,\tau_y\,)$$

$$\tau_{cr} = \sqrt{0.8\,\tau_y\ \tau_E} \qquad (\,0.8\,\tau_y < \tau_E \qquad)$$

《 A1 》

$$\sigma_E = k \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$$

$E = 2 \times 10^5$ (N/mm^2)

ν (*Poisson's ratio*) $= 0.3$

(***a***) $b = 400$, $t = 12$, $k = 4.0$

$$\sigma_E = 722{,}315 \left(\frac{t}{b}\right)^2 = 650\ N/mm^2$$

$$R = \sqrt{\frac{\sigma_y}{\sigma_E}} = \sqrt{\frac{235}{650}} = 0.60 < 0.70$$

$\sigma_{cr}/\sigma_y = 1.0 \rightarrow \underline{\sigma_{cr} = \sigma_y = 235\ N/mm^2}$

$\underline{\sigma_a = \sigma_{cr}/1.7 \cong 140\ N/mm^2}$

(***b***) $b = 300$, $t = 20$, $k = 0.426$

$$\sigma_E = 76{,}927 \left(\frac{t}{b}\right)^2 = 341\ N/mm^2$$

$$R = \sqrt{\frac{\sigma_y}{\sigma_E}} = \sqrt{\frac{235}{341}} = 0.83 > 0.70$$

$\sigma_{cr}/\sigma_y = 0.5/R^2 = 0.726$

$\underline{\sigma_{cr} = 0.726\ \sigma_y = 170.6\ N/mm^2}$

$\underline{\sigma_a = \sigma_{cr}/1.7 = 100.4\ N/mm^2}$

Based on JHBS

(***a***) $t\,(=12) > \dfrac{b\,(=400)}{38.7} = 10.3 \rightarrow \underline{\sigma_a = 140\ N/mm^2}$

(***b***) $\dfrac{b\,(=300)}{16} = 18.8 < 20 < \dfrac{b}{12.8} (= 23.4)$

$\rightarrow \sigma_a = 23{,}000(t/b)^2 = \underline{102.2\ N/mm^2}$

《 A2 》

1) ***Column strength***

$A = 2 \times 30 \times 2.4 + 2 \times 40 \times 1 = 224\ cm^2$ ($22{,}400\ mm^2$)

$$I_x = 2 \times 30 \times 2.4 \times 21.2^2 + 2 \times \frac{40^3 \times 1}{12} = 75{,}386\ cm^4$$

$$I_y = 2 \times 40 \times 1.0 \times 14.5^2 + 2 \times \frac{30^3 \times 2.4}{12} = 27{,}620\ cm^4 < I_x$$

$$P_E = \frac{\pi^2}{{L_e}^2} E I_y = \frac{\pi^2}{(5{,}000)^2} \times 2 \times 10^5 \times 2.762 \times 10^8 = 21{,}785{,}772 (N)$$

$$\sigma_E = \frac{P_E}{A} = \frac{21{,}785{,}772}{22{,}400} = 972.6\ (\,N/mm^2\,)$$

$$\lambda_C = \sqrt{\frac{\sigma_y}{\sigma_E}} = \sqrt{\frac{355}{972.6}} = 0.604\ (\,0.2 < \lambda_C < 1.0\,)$$

$\sigma_{cr}/\sigma_y = 1.109 - 0.547\,\lambda_C = 0.779 \rightarrow \sigma_{cr}\,(C) = 0.779\,\sigma_y$

2) ***Plate*** $(\mathbf{400 \times 10})$ ***strength***

$$\sigma_E = 4 \times \frac{\pi^2 E}{12(1-\nu^2)} \times \left(\frac{t}{b}\right)^2 = 722{,}315 \left(\frac{t}{b}\right)^2$$

$$= 722{,}315 \left(\frac{10}{400}\right)^2 = 451.4\ N/mm^2$$

$$R = \sqrt{\frac{355}{451.4}} = 0.887 > 0.7$$

$\sigma_{cr}/\sigma_y = 0.5/R^2 = 0.636 \rightarrow \sigma_{cr}\,(P) = 0.636\,\sigma_y$

3) ***Coupled strength***

$\sigma_{cr} = \sigma_{cr}\,(C) \times \sigma_{cr}\,(P)/\sigma_y = 0.495\,\sigma_y = 175.9\ N/mm^2$

$\underline{\sigma_a = \sigma_{cr}/1.7 = 103.5\ N/mm^2}$

《 A3 》

$$\sigma_E = (k_R = 4n^2) \cdot \frac{\pi^2 E}{12(1-V^2)} \left(\frac{t}{b}\right)^2$$

$b = 2{,}400mm$, $t = 24mm$, $n = 4$

$$= 11{,}557{,}040 \left(\frac{t}{b}\right)^2 = 1{,}155.7\ (\,N/mm^2\,)$$

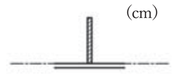
$$R = \sqrt{\frac{\sigma_y}{\sigma_E}} = \sqrt{\frac{235}{1{,}155.7}} = 0.45 < 0.5$$

$\sigma_{cr}/\sigma_y = 1.0 \rightarrow \underline{\sigma_{cr} = \sigma_y = 235\ (\,N/mm^2\,)}$

$\underline{\sigma_a = \sigma_{cr}/1.7 \cong 140\ (\,N/mm^2\,)}$

※ ***Check of longitudinal rib*** $(\mathbf{1 - PL\ 200 \times 22})$

(cm)



$$I_\ell = \frac{20^3 \times 2.2}{3} = 5{,}866.7\ cm^4$$

$$A_\ell = 20 \times 2.2 = 44\ cm^2 > \frac{bt}{10n} = \frac{240 \times 2.4}{10 \times 4} = 14.4\ cm^2$$

$$\delta_\ell = \frac{A_\ell}{bt} = \frac{44}{240 \times 2.4} = 0.0764$$

$$\gamma_\ell = \frac{I_\ell}{bt^3/11} = \frac{5{,}866.7}{(240 \times 2.4^3)/11} = 19.5$$

$$\alpha_0 = \sqrt[4]{1 + n \times \gamma_\ell} = \sqrt[4]{1 + 4 \times 19.5} = 2.98$$

$(\,\alpha\,(=1.2) < \alpha_0\,)$

$$t_0 = \frac{2{,}400}{28 \times 4} = 21.4\ mm$$

$$\gamma_{\ell,reg.} = 4\alpha^2 n\left(\frac{t_0}{t}\right)^2 (1+n\delta_\ell) - \frac{(\alpha^2+1)^2}{n}$$

$$= 4\times1.2^2\times4\times\left(\frac{21.4}{24}\right)^2\times(1+4\times0.0764)$$

$$-\frac{(1.2^2+1)^2}{4} = 22.4$$

$$\underline{I_\ell(=5{,}866.7\ cm^4) < \frac{bt^3}{11}\gamma_{\ell,reg.} = 6{,}765.2\ cm^4}$$

Out !!

※ ***Change size of longitudinal rib to (220 × 22)***

$$I_\ell = \frac{22^3\times2.2}{3} = 7{,}808.5\ cm^4$$

$$A_\ell = 22\times2.2 = 48.4\ cm^2 > \frac{bt}{10^n} = 14.4\ cm^2$$

$$\delta_\ell = \frac{A_\ell}{bt} = \frac{48.4}{240\times2.4} = 0.084$$

$$\gamma_\ell = \frac{I_\ell}{bt^3/11} = \frac{7{,}808.5}{(240\times2.4^3)/11} = 25.9$$

$$\alpha_0 = \sqrt[4]{1+n\times\gamma_\ell} = \sqrt[4]{1+4\times25.9} = 3.20 > \alpha\ (=1.2)$$

$$\gamma_{\ell,reg.} = 4\times1.2^2\times4\times\left(\frac{21.4}{24}\right)^2\times(1+4\times0.084)$$

$$-\frac{(1.2^2+1)^2}{4} = 23.0$$

$$\underline{I_\ell(=7{,}808.5\ cm^4) > \frac{bt^3}{11}\gamma_{\ell,reg.} = 6{,}937\ cm^4}$$

Ok !!

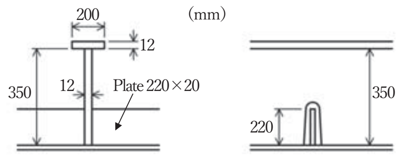
※ ***Check of ribs.*** (***plate 220 × 22 : SM400***)

$$t\ (=22mm) > \frac{b\ (=220)}{12.8} = 17.2\ mm$$

$$\rightarrow \underline{\sigma_a = 140\ N/mm^2} \leftarrow \boldsymbol{rib}$$

※※

Cross beam



$$I_t = \frac{35^3\times1.2}{3} + 20\times1.2\times35.6^2 = 47{,}567\ cm^4$$

$$I_t \geqq \frac{bt^3}{11}\times\frac{1+n\,\gamma_{\ell,reg.}}{4\alpha^3}$$

$$I_t\ (=47{,}567\ cm^4) \geqq \frac{240\times2.4^3}{11}\times\frac{1+4\times23}{4\times1.2^3}$$

$$= 4{,}058.2\ cm^4$$

《 A4 》

(***a***) $\sigma_E = 4n^2\dfrac{\pi^2E}{12(1-V^2)}\left(\dfrac{t}{b}\right)^2$

$(n = 15\ ,\ b = 6{,}000mm\ ,\ t = 16mm)$

$$= 4\times15^2\times\frac{\pi^2E}{12(1-\nu^2)}\times\left(\frac{16}{6{,}000}\right)^2 = 1{,}155.7\ N/mm^2$$

$$R_p = \sqrt{\frac{\sigma_y}{\sigma_E}} = \sqrt{\frac{355}{1{,}155.7}} = 0.554 > 0.5$$

$$\sigma_{cr}/\sigma_y = 1.5 - R_p = 0.946 \rightarrow \sigma_{cr} = 0.946\,\sigma_y = 335.8\ N/mm^2$$

$$\underline{\sigma_a = \sigma_{cr}/1.7 = 197.5\ N/mm^2}$$

by JHBS

$$\frac{b}{46fn}\left(=\frac{6{,}000}{46\times1\times15} = 8.7\ mm\right) < t\ (=16mm)$$

$$< \frac{b}{22fn}\left(=\frac{6{,}000}{22\times1\times15} = 18.2\ mm\right)$$

$$\sigma_a = 210 - 4.6\left(\frac{b}{tfn} - 22\right) = 210 - 4.6\left(\frac{6{,}000}{16\times1\times15} - 22\right)$$

$$= \underline{196.2\ N/mm^2}$$

(***b***) *Longitudinal rib*

Assume plate 190 × 20 (*SM490Y*)

$$A_\ell = 19\times2.0 = 38\ cm^2\ ,\quad \delta_\ell = \frac{A_\ell}{bt} = \frac{38}{6{,}000\times1.6} = 0.0396$$

$$I_\ell = \frac{19^3\times2.0}{3} = 4{,}573\ cm^4$$

$$\gamma_\ell = \frac{I_\ell}{bt^3/11} = \frac{4{,}573}{(600\times1.6^3)/11} = 20.5$$

$$\alpha_0 = \sqrt[4]{1+n\times\gamma_\ell} = \sqrt[4]{1+15\times20.5} = 4.19 > \alpha\ (=0.33)$$

Since $\boldsymbol{R_p > 0.5}$

$$\gamma_{\ell,req.} = 4\alpha^2 n\,(1+n\delta_\ell) - \frac{(\alpha^2+1)^2}{n}$$

$$= 4\times0.33^2\times15\times(1+15\times0.0396) - \frac{(0.33^2+1)^2}{15} = 10.3$$

$$A_\ell(=38\ cm^2) \geqq \frac{bt}{10n} = \frac{600\times1.6}{10\times15} = 6.4\ cm^2$$

$$I_\ell(=4{,}573\ cm^4) \geqq \frac{bt^3}{11}\gamma_{\ell,req.} = \frac{600\times1.6^3}{11}\times10.3 = 2{,}301\ cm^4$$

Since a little bit conservative , we select more smaller plate for ribs. It is ***<u>160 × 16 (SM490Y).</u>***

$$A_\ell = 16\times1.6\times25.6\ cm^2\quad ,\quad \delta_\ell = \frac{25.6}{600\times1.6} = 0.027$$

$$I_\ell = \frac{16^3\times1.6}{3} = 2{,}185\ cm^4$$

$$\gamma_\ell = \frac{I_\ell}{bt^3/11} = \frac{2{,}185}{(600\times1.6^3)/11} = 9.8$$

$$\alpha_0 = \sqrt[4]{1+n\times\gamma_\ell} = \sqrt[4]{1+15\times9.8} = 3.49 > \alpha\ (=0.33)$$

$$\gamma_{\ell,req.} = 4\alpha^2 n(1+n\delta_\ell) - \frac{(\alpha^2+1)^2}{n}$$

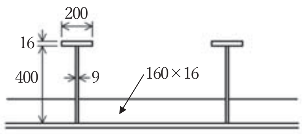
$$= 4\times0.33^2\times15\times(1+15\times0.027) - \frac{(0.33^2+1)^2}{15} = 9.1$$

$$A_\ell (= 25.6\ cm^2)\ >\ \frac{bt}{10n} = \frac{600 \times 1.6}{10 \times 15} = 6.4\ cm^2$$

$$\underline{I_\ell (= 2{,}185\ cm^4)}\ >\ \frac{bt^3}{11}\gamma_{\ell,req.} = \frac{600 \times 1.6^3}{11} \times 9.1 = \underline{2{,}033\ cm^4}$$

Plate 160 × 16 is employed for longitudinal ribs.

(*c*) ***Cross beam***

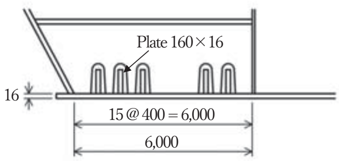


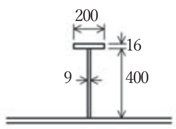
$$I_t = 20 \times 1.6 \times 40.8^2 + \frac{40^3 \times 0.9}{3} = 72{,}468\ cm^4$$

$$I_t\ (= 72{,}468\ cm^4\ )$$

$$> \frac{bt^3}{11} \times \frac{1 + n\,\gamma_{\ell,reg.}}{4\alpha^3} = \frac{600 \times 1.6^3}{11} \times \frac{1 + 15 \times 9.1}{4 \times 0.33^3}$$

$$= 70{,}533\ cm^4$$





《 A5 》

$$a/h\ < 1.0\ \ \rightarrow\ k_\tau = 4.00 + 5.34\ (h/a)^2 = 7.0$$

$$\tau_E = k_\tau \frac{\pi^2 E}{12(1 - V^2)} \cdot \left(\frac{t}{h}\right)^2 = 1{,}264{,}051 \left(\frac{t}{h}\right)^2 = \underline{80.9\ (\ N/mm^2\ )}$$

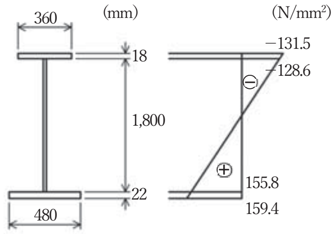
《 A6 》

$$\tau_E \leqq 0.8\ \tau_y = 108\ N/mm^2\ \ \rightarrow\ \ \tau_{cr} = \tau_E = 80.9\ N/mm^2$$

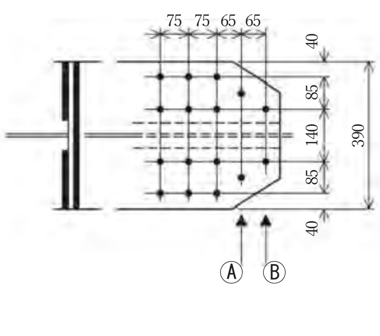
$$\frac{\tau_{ult}}{\tau_y} = \frac{80.9}{135} + \frac{\sqrt{3}}{2} \cdot \frac{\left(1 - \frac{80.9}{135}\right)}{\sqrt{1 + 0.75^2}} = 0.599 + 0.278 = 0.877$$

$$\tau_{ult} = 0.877\ \tau_y = 118.4\ N/mm^2$$

(Q1) Design the friction-type bolt (M22, F10T, 2-plane friction) connection of the following I-section. The material grade is SM490Y (σ_a=210 N/mm²)
The shear force is 295 kN.



(Q2) Find the net cross-sectional area at the Sections Ⓐ and Ⓑ.



《 A1 》

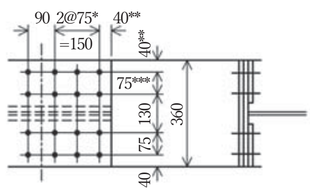
(1) *Connection of the upper flange.*

$$\sigma_U = -131.5\ <\ 0.75 \times 210 = 157.5\ N/mm^2$$

$\rightarrow$ *Design using 75% of full strength.*

・Number of bolts and arrangements.

$$\mathrm{M} = \frac{157.5 \times 360 \times 18}{96{,}000} = 10.6\ \rightarrow\ 12\ bolts$$



・Splice plate (SM490Y)

(*mm*)

1 − spl pl	360	×	9	=	3,240
2 − spl pl	155	×	10	=	3,100
	A spl			=	6,340

$$\sigma_{spL} = 157.5 \times \frac{360 \times 18}{6{,}340} = 161.0\ < 210\ (\ N/mm^2\ )$$

$*$ 75 ≤ *pitch*(= 75) < 150

$**$ *min. edge* (= 32) < 40

$***$ 75 ≤ *gauge* (= 75) < 24 + (= 24 × 9 = 216)

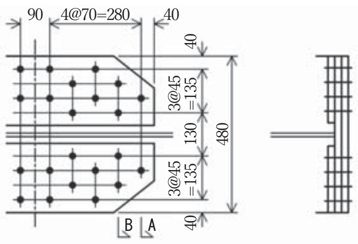
(2) *Connection of the lower flange.*

$$\sigma_L = 159.4 \;>\; 0.75 \times 210 = 157.5\ N/mm^2$$

$\rightarrow$ *Design using the design stress.*

・Number of bolt and arrangement.

$$M = \frac{159.4 \times 480 \times 22}{96{,}000} = 17.5 \;\rightarrow\; 18\ bolts$$



・Splice plate (**SM490Y**)

Check of plate to be connected.

－A section－

$$A_n = (480 - 2 \times 25) \times 22 = 9{,}460\ mm^2$$

$$\sigma_L = 159.4 \times \frac{480 \times 22}{9{,}460} = 177.9 \;<210\ (N/mm^2)$$

－B section－

$$A_n = (480 - 4 \times 25) \times 22 = 8{,}360\ mm^2$$

$$\sigma_L = 159.4 \times \frac{480 \times 22}{8{,}360} \times \left(\frac{16}{18}\right)^{*} = 180.0 \;<210\ (N/mm^2)$$

$*$ 2 − *bolt force already transferred to splice plate.*

$$A_{req.} = 480 \times 22 \times \frac{159.4}{210} = 8{,}016\ (mm^2)$$

1 − spl pl (480 − 4 × 25) × 14 = 5,320

2 − spl pl (215 − 2 × 25) × 14 = 4,620

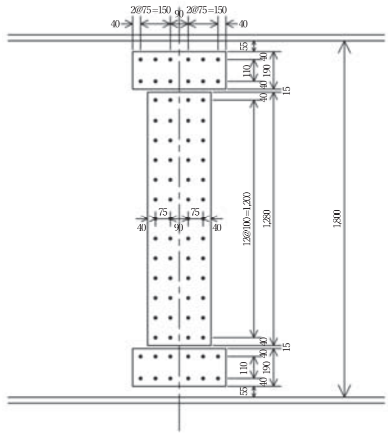
A spl = 9,940 (mm^2) > $A_{req.}$

$$\sigma_{spL} = 159.4 \times \frac{480 \times 22}{9{,}940} = 169.3 \;<210\ (N/mm^2)$$

$$\left(\omega = d - p^2/4g = 25 - \frac{70^2}{4 \times 45} = -2.22 \;<0\right)$$

(3) *Connection of the web.*

・Number of bolt and arrangement.



(***a***) *First row* (p : *Working force*)

$$P_1 = \frac{157.5 + 133.5}{2} \times (95 + 55) \times 9 = 196{,}425\ (N)$$

$$n_1 = \frac{196{,}425}{96{,}000} = 2.1 \;\rightarrow\; 3\ bolts$$

(***b***) *Third row*

$$P_3 = \frac{117.2 + 101.6}{2} \times (47.5 + 50.0) \times 9 = 95{,}999\ (N)$$

$$n_3 = \frac{95{,}999}{96{,}000} = 1.0 \;\rightarrow\; 2\ bolts$$

Total number of bolt is 38.

Safety under shear.

$$P_s = \frac{295 \times 10^3}{38} = 7{,}763\ (N) \;<\; P_a = 96{,}000\ (N)$$

Safety under combined moment and shear.

$$P_{P_1} = \frac{196{,}425}{3} = 65{,}475\ (N)$$

$$P = \sqrt{{P_{P_1}}^2 + {P_s}^2} = 65{,}934\ (N) \;<\; P_a = 96{,}000\ (N)$$

・Splice plate (**SM490Y**)

4 − spl pl 190 × 9 = 1,710 $*$

2 − spl pl 1,280 × 9 = 11,520 $*$

$*$ *Cross − sectional area of one plate.*

(***a***) *Moment of inertia of splice plate*

$$I_S = 2 \times \left(17.1 \times 66.4^2 + \frac{19.0^3 \times 0.9}{12} + 17.1 \times 83.6^2 \times \frac{19.0^3 \times 0.9}{12}\right) + 2 \times \left(115.2 \times 8.6^2 + \frac{128.0^3 \times 0.9}{12}\right)$$

$$= 723{,}479\ (cm^4) \;>\; I_W = \frac{180^3 \times 0.9}{12} = 437{,}400\ (cm^4)$$

(***b***) *Moment acting on splice plate*

$$M_S = \sigma_L \times \frac{I_S}{y_L}$$

$$= 157.5 \times \frac{1{,}800^3 \times 9/12 + 1{,}800 \times 9 \times 86^2}{986}$$

(***c***) *Fiber stress in splice plate*

$$\sigma_{spL} = \frac{7.18 \times 10^8}{7.235 \times 10^9} \times 931$$

$$= 92.4\ N/mm^2 \;< \sigma_{ta} = 210\ (N/mm^2)$$

《 A2 》

(***a***) *At section* Ⓐ

$$A_g = 39 \times 2.8 = 109.2\ cm^2$$

$$w\ = d - \frac{d^2}{4_g} = 2.5 - \frac{6.5^2}{4 \times 4.25} = 0.015\ >\ 0$$

$$A_n = A_g - 2\ \times\ (\,2.5 + 2w\,)\ \times\ 2.8$$

$$= 109.2 - 2\ \times\ (\,2.5 + 2 \times 0.015\,)\ \times\ 2.8$$

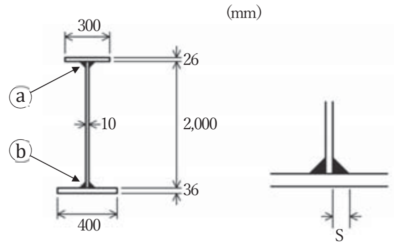
$$= 95\ cm^2$$

(***b***) *At section* Ⓑ

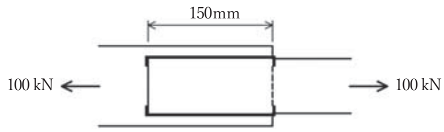
$$A_n = A_g - 2\ \times\ 2.5\ \times\ 2.8$$

$$= 109.2 - 14 = 95.2\ cm^2$$

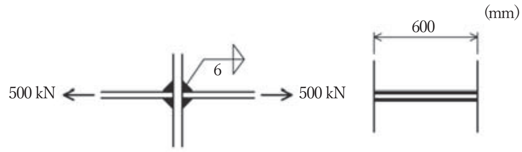
(Q1) Find the fillet welding size (S)



(Q2) Find the required size (S) of the fillet weld. The material grade is SM400.



(Q3) Check the safety.

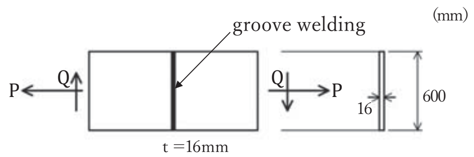


(Q4) A groove welding part is subjected to tension (P) and shear force (Q).
The material grade is SM400
(σ_a = 140 N/mm², τ_a = 80 N/mm²).
Check the safety.

(**1**) $P = 1{,}000\ kN$

(**2**) $Q = 650\ kN$

(**3**) $P = 1{,}000\ kN$ & $Q = 650\ kN$



《 A1 》

$$\sqrt{2t_{max.}}\ \leqq\ S\ <\ t_{min.}$$

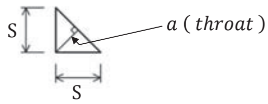
ⓐ $\sqrt{2 \times 26}\ = 7.2\ mm\ \leqq\ S\ <\ t_{min.}\ (= 10\ mm\,)$

$\rightarrow\ \underline{S\ = 8\ mm}$

ⓑ $\sqrt{2 \times 36}\ = 8.5\ mm\ \leqq\ S\ <\ t_{min.}\ (= 10\ mm\,)$

$\rightarrow\ \underline{S\ = 9\ mm}$

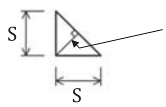
《 A2 》



$$2 \times 150 \times \frac{S}{\sqrt{2}} \times \underset{(=\tau_a)}{80} = 100 \times 10^3$$

$$S\ >\ 5.89\ mm\ \rightarrow\ \underline{S\ = 6\ mm}$$

《 A3 》



$$a = \frac{6}{\sqrt{2}} = 4.24\ mm$$

$(\,S = 6\ mm\,)$

$$\tau = \frac{500 \times 10^3}{2 \times 4.24 \times 600} = 98.3\ N/mm^2\ \ >\ \underset{(=\tau_a)}{80}\ N/mm^2$$

<u>***Not safe*** !!</u>

【11-4-2】(4)

《 A4 》

(**1**) $P = 1{,}000\ kN$

$$\sigma = \frac{1{,}000 \times 10^3}{600 \times 16} = 104.2\ N/mm^2 \ < \ 140\ N/mm^2$$

(**2**) $Q = 650\ kN$

$$\tau = \frac{650 \times 10^{-3}}{600 \times 16} = 67.7\ N/mm^2 \ < \ 80\ N/mm^2$$

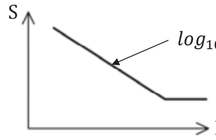
(**3**) $P = 1{,}000\ kN$ & $Q = 650\ kN$

$$\left(\frac{104.2}{140}\right)^2 + \left(\frac{67.7}{80}\right)^2 = \underline{1.27 \ > \ 1.2}$$

Not safe

【11-4-3】(1)

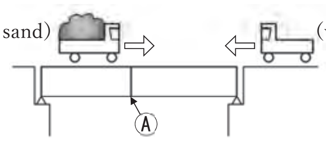
(Q1) The following S-N curve is obtained.



(**1**) *Find S , when* $N = 2 \times 10^6$

(**2**) *Find S , when* $N = 10^6$

(Q2) The following bridge is subjected to truck crossing. The One-way is with sand and the return way is without sand.



The section Ⓐ has the detail with a fatigue grade of (F), and is subjected to

stress due to a dead load. $= 40\ N/mm^2$

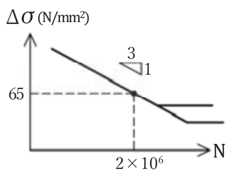
stress due to the truck (with sand). $= 90\ N/mm^2$

stress due to the truck (without sand). $= 70\ N/mm^2$

【11-4-3】(2)

Daily, 300 trucks cross the bridge.
Find the fatigue life using Miner's law.

$$\mathrm{D} = \Sigma \frac{n_i}{N_i} = 1.0$$



【11-4-3】(3)

《 A1 》

(**1**) $S = 89.1\ N/mm^2$

(**2**) $S = 112.2\ N/mm^2$

(*Substitute N value into eq.*)

《 A2 》

$$\Delta\sigma^3 \cdot N = 2 \times 10^6 \times 65^3$$

$\Delta\sigma_1 = 90\ N/mm^2$

$$N_1 = 2 \times 10^6 \left(\frac{65}{90}\right)^3 = 753{,}429$$

$\sigma_2 = 70\ N/mm^2$

$$N_2 = 2 \times 10^6 \left(\frac{65}{70}\right)^3 = 1{,}601{,}312$$

$$\frac{n}{N_1} + \frac{n}{N_2} = n\,(1{,}312 \times 10^{-6} + 0.6245 \times 10^{-6})$$

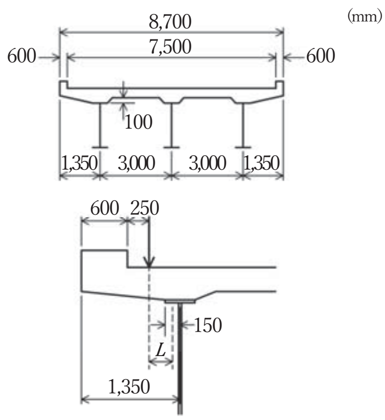
$$= 1.9465 \times 10^{-6} \cdot n = 1$$

$$n = 514{,}139$$

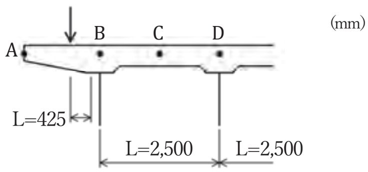
$$\frac{514{,}139}{\underset{\substack{\uparrow \\ Cycle}}{300} \times \underset{\substack{\uparrow \\ day}}{365}} = 4.695^{year} \cong \underset{\substack{\uparrow \\ fatigue\ life}}{\underline{4.7^{year}}}$$

【11-5-1】(1)

(Q1) Find min. RC thickness (k_1 =1.25, k_2 =1.00)



(Q2) Find the design moment(The main direction and the distributing reinforcement) M_A , M_B , M_C and M_D per unit length due to a live load.



【11-5-1】(2)

《 A1 》

(1) *Cantilevered slab*

$$L = 1{,}350 - 600 - 250 - \frac{150}{2} = 425\ mm\ (= 0.425m)$$

$$d_0 = 80L + 200 = 234.\underline{0}\ mm \ \rightarrow\ 234\ mm$$

$\uparrow$ *rounding*

$$d\ = k_1 k_2 d_0 = 1.25 \times 1.00 \times 234 = 29\underline{2}.5\ mm \ \rightarrow \underline{\ 290\ mm\ }$$

$\uparrow$ *rounding*

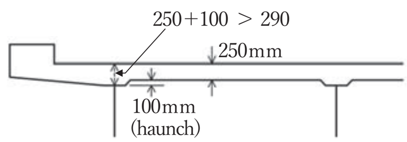
(2) *Continuous slab*

$$d_0 = 30L + 110 = 30 \times 3.0 + 110 = 200.\underline{0}\ mm \ \rightarrow\ 200\ mm$$

$\uparrow$ *rounding*

$$d\ = k_1 k_2 d_0 = 1.25 \times 1.00 \times 200 = 25\underline{0}.0\ mm \ \rightarrow \underline{\ 250\ mm\ }$$

$\uparrow$ *rounding*



<u>slab thickness 250mm is selected</u>

【11-5-1】(3)

(1) $At.\ A\ \ (L = 0.425\ m)$

$main.\quad M_{L+i} = 0$

$dist.\quad M_{L+i} = (0.15\ L + 0.13)\ P = 0.194\ P$

(2) $At.\ B\ \ (L = 0.425\ m)$

$$main.\quad M_{L+i} = \frac{PL}{1.30\ L + 0.25} = 0.530\ P$$

$dist.\quad M_{L+i} = 0$

(3) $At.\ C\ \ (L = 2.5\ m)$

$main.\quad M_{L+i} = 0.8\ (\ 0.12\ L + 0.07\ )\ P = 0.296\ P$

$dist.\quad M_{L+i} = 0.8\ (\ 0.10\ L + 0.04\ )\ P = 0.232\ P$

(4) $At.\ D\ \ (L = 2.5\ m)$

$main.\quad M_{L+i} = -M_{L+i}(\ at\ C\ ) = -0.296\ P$

$dist.\quad M_{L+i} = 0$

$\left(M_{L+i} :\ kN \cdot m/m \quad ,\quad P = 100kN \right)$

[***note***]

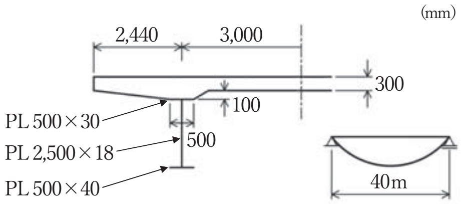
Based on JBHS, increase the coefficient is specified as follows, when slab span is perpendicular to the vehicle travel direction.

【11-5-1】(4)

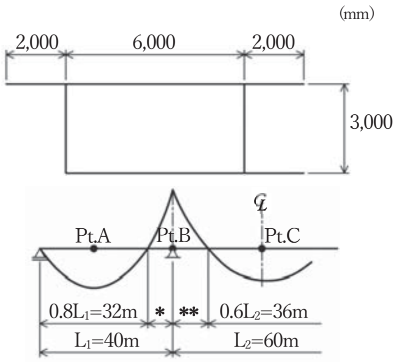
slab span	$L\ \leqq\ 2.5$	$L\ >\ 2.5$
coefficient	1.0	$1.0 + (L - \ 2.5)/12$

In this question (Q2), since L ≦ 2.5m increment coefficient in 1.0

(Q1) Find the effective width of the cross section (composite section) given below.



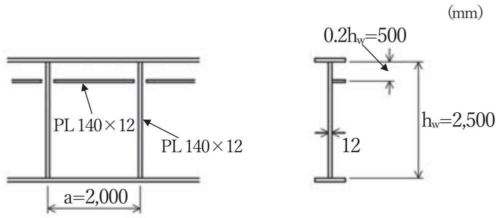
(Q2) Find the effective width of the box girder given below.



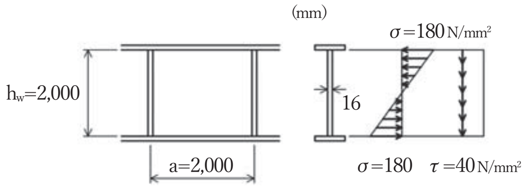
$*\ \ 0.2\,L_1 = 8\,m$ $\qquad L_u\ (\,at\ support\ \ B\,)$

$**\ \ 0.2\,L_2 = 12\,m$ $\qquad = 0.2\,(\,L_1 + L_2\,) = 20\,m$

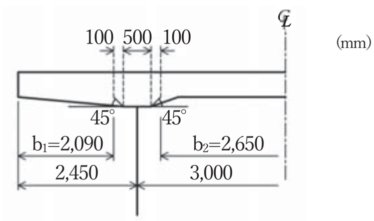
(Q3) Design the horizontal and vertical stiffeners. The material grade is SM490Y.



(Q4) Check the stability of the web under normal and shear stresses. The material grade is SM400.



《 A1 》



(***a***) *Cantilevered slab*

$b_1/L_u = 2.09/40 = 0.052\ \ >\ 0.05$

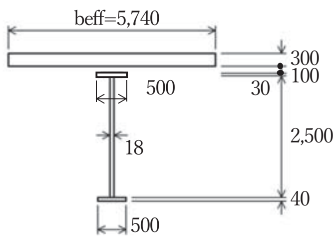
$\lambda_1 = \{1.1 - 2\,(b_1/L_u)\}\,b_1 = 0.996\,b_1\ = 2{,}082\ mm$

(***b***) *Span slab*

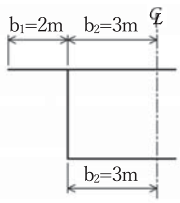
$b_2/L_u = 2.65/40 = 0.066\ \ >\ 0.05$

$\lambda_2 = \{1.1 - 2\,(b_2/L_u)\}\,b_2 = 0.986\,b_2\ = 2{,}958\ mm$

$\lambda = 2{,}082 + 100 + 500 + 100 + 2{,}958 = \underline{5{,}740\ mm}$



《 A2 》



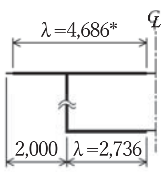
(**1**) $P_t.$ ***A***

(***a***) $b_1/L_u = 2/32 = 0.0625\ \ >\ 0.05$

$\lambda_1 = \{1.1 - 2\,(b_1/L_u)\}\,b_1 = 0.975\,b_1\ = 1{,}950\ mm$

(***b***) $b_2/L_u = 3/32 = 0.0938\ \ >\ 0.05$

$\lambda_2 = \{1.1 - 2\,(b_2/L_u)\}\,b_2 = 0.912\,b_2\ = 2{,}736\ mm$



$*\,4{,}686 = 1{,}950 + 2{,}736$

(**2**) $P_t.$ ***B*** (*at intermediate support*)

(***a***) $0.02\ <\ b_1/L_u\,(= 2/20\,)\ <\ 0.3$

$\lambda_1 = \{1.06 - 3.2\,(b_1/L_u) + 4.5\,(b_1/L_u)^2\}\,b_1$

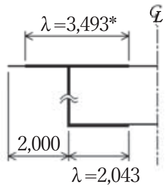
$= 0.725\,b_1\ = 1{,}450\ mm$

【11-5-2】(5)

(b) $0.02 < b_2/L_u\ (3/20 = 0.15) < 0.3$

$$\lambda_2 = \{1.06 - 3.2\,(b_2/L_u) + 4.5\,(b_2/L_u)^2\}\,b_2$$

$$= 0.681\,b_2 = 2{,}043\ mm$$



$* 3{,}493 = 1{,}450 + 2{,}043$

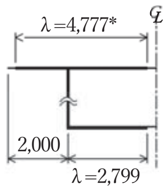
(3) $P_t.\ \boldsymbol{C}$

(a) $b_1/L_u = 2/36 = 0.0556 > 0.5$

$$\lambda_1 = \{1.1 - 2\,(b_1/L_u)\}\,b_1 = 0.989\,b_1 = 1{,}978\ mm$$

(b) $b_2/L_u = 3/36 = 0.0833 > 0.5$

$$\lambda_2 = \{1.1 - 2\,(b_2/L_u)\}\,b_2 = 0.933\,b_2 = 2{,}799\ mm$$



$* 4{,}777 = 1{,}978 + 2{,}799$

【11-5-2】(6)

《 A3 》

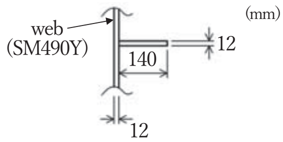
(1) *Horizontal stiffener (SM490Y)*

(cm)

$$\gamma_{h,req.} = 30\,(a/h_w) = 30 \times \frac{200}{250} = 24$$

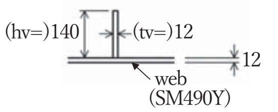
$$I_{h,req.} > \frac{h_w\,t_w^{\ 3}}{11}\gamma_{h,req.} = \frac{250 \times 1.2^3}{11} \times 24 = 943\ cm^4$$

$$I_h = \frac{h_w^{\ 3}\,t_w}{3} = \frac{14^3 \times 1.2}{3} = 1{,}098\ cm^4 > I_{h,req.}$$



<u>$PL\ 140 \times 12\ (SM490Y)$</u>

(2) *Vertical stiffener (SM400)*



$$t_v\ (= 12m) > \frac{h_v}{13} = 10.8\ mm$$

$$\underset{(140)}{h_v} > \frac{2{,}500}{30} + 50 = 133.3\ mm$$

(cm)

$$\gamma_{v,req.} = 8.0\,(h_w/a)^2 = 8 \times \left(\frac{250}{200}\right)^2 = 12.5$$

$$I_{v,req.} > \frac{h_w\,t_w^{\ 3}}{11}\gamma_{v,req.} = \frac{250 \times 1.2^3}{11} \times 12.5 = 491\ cm^4$$

$$I_v = \frac{h_v^{\ 3}\,t_v}{3} = 1{,}098\ cm^4 > I_{v,req.}$$

【11-5-2】(7)

《 A4 》

$$a/h_w = 1 < 1.5$$

$$\left(\frac{h_w}{100\,t}\right)^4 \left[\left(\frac{\sigma}{345}\right)^2 + \left\{\frac{\tau}{58 + 77\,(h_w/a)^2}\right\}^2\right]$$

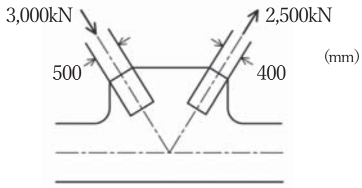
$$= \left(\frac{2{,}000}{100 \times 16}\right)^4 \left[\left(\frac{180}{345}\right)^2 + \left\{\frac{40}{58 + 77\,(2{,}000/2{,}000)^2}\right\}^2\right]$$

$$= \underline{0.879 < 1.0}$$

OK !!

【11-5-3】(1)

(Q1) Find gusset plate thickness (t_g)



(Q2) Check the safety of the following chord member under compression (N).
The material grade is SM490Y.

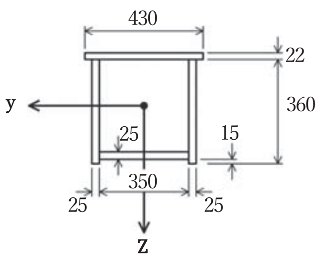
$N = -5{,}000\ kN$

effective buckling length (L_e)

$L_{e,y} = 7{,}600\ mm$ (*in − plane*)

$L_{e,z} = 7{,}600\ mm$ (*out − of plane*)

$L_{e,y} = L_{e,z} = 7{,}600\ mm$ (← *panel length*)



$b/t\ (= 350/22 = \underline{15.9}\ ,\ 320/25 = \underline{12.8}) < 31.6\ (SM490)$

《 A1 》

$$t_g = 2 \times \frac{3{,}000}{500} = 12\ mm$$

$$t_g = 2 \times \frac{2{,}500}{400} = 12.5\ mm$$

$$\rightarrow\ t_g \geqq 13\ mm$$

《 A2 》

		A	y	Ay	Ay² (cm)
1 − Flg PL	430 × 22	94.6	19.1	1,807	34,511
2 − W PL	360 × 25	180.0	−	−	19,440
1 − Flg PL	350 × 25	87.5	−15.25	−1,334	20,349
		362.1		473	74,300 (cm^4)
					−612
				$I_y =$	73,688 (cm^4)

$$\delta = \frac{473}{362.1} = 1.3\ (cm)$$

$$I_z = \frac{43^3 \times 2.2}{12} + \frac{35^3 \times 2.5}{12} + 2 \times 36 \times 2.5 \times 18.75^2$$

$$= 86{,}789\ (cm^4)\ >\ I_y$$

$$A_w = 180\ (cm^2)\ >\ 0.4\,A = 0.4\ \times\ 362.1 = 144.8\ (cm^2)$$

$$r_y = \sqrt{I_y/A} = 14.3\ (cm)\ ,\quad r_z = \sqrt{I_z/A} = 15.5\ (cm)$$

$$\lambda_y\left(= L_{e,y}/r_y\right) = 53.1\quad >\ \lambda_z\left(= L_{e,z}/r_z\right) = 49.0$$

$$\sigma_{ca} = 210 - 1.5\left(\frac{L_{e,y}}{r_y} - 14\right) = 151.4\ (N/mm^2)$$

$$\sigma = 5{,}000 \times 10^3/(362.1 \times 10^2) = 138.1\ (N/mm^2)\ <\ \sigma_{ca}$$

(***Safe*** !!)

(Q1) Calculate natural frequency (f) and circular frequency (ω), when

($\boldsymbol{a}$) $T = 1\,sec.$

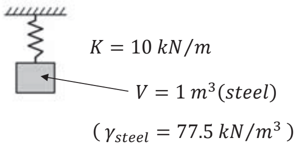
($\boldsymbol{b}$) $T = 2\,sec.$

(Q2) Is the following correct or not?
Natural frequency (f) of stiff structures
(ex. beam difficult to bend)
is higher than flexible one
(ex. easy to bend)

(Q3) Natural circular frequency (ω) of the mass – spring system is given,

$$\omega = \sqrt{\frac{k}{m}}$$

When,



g : gravity of acceleration (=9.8m/sec²)

Find (ω) and (f).

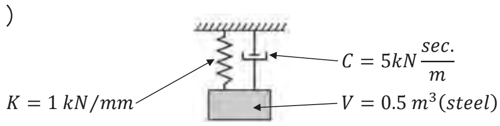
(Q4) Natural period (T) of the pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

L : length of pendulum

Find T, when (a) L=0.5m, (b) L=2m.

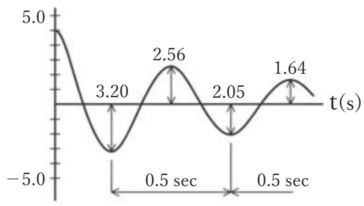
(Q5)



Calculate damping coefficient (h)

$$h = \frac{1}{\sqrt{2km}}$$

(Q6) Find damping coefficient (h)
displacement (cm)



《 A1 》

$\mathrm{f} = 1/T$, $\omega = 2\pi f$

($\boldsymbol{a}$) $\mathrm{f} = 1\,cycle/s$, $\omega = 6.28\,rad./s$

($\boldsymbol{b}$) $\mathrm{f} = 0.5\,c/s$, $\omega = 3.14\,rad./s$

《 A2 》

YES

《 A3 》

$$w(weight) = 77.5\ kN/m^3 \times 1\ m^3 = 77.5\ kN$$

$$m(mass) = \frac{w}{g} = \frac{77.5\ kN}{9.8m/sec^2} = 7.91\frac{kN}{m}\cdot sec^2$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{10\ kN/m}{7.91\ (kN/m)sec^2}} = 1.124\ rad./sec$$

$$\mathrm{f} = \frac{\omega}{2\pi} = 0.179\ c/s \quad (or\ H_z)$$

《 A4 》

$$(\boldsymbol{a})\ \ T = 2\pi\sqrt{\frac{0.5m}{9.8\ (m/sec^2)}} = 1.42\ sec.$$

$$(\boldsymbol{b})\ \ T = 2\pi\sqrt{\frac{2}{9.8}} = 2.84\ sec.$$

《 A5 》

$$w = 77.5\ kN/m^3 \times \underset{(=V)}{\underline{0.5\ m^3}} = 38.75\ KN$$

$$m = \frac{w}{g} = 3.954\frac{kN}{m}\cdot sec^2$$

$$C = 5\ \mathrm{kN}\cdot\frac{sec}{m}$$

$$\mathrm{h} = \frac{C}{\sqrt{2km}} = \frac{5\left\{\frac{kN}{m}\cdot sec\right\}}{\sqrt{2\cdot 1{,}000\left(\frac{kN}{m}\right)\cdot 3.954\left\{\left(\frac{kN}{m}\right)sec^2\right\}}}$$

$$= 5.623 \times 10^{-2}$$

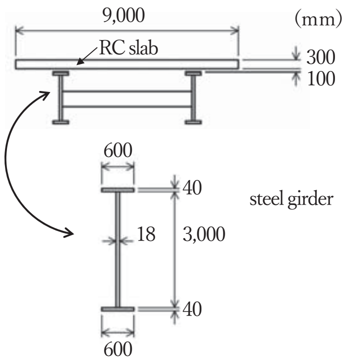
《 A6 》

$$\frac{X_m}{X_{m+1}} = \mathrm{e}^{2\pi h/\sqrt{1-h^2}} = \frac{3.20}{2.05}$$

$$\mathrm{In}\left|\frac{X_m}{X_{m+1}}\right| = \frac{2\pi h}{\sqrt{1-h^2}} = \mathrm{In}\left(\frac{3.20}{2.05}\right) = 0.446$$

h= 0.071

(Q1)



(a) Find cross sectional area (A_s) and moment of inertia (I_s) of steel girder.

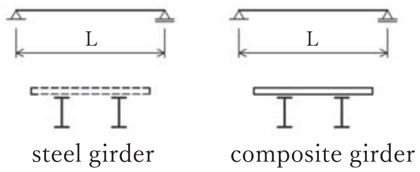
(b) Find cross sectional area (A_v) and moment of inertia (I_v) of composite girder ($n=E_s/E_c=7.0$).

(c) Find total weight (w [per unit length]) of composite girder.

(d) Find first natural frequency (f_1) and period (T_1), when simple span (L) is 40m.

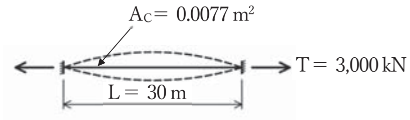
(d)−1 Assume steel girder only resists bending (non − composite girder)

(d)−2 Assume composite girder resists bending



(Q2) When span L=60m (composite girder), find (f_1).

(Q3) Find (f_1) of the bar.



$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\rho A_c}} \qquad \rho = \frac{\gamma_S}{g}$$

《 A1 》

$(\boldsymbol{a})$ $A_s = 2\times 60\times 4 + 300\times 1.8 = 1{,}020\ cm^2$ $(one\ girder)$

$$I_s = 2\times 60\times 4\times 152^2 + \frac{300^3\times 1.8}{12}$$

$$= 15{,}139{,}920\ cm^4\ (one\ girder)$$

$(0.1514 m^4)$

$(\boldsymbol{b})$ (cm)

	A	y	Ay	Ay²
1-D PL 9,000×300 (n=7)	3,857	179*	690,403	123,582,137 289,286**
2-Steel girder	2,040	—	—	30,279,840
Σ	5,897		690,403	154,151,263
				− 80,862,082***
				I_V=73,289,181 cm⁴
				(0.7329m⁴)

$$\delta = \frac{690{,}403}{5{,}897} = 117.1$$

$A_V = 5{,}897\ cm^2$

$I_V = 73{,}289{,}181\ cm^4$

$* (179) = 150 + 4 + 10 + 15$

15: a half of RC slab thickness

4: thickness of flange

150: a half of web depth

$$** (289{,}286) = \frac{900\times 30^3}{12}/7$$

$$*** (80{,}862{,}082) = \delta^2 \cdot A_V$$

$(\boldsymbol{c})$

$Slab : 9m\times 0.3m\times 24.5\ kN/m^3 = 66.15\ kN/m$

$Steel : 0.204m^2\times 77.5\ kN/m^3 = 15.81\ kN/m$

$(\boldsymbol{d})$

$$\omega_1 = \left(\frac{\pi}{L}\right)^2\sqrt{\frac{EI}{m}}$$

$$m = \frac{w}{g} = \frac{(66.15+15.81)\{kN/m\}}{9.8\ \{m/sec^2\}} = 83.63\left\{kN\left(\frac{sec}{m}\right)^2\right\}$$

$E = 2.0\times 10^8\ kN/m^2$

$(\boldsymbol{d})-\mathbf{1}$

$I = I_S = 0.1514\ m^4$, $L = 40\ m$

$$\omega_1 = \left(\frac{\pi}{40\{m\}}\right)^2\sqrt{\frac{2.0\times 10^8\{kN/m^2\}\times 0.1514\{m^4\}}{83.63\left\{kN\left(\frac{sec}{m}\right)^2\right\}}}$$

$= 3.7\ rad/sec$

$$f_1 = \frac{\omega_1}{2\pi} = \underline{0.59\ H_z(c/s)}\ ,\ T_1 = \underline{1.69\ sec}$$

$(\boldsymbol{d})-\mathbf{2}$

$I = I_V = 0.7329\ m^4$, $L = 40\ m$

$$\omega_1 = \left(\frac{\pi}{40}\right)^2\sqrt{\frac{2.0\times 10^8\times 0.7329}{83.63}}$$

$= 8.158\ rad/sec$

$$f_1 = \frac{\omega_1}{2\pi} = \underline{1.299\ H_z(c/s)}\ ,\ T_1 = \underline{0.77\ sec}$$

【12-1-2】(5)

《 A2 》

$$\omega_1 = \left(\frac{\pi}{60}\right)^2 \sqrt{\frac{2.0 \times 10^8 \times 0.7329}{83.63}} = 3.626\ rad/sec$$

$$f_1 = \frac{\omega_1}{2\pi} = \underline{0.577\ H_z(c/s)}\ ,\ \ T_1 = 1.73\ sec$$

《 A3 》

$$\rho = \frac{\gamma_s}{g} = \frac{77.5\ \{kN/m^3\}}{9.8\ \{m/sec^2\}} = 7.908\ \left\{\left(\frac{kN}{m^4}\right) sec^2\right\}$$

$$f_1 = \frac{1}{2 \times 30\{m\}} \sqrt{\frac{3{,}000\ \{kN\}}{7.908\ \left\{\left(\frac{kN}{m^4}\right) sec^2\right\} \times\ 0.0077\ \{m^2\}}}$$

$$= \underline{3.70\ H_z}$$

Special Lecture for Structural Analysis

Professor of Nagaoka University of Technology
Eiji IWASAKI

1. Concept of finite element method

1.1 Solution method of structural mechanics

Solution of structural mechanics requires to satisfy following equations

- Equation of equilibrium (force and moment balance)
- Relation of stress (force) and strain (deformation)
- Deformation geometry (compatibility condition, support condition)

Solution methods of structural mechanics can use two type as followings

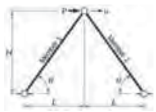
- Force method
- Displacement method

Force method that forces are unknown variables, is quite useful for solving simple problems with a few unknown forces. It is useful to solve small problems by hand calculation. It is probably a familiar method.

Displacement method that displacements are unknown variable is a very systematic procedure for solving problems. This method is used in all finite element computer programs. However, it is not suitable for hand calculation.

Q1. Solve axial forces and displacement

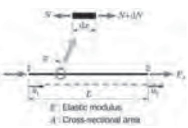
Solve the axial forces of each member and displacement at top point of the truss structure on the right figure.



E : Elasticity modulus
A : Cross-sectional area

1.2 Truss element

Consider a uniform prismatic elastic bar of length L, with elastic modulus E and cross-sectional area A.



Axial strain ε_x, axial stress σ_x and axial force N are

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \sigma_x = E\varepsilon_x \qquad N = \int_A \sigma_x dA = EA\varepsilon_x$$

where u is axial displacement of a point at x.

Equation of equilibrium at small region

$$\frac{dN}{dx} = 0 \Longrightarrow EA\frac{\partial^2 u}{\partial x^2} = 0 \xrightarrow{\text{General solution}} u = C_1 + C_2 x$$

where C_1 and C_2 are constants of integration

By boundary conditions at $x = 0$ and $x = L$,

$$u = u_1 \text{ at } x = 0 \Longrightarrow u_1 = C_1$$
$$u = u_2 \text{ at } x = L \Longrightarrow u_2 = C_1 + C_2 L$$
$$\Longrightarrow C_1 = u_1 \qquad C_2 = \frac{u_2 - u_1}{L}$$

Therefore,

$$u = N_1 u_1 + N_2 u_2 \qquad \text{where} \qquad N_1 = 1 - \frac{x}{L} \qquad N_2 = \frac{x}{L}$$

Note : N_1 and N_2 are not axial force at ends 1 and 2

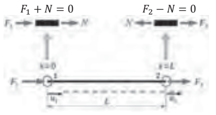
N_1 and N_2 are called as *shape functions* in finite element analysis.

1.2 Truss element

By previous displacement u, axial force N is

$$N = EA\frac{u_2 - u_1}{L}$$

$F_1 + N = 0$ $\qquad$ $F_2 - N = 0$



Equations of equilibrium at ends 1 and 2

$$F_1 + \frac{EA}{L}(u_2 - u_1) = 0$$
$$F_2 - \frac{EA}{L}(u_2 - u_1) = 0$$

Ordinary, the finite element analysis is used matrix description such as follows

$$\frac{EA}{L}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

The square matrix in above equation is called as *element stiffness matrix*.

1.3 Beam element

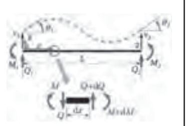
Consider a uniform plane beam member, with elastic modulus E and centroidal moment of inertia I of its cross-sectional area.

Axial strain ε_x, axial stress σ_x and bending moment M are

$$\varepsilon_x = \frac{\partial u_P}{\partial x} = -y\frac{\partial^2 v}{\partial x^2} \qquad \sigma_x = E\varepsilon_x$$

$$M = -\int_A \sigma y dA = EI\frac{\partial^2 v}{\partial x^2}$$

where v is deflection of a point at x.



Equation of equilibrium at small region

$$\frac{d^2 M}{dx^2} = 0 \Longrightarrow EI\frac{\partial^4 v}{\partial x^4} = 0$$

General solution

$$v = C_1 + C_2 x + C_3 x^2 + C_4 x^3$$

where from C_1 to C_4 are constants of integration

$$\frac{dQ}{dx} = 0 \qquad \frac{dM}{dx} + Q = 0$$

1.3 Beam element

By boundary conditions at $x = 0$ and $x = L$,

$$v = v_1 \text{ at } x = 0 \Longrightarrow v_1 = C_1$$
$$v' = \theta_1 \text{ at } x = 0 \Longrightarrow \theta_1 = C_2$$
$$v = v_2 \text{ at } x = L \Longrightarrow v_2 = C_1 + C_2 L + C_3 L^2 + C_4 L^3$$
$$v' = \theta_2 \text{ at } x = L \Longrightarrow \theta_2 = C_2 + 2C_3 L + 3C_4 L^2$$

$$\Longrightarrow \quad C_1 = v_1 \qquad C_2 = \theta_1 \qquad C_3 = 3\frac{v_2 - v_1}{L^2} - \frac{2\theta_1 + \theta_2}{L} \qquad C_4 = 2\frac{v_1 - v_2}{L^3} + \frac{\theta_1 + \theta_2}{L^2}$$

Therefore,

$$v = N_3 v_1 + N_4 v_2 + N_5 \theta_1 + N_6 \theta_2$$

where $\quad N_3 = 1 - 3\xi^2 + 2\xi^3 \qquad N_5 = (\xi - 2\xi^2 + \xi^3)L$

$\qquad\quad N_4 = 3\xi^2 - 2\xi^3 \qquad N_6 = (-\xi^2 + \xi^3)L \qquad \xi = \frac{x}{L}$

From N_1 to N_4 are called as *shape functions* in finite element analysis.

Bending moment M and shear force Q are

$$M = EIv'' = \frac{EI}{L^2}(12\xi - 6)(v_1 - v_2) + \frac{EI}{L}(6\xi - 4)\theta_1 + \frac{EI}{L}(6\xi - 2)\theta_2$$

$$Q = -M' = -\frac{12EI}{L^3}(v_1 - v_2) - \frac{6EI}{L^2}(\theta_1 + \theta_2)$$

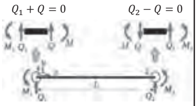
1.3 Beam element

Equations of equilibrium at ends 1 and 2

$$M_1 - \frac{6EI}{L^2}(v_1 - v_2) - \frac{4EI}{L}\theta_1 - \frac{2EI}{L}\theta_2 = 0$$
$$M_2 - \frac{6EI}{L^2}(v_1 - v_2) - \frac{2EI}{L}\theta_1 - \frac{4EI}{L}\theta_2 = 0$$
$$Q_1 - \frac{12EI}{L^3}(v_1 - v_2) - \frac{6EI}{L^2}(\theta_1 + \theta_2) = 0$$
$$Q_2 + \frac{12EI}{L^3}(v_1 - v_2) + \frac{6EI}{L^2}(\theta_1 + \theta_2) = 0$$

$M_1 + M = 0$ $\quad$ $M_2 - M = 0$

$Q_1 + Q = 0$ $\quad$ $Q_2 - Q = 0$



Ordinary, the finite element analysis is used matrix description such as follows

$$\frac{EI}{L^3}\begin{pmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{pmatrix}\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{Bmatrix}$$

The square matrix in above equation is called as *element stiffness matrix*.

1.4 Combination of truss and beam elements

Combine equations of truss and beam elements

$$\begin{pmatrix} T & 0 & 0 & T & 0 & 0 \\ 0 & B & B & 0 & B & B \\ 0 & B & B & 0 & B & B \\ T & 0 & 0 & T & 0 & 0 \\ 0 & B & B & 0 & B & B \\ 0 & B & B & 0 & B & B \end{pmatrix}\begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ Q_1 \\ M_1 \\ F_2 \\ Q_2 \\ M_2 \end{Bmatrix}$$

where "T'' and "B'' in matrix indicates corresponding components for displacement and force at truss and beam elements.

In the finite element method, the above relation between displacement and force is also expressed by simply following as

$$\boldsymbol{kd} = \boldsymbol{f} \qquad \text{or} \qquad \begin{pmatrix} \boldsymbol{k}_{11} & \boldsymbol{k}_{12} \\ \boldsymbol{k}_{21} & \boldsymbol{k}_{22} \end{pmatrix}\begin{Bmatrix} \boldsymbol{d}_1 \\ \boldsymbol{d}_2 \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{f}_1 \\ \boldsymbol{f}_2 \end{Bmatrix}$$

where $\boldsymbol{d}$ and $\boldsymbol{f}$ list the nodal displacement and nodal force components,

$\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ list displacement components at node 1 and 2, respectively,

$\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ list force components at node 1 and 2, respectively.

1.5 Formulas for element matrices

Formulas for element matrices are also obtained by the principle of virtual work.

$$\int_V \sigma_x \delta\varepsilon_x dV = F_1\delta u_1 + Q_1\delta v_1 + M_1\delta\theta_1 + F_2\delta u_2 + Q_2\delta v_2 + M_2\delta\theta_2$$

where δu, δv etc indicate virtual displacements.

If displacements satisfies the above equation, they also satisfy the conditions of equilibrium.

In previous bar and beam element, strain and stress are

$$\varepsilon_x = u' - yv'' \qquad \sigma_x = E\varepsilon_x$$

where displacements u and v are expressed by quantities of nodes 1 and 2 as follows

$$u = N_1u_1 + N_2u_2 \equiv \boldsymbol{N}_t\boldsymbol{d} \qquad v = N_3v_1 + N_4v_2 + N_5\theta_1 + N_6\theta_2 \equiv \boldsymbol{N}_b\boldsymbol{d}$$

where

$$\boldsymbol{N}_t = \{N_1 \quad 0 \quad 0 \quad N_2 \quad 0 \quad 0\} \qquad \boldsymbol{N}_b = \{0 \quad N_3 \quad N_5 \quad 0 \quad N_4 \quad N_6\}$$

The equation of principle of virtual work is rewritten as follows

$$\delta\boldsymbol{d}^T \int_V \boldsymbol{B}^T E\boldsymbol{B}dV\,\boldsymbol{d} = \delta\boldsymbol{d}^T\boldsymbol{f} \quad \Rightarrow \quad \boldsymbol{kd} = \boldsymbol{f}$$

where

$$\boldsymbol{k} = \int_V \boldsymbol{B}^T E\boldsymbol{B}dV \qquad \boldsymbol{B} = \boldsymbol{N}_t' - y\boldsymbol{N}_b''$$

$\boldsymbol{B}^T$ indicates row and column transposition of $\boldsymbol{B}$.

Q2. Derive truss element stiffness matrix by the principle of virtual work

The element stiffness matrix $\boldsymbol{k}$ is defined by following equation.

$$\boldsymbol{k} = \int_V \boldsymbol{B}^T E\boldsymbol{B}dV = \int_0^L\int_A \boldsymbol{B}^T E\boldsymbol{B}dA\,dx \qquad \text{where} \qquad \boldsymbol{B} = \boldsymbol{N}_t' = \frac{d\boldsymbol{N}_t}{dx}$$

In truss element, $\boldsymbol{N}_t$ is

$$\boldsymbol{N}_t = \{N_1 \quad N_2\} \qquad N_1 = 1 - \frac{x}{L} \qquad N_2 = \frac{x}{L}$$

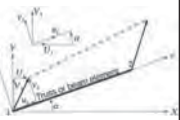
Derive stiffness matrix $\boldsymbol{k}$ in truss element.

1.6 Element of arbitrary orientation

Consider *x* and *y* axes are arbitrary oriented.

Nodal displacements u_1 and v_1 in the *x-y* coordinate are obtained by displacement components in the X-Y coordinate.

$$u_1 = U_1\cos\alpha + V_1\sin\alpha \qquad \theta_1 = \Theta_1$$
$$v_1 = -U_1\sin\alpha + V_1\cos\alpha$$



Similarly, displacements at node 2 are

$$u_2 = U_2\cos\alpha + V_2\sin\alpha \qquad \theta_2 = \Theta_2$$
$$v_2 = -U_2\sin\alpha + V_2\cos\alpha$$

x-y coordinate and X-Y coordinate are called as local element coordinate system and global structural coordinate system, respectively.

Above relationship is also expressed by simply following as

$$\boldsymbol{d} = \boldsymbol{TD}$$

where

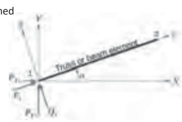
$$\boldsymbol{d} = \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} \qquad \boldsymbol{T} = \begin{pmatrix} c & s & 0 & 0 & 0 & 0 \\ -s & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & 0 & 0 & -s & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad \boldsymbol{D} = \begin{Bmatrix} U_1 \\ V_1 \\ \Theta_1 \\ U_2 \\ V_2 \\ \Theta_2 \end{Bmatrix}$$

$$c = \cos\alpha \qquad s = \sin\alpha$$

1.6 Element of arbitrary orientation

Nodal forces F_1 and Q_1 in the *x-y* coordinate are obtained by displacement components in the X-Y coordinate.

$$F_1 = P_{X1}\cos\alpha + P_{Y1}\sin\alpha$$
$$Q_1 = -P_{X1}\sin\alpha + P_{Y1}\cos\alpha$$



Similarly, forces at node 2 are

$$F_2 = P_{X2}\cos\alpha + P_{Y2}\sin\alpha$$
$$Q_2 = -P_{X2}\sin\alpha + P_{Y2}\cos\alpha$$

Above relationship is also expressed by simply following as

$$\boldsymbol{f} = \boldsymbol{TF}$$

where

$$\boldsymbol{f} = \begin{Bmatrix} F_1 \\ Q_1 \\ M_1 \\ F_2 \\ Q_2 \\ M_2 \end{Bmatrix} \qquad \boldsymbol{F} = \begin{Bmatrix} F_{X1} \\ F_{Y1} \\ M_1 \\ F_{X2} \\ F_{Y2} \\ M_2 \end{Bmatrix}$$

In x-y coordinate, related equation between nodal displacements and nodal forces are

$$\boldsymbol{f} = \boldsymbol{kd}$$

In X-Y coordinate,

$$\boldsymbol{F} = \boldsymbol{T}^{-1}\boldsymbol{f} = \boldsymbol{T}^T\boldsymbol{f} = \boldsymbol{T}^T\boldsymbol{kd} = \boldsymbol{T}^T\boldsymbol{kTD} \equiv \boldsymbol{KD}$$

Therefore, stiffness matrix in X-Y coordinate are

$$\boldsymbol{K} = \boldsymbol{T}^T\boldsymbol{kT}$$

where $\quad \boldsymbol{T}^{-1} = \boldsymbol{T}^T$

Q3. Derive truss element stiffness matrix on the global structural coordinate

The truss element stiffness matrix is written as follows on the element coordinates system

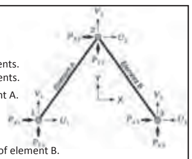
$$\boldsymbol{k} = \frac{EA}{L}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

Transformation matrix is

$$\boldsymbol{T} = \begin{pmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{pmatrix}$$

Derive truss element matrix on the global structural coordinate system.

1.7 Assembly of elements



Consider assembly two elements truss or beam elements. However, this concept can use other many type elements.

Relation between displacements and forces of element A.

$$\begin{pmatrix} \boldsymbol{K}_{11}^A & \boldsymbol{K}_{12}^A \\ \boldsymbol{K}_{21}^A & \boldsymbol{K}_{22}^A \end{pmatrix}\begin{Bmatrix} \boldsymbol{D}_1^A \\ \boldsymbol{D}_2^A \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_1^A \\ \boldsymbol{F}_2^A \end{Bmatrix}$$

Similarly, relation between displacements and forces of element B.

$$\begin{pmatrix} \boldsymbol{K}_{22}^B & \boldsymbol{K}_{23}^B \\ \boldsymbol{K}_{32}^B & \boldsymbol{K}_{33}^B \end{pmatrix}\begin{Bmatrix} \boldsymbol{D}_2^B \\ \boldsymbol{D}_3^B \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_2^B \\ \boldsymbol{F}_3^B \end{Bmatrix}$$

where these equations are written by global structural coordinate.

By equilibrium and compatibility conditions at each nodes

$\boldsymbol{D}_1^A \equiv \boldsymbol{D}_1 \quad \boldsymbol{D}_2^A = \boldsymbol{D}_2^B \equiv \boldsymbol{D}_2 \quad \boldsymbol{D}_3^B \equiv \boldsymbol{D}_3$ (Compatibility conditions)

$\boldsymbol{F}_1^A \equiv \boldsymbol{F}_1 \quad \boldsymbol{F}_2^A + \boldsymbol{F}_2^B \equiv \boldsymbol{F}_2 \quad \boldsymbol{F}_3^B \equiv \boldsymbol{F}_3$ (Equilibrium conditions)

From these equations

$$\boldsymbol{F}_1 = \boldsymbol{K}_{11}^A\boldsymbol{D}_1 + \boldsymbol{K}_{12}^A\boldsymbol{D}_2$$
$$\boldsymbol{F}_2 = \boldsymbol{K}_{21}^A\boldsymbol{D}_1 + (\boldsymbol{K}_{22}^A + \boldsymbol{K}_{22}^B)\boldsymbol{D}_2 + \boldsymbol{K}_{23}^B\boldsymbol{D}_3$$
$$\boldsymbol{F}_3 = \boldsymbol{K}_{32}^B\boldsymbol{D}_2 + \boldsymbol{K}_{33}^B\boldsymbol{D}_3$$

$$\Rightarrow \begin{pmatrix} \boldsymbol{K}_{11}^A & \boldsymbol{K}_{12}^A & \boldsymbol{0} \\ \boldsymbol{K}_{21}^A & \boldsymbol{K}_{22}^A + \boldsymbol{K}_{22}^B & \boldsymbol{K}_{23}^B \\ \boldsymbol{0} & \boldsymbol{K}_{32}^B & \boldsymbol{K}_{33}^B \end{pmatrix}\begin{Bmatrix} \boldsymbol{D}_1 \\ \boldsymbol{D}_2 \\ \boldsymbol{D}_3 \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_1 \\ \boldsymbol{F}_2 \\ \boldsymbol{F}_3 \end{Bmatrix}$$

1.7 Assembly of elements

Assembled global equation between nodal displacements and nodal forces are following

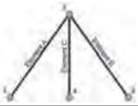
$$\begin{pmatrix} \boldsymbol{K}_{11}^A & \boldsymbol{K}_{12}^A & \boldsymbol{0} \\ \boldsymbol{K}_{21}^A & \boldsymbol{K}_{22}^A + \boldsymbol{K}_{22}^B & \boldsymbol{K}_{23}^B \\ \boldsymbol{0} & \boldsymbol{K}_{32}^B & \boldsymbol{K}_{33}^B \end{pmatrix} \begin{Bmatrix} \boldsymbol{D}_1 \\ \boldsymbol{D}_2 \\ \boldsymbol{D}_3 \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_1 \\ \boldsymbol{F}_2 \\ \boldsymbol{F}_3 \end{Bmatrix}$$

The above global stiffness matrix is included in each element stiffness matrices corresponding nodal components of row and column.

Therefore, global stiffness matrix can obtained systematic or automatically from each element stiffness matrices without using nodal equilibrium and compatibility conditions.

In structure that consists three members and four nodes as shown right figure, the global equation is following

$$\begin{pmatrix} \boldsymbol{K}_{11}^A & \boldsymbol{K}_{12}^A & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{K}_{21}^A & \boldsymbol{K}_{22}^A + \boldsymbol{K}_{22}^B + \boxed{\boldsymbol{K}_{22}^C} & \boldsymbol{K}_{23}^B & \boxed{\boldsymbol{K}_{24}^C} \\ \boldsymbol{0} & \boldsymbol{K}_{32}^B & \boldsymbol{K}_{33}^B & \boldsymbol{0} \\ \boldsymbol{0} & \boxed{\boldsymbol{K}_{42}^C} & \boldsymbol{0} & \boxed{\boldsymbol{K}_{44}^C} \end{pmatrix} \begin{Bmatrix} \boldsymbol{D}_1 \\ \boldsymbol{D}_2 \\ \boldsymbol{D}_3 \\ \boldsymbol{D}_4 \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_1 \\ \boldsymbol{F}_2 \\ \boldsymbol{F}_3 \\ \boldsymbol{F}_4 \end{Bmatrix}$$



1.8 Singularity of stiffness matrix

Equation of equilibrium $\boldsymbol{KD} = \boldsymbol{F}$ will be solved with respect to the displacements $\boldsymbol{D}$ given forces $\boldsymbol{F}$. At this time, appropriate support conditions must be imposed on the displacement.

Element stiffness matrix is singular.

Matrix $\boldsymbol{A}$ is singular if $|\boldsymbol{A}| = 0$.

For example, truss stiffness matrix

$$|\boldsymbol{k}| = \left|\frac{EA}{L}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\right| =$$

For example, beam stiffness matrix

$$|\boldsymbol{k}| = \left|\frac{EI}{L^3}\begin{pmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{pmatrix}\right| =$$

Global stiffness matrix that assembled all elements is also singular.

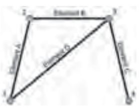
If appropriate support conditions are imposed in the equation of equilibrium, stiffness matrix is not singular.

If matrix is singular, equation can not be solved.

Therefore, support condition is very important.

Q4. Derive global equation

Derive global equation for structure that consists four elements.



1.9 How to set support conditions

Consider one beam element for simplicity explanation.

$$\begin{pmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{pmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ Q_1 \\ M_1 \\ F_2 \\ Q_2 \\ M_2 \end{Bmatrix}$$

In simply support conditions, that is $u_1 = v_1 = v_2 = 0$, F_1, Q_1 and Q_2 mean reaction forces. These reaction forces are unknown values.

Instead of the conditions of displacement equal to zero, reduce the size of stiffness matrix.

$$\begin{pmatrix} 0 & -\frac{EA}{L} & 0 \\ \frac{6EI}{L^2} & 0 & \frac{6EI}{L^2} \\ \frac{4EI}{L} & 0 & \frac{2EI}{L} \\ 0 & \frac{EA}{L} & 0 \\ -\frac{6EI}{L^2} & 0 & -\frac{6EI}{L^2} \\ \frac{2EI}{L} & 0 & \frac{4EI}{L} \end{pmatrix} \begin{Bmatrix} \theta_1 \\ u_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ Q_1 \\ M_1 \\ F_2 \\ Q_2 \\ M_2 \end{Bmatrix}$$

1.9 How to set support conditions

F_1, Q_1 and Q_2 mean reaction forces. These reaction forces are unknown values.

Next, the previous equation is separated into two parts.

$$\begin{pmatrix} \frac{4EI}{L} & 0 & \frac{2EI}{L} \\ 0 & \frac{EA}{L} & 0 \\ \frac{2EI}{L} & 0 & \frac{4EI}{L} \end{pmatrix} \begin{Bmatrix} \theta_1 \\ u_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} M_1 \\ F_2 \\ M_2 \end{Bmatrix}$$

$$\begin{pmatrix} 0 & -\frac{EA}{L} & 0 \\ \frac{6EI}{L^2} & 0 & \frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & 0 & -\frac{6EI}{L^2} \end{pmatrix} \begin{Bmatrix} \theta_1 \\ u_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ Q_1 \\ Q_2 \end{Bmatrix}$$

Previous equation

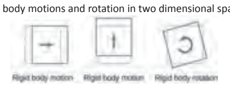
$$\begin{pmatrix} 0 & -\frac{EA}{L} & 0 \\ \frac{6EI}{L^2} & 0 & \frac{6EI}{L^2} \\ \frac{4EI}{L} & 0 & \frac{2EI}{L} \\ 0 & \frac{EA}{L} & 0 \\ -\frac{6EI}{L^2} & 0 & -\frac{6EI}{L^2} \\ \frac{2EI}{L} & 0 & \frac{4EI}{L} \end{pmatrix} \begin{Bmatrix} \theta_1 \\ u_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ Q_1 \\ M_1 \\ F_2 \\ Q_2 \\ M_2 \end{Bmatrix}$$

Solving the first equation obtains the displacement.

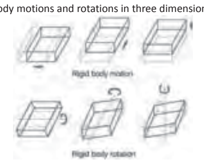
Substituting these displacements into the second equation obtains reaction forces.

1.10 Support conditions

Rigid body motions and rotation in two dimensional space.



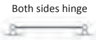
Rigid body motions and rotations in three dimensional space.



These rigid body displacements must be restrained by supporting conditions.

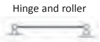
1.10 Support conditions

Both sides hinge



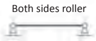
No singular

Hinge and roller



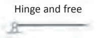
No singular

Both sides roller



Singular

Hinge and free



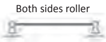
Singular

If rigid body motion and rotation are constrained by appropriate support condition, stiffness matrix is no singular. Then displacements can solve from equilibrium equation.

When the equation can not be solved by FEM, the support conditions are often not appropriate.

Q5. Confirm the singularity of stiffness matrix

Both sides roller



$u_1 \neq 0$ $\quad$ $u_2 \neq 0$

$v_1 = 0$ $\quad$ $v_2 = 0$

$\theta_1 \neq 0$ $\quad$ $\theta_2 \neq 0$

In the above both sides roller supported beam, confirm the singularity of stiffness matrix.

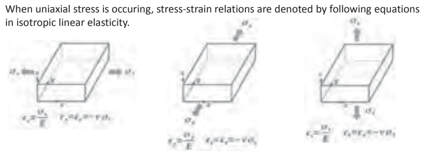
Ex.1 Truss bridge

Introduce what elements are used in the truss bridge model.

2. Typical finite elements

2.1 Plane elasticity problem

When uniaxial stress is occuring, stress-strain relations are denoted by following equations in isotropic linear elasticity.



When three axes of stress are happen, the equation are as follows by superposition.

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_z - \nu\sigma_x)$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x - \nu\sigma_y)$$

2.1 Plane elasticity problem

If the thickness of direction of z-axis is very thin, it can be assumed to be a plane stress situation. Therefore, $\sigma_z \approx 0$

Previous relation between stress and strain are

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \qquad \varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \qquad \varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y)$$

Solving the first two equations for stresses

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) \qquad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

On the other hand, the shear stress is

$$\tau_{xy} = G\gamma_{xy}$$

where G is shear elasticity modulus.

$$G = \frac{E}{2(1+\nu)}$$

Strains are obtained by displacements

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

2.1 Plane elasticity problem

Previous stress-strain relations are described by follows.

$$\boldsymbol{\sigma} = \boldsymbol{E}\boldsymbol{\varepsilon} \quad \text{where} \quad \boldsymbol{\sigma} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \quad \boldsymbol{E} = \frac{E}{1-\nu^2}\begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{pmatrix} \quad \boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Strain-displacement relations are also described by follows.

$$\boldsymbol{\varepsilon} = \boldsymbol{\partial u} \quad \text{where} \quad \boldsymbol{\partial} = \begin{pmatrix} \partial/\partial x & 0 \\ 0 & \partial/\partial y \\ \partial/\partial y & \partial/\partial x \end{pmatrix} \quad \boldsymbol{u} = \begin{Bmatrix} u \\ v \end{Bmatrix}$$

The equation of virtual strain energy is

$$\delta U = \int_V (\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy})dV = \int_V \delta\boldsymbol{\varepsilon}^T \boldsymbol{\sigma} dV$$

The equation of virtual work by load is

$$\delta W = \int_V (q_x \delta u + q_y \delta v)dV + \int_S (p_x \delta u + p_y \delta v)dS = \int_V \delta\boldsymbol{u}^T \boldsymbol{q} dV + \int_S \delta\boldsymbol{u}^T \boldsymbol{p} dS$$

The equation of principle of virtual work is

$$\delta U = \delta W$$

2.2 Plate bending problem

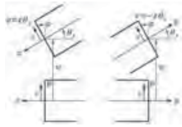
Consider bending deformation of the plate in the x-y plane

When the rotation θ_x, θ_y are occurred by the bending deformation, the displacement at a position away from the x-y plane by z is

$$u = z\theta_y \qquad v = -z\theta_x$$

where θ_x and θ_y are rotation about x-axis and y-axis, respectively.

By the above displacements, strains are



$$\varepsilon_x = \frac{\partial u}{\partial x} = z\frac{\partial \theta_y}{\partial x}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = -z\frac{\partial \theta_x}{\partial y}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = z\left(\frac{\partial \theta_y}{\partial y} - \frac{\partial \theta_x}{\partial x}\right)$$

And out of plane shear strains are

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = -\theta_x + \frac{\partial w}{\partial y}$$

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = \frac{\partial w}{\partial x} + \theta_y$$

where w is displacement of z-axis direction

2.2 Plate bending problem

Strain-displacement relations are also described by following equations.

$$\boldsymbol{\varepsilon} = \boldsymbol{\partial u}$$

where

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \quad \boldsymbol{\partial} = \begin{pmatrix} 0 & 0 & z\frac{\partial}{\partial x} \\ 0 & -z\frac{\partial}{\partial y} & 0 \\ 0 & -z\frac{\partial}{\partial x} & z\frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & -1 & 0 \\ \frac{\partial}{\partial x} & 0 & 1 \end{pmatrix} \quad \boldsymbol{u} = \begin{Bmatrix} w \\ \theta_x \\ \theta_y \end{Bmatrix}$$

Stress-strain relations are denoted by following equations in plate bending problem with shear deformation, where x and y axes are set in-plane, z axis is set out of plane.

$$\boldsymbol{\sigma} = \boldsymbol{E}\boldsymbol{\varepsilon}$$

where

$$\boldsymbol{\sigma} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} \quad \boldsymbol{E} = \frac{E}{1-\nu^2}\begin{pmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & (1-\nu)/2 & 0 & 0 \\ 0 & 0 & 0 & (1-\nu)/2 & 0 \\ 0 & 0 & 0 & 0 & (1-\nu)/2 \end{pmatrix}$$

2.2 Plate bending problem

The equation of virtual strain energy is

$$\delta U = \int_V (\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{zx}\delta\gamma_{zx})dV = \int_V \delta\boldsymbol{\varepsilon}^T \boldsymbol{\sigma} dV$$

The equation of virtual work by load is

$$\delta W = \int_V (q_z \delta w + m_x \delta\theta_x + m_y \delta\theta_y)dV + \int_S (p_z \delta w + m_x \delta\theta_x + m_y \delta\theta_y)dS$$

$$= \int_V \delta\boldsymbol{u}^T \boldsymbol{q} dV + \int_S \delta\boldsymbol{u}^T \boldsymbol{p} dS$$

The equation of principle of virtual work is

$$\delta U = \delta W$$

The element that combines the plane element and the plate bending element is called a *shell element*.

2.4 Interpolation of displacement

Displacement $\boldsymbol{u}$ is interpolated by nodal displacements as followings

$$\boldsymbol{u} = N_1\boldsymbol{u}_1 + N_2\boldsymbol{u}_2 + \cdots \equiv \boldsymbol{N}\boldsymbol{d}$$

where $\boldsymbol{u}_i$ is displacement as i-th node, and N_i is the shape function for i-th node.

$$\boldsymbol{N} = \{N_1 \quad N_2 \quad \cdots\} \qquad \boldsymbol{d} = \begin{Bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \\ \vdots \end{Bmatrix}$$

By above interpolation,

$$\boldsymbol{\varepsilon} = \boldsymbol{\partial u} = \boldsymbol{\partial N d} \equiv \boldsymbol{B d} \qquad \text{where} \qquad \boldsymbol{B} = \boldsymbol{\partial N} \qquad \text{is called B-matrix in FEM}$$

$$\boldsymbol{\sigma} = \boldsymbol{E\varepsilon} = \boldsymbol{EBd}$$

Therefore, the principle of virtual work $\delta U = \delta W$ is

$$\delta\boldsymbol{d}^T\boldsymbol{kd} = \delta\boldsymbol{d}^T\boldsymbol{f} \quad \Rightarrow \quad \boldsymbol{kd} = \boldsymbol{f}$$

where

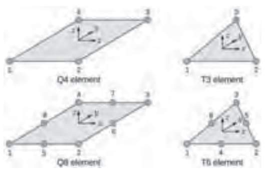
$$\boldsymbol{k} = \int_V \boldsymbol{B}^T\boldsymbol{EB}dV \qquad \boldsymbol{f} = \int_V \boldsymbol{N}^T\boldsymbol{q}dV + \int_S \boldsymbol{N}^T\boldsymbol{p}dS$$

2.5 Types of finite elements

In plane stress problem and plate bending problem, quadrilateral and triangle finite elements are used.

Types of commonly used finite element are as follows

- Q4 element
- Q8 element
- T3 element
- T6 element



2018/1/17

2.6 Feature of some elements

T3 element

Displacements in the finite element are interpolated as follows

$$u = \beta_1 + \beta_2 x + \beta_3 y$$
$$v = \beta_4 + \beta_5 x + \beta_6 y$$

where u and v are displacements for x and y axes directions, respectively.

β_i $(i = 1 \cdots 6)$ are represented by nodal displacements.

In plane problem, strains are

$$\varepsilon_x = \frac{\partial u}{\partial x} = \beta_2$$
$$\varepsilon_y = \frac{\partial v}{\partial y} = \beta_6$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \beta_3 + \beta_5$$

Strains do not vary within the element, this element is called "*constant strain element*".

2.6 Feature of some elements

T6 element

Displacements in the finite element are interpolated as follows

$$u = \beta_1 + \beta_2 x + \beta_3 y + \beta_4 x^2 + \beta_5 xy + \beta_6 y^2$$
$$v = \beta_7 + \beta_8 x + \beta_9 y + \beta_{10} x^2 + \beta_{11} xy + \beta_{12} y^2$$

where u and v are displacements for x and y axes directions, respectively.

β_i $(i = 1 \cdots 12)$ are represented by nodal displacements.

In plane problem, strains are

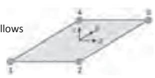
$$\varepsilon_x = \frac{\partial u}{\partial x} = \beta_2 + 2\beta_4 x + \beta_5 y$$
$$\varepsilon_y = \frac{\partial v}{\partial y} = \beta_9 + \beta_{11} x + 2\beta_{12} y$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \beta_3 + \beta_8 + (\beta_5 + 2\beta_{10})x + (2\beta_6 + \beta_{11})y$$

Strains can vary linearly within the element, this element is called "*linear strain element*".

2018/1/17

2.6 Feature of some elements

Q4 element



Displacements in the finite element are interpolated as follows

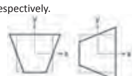
$$u = \beta_1 + \beta_2 x + \beta_3 y + \beta_4 xy$$
$$v = \beta_5 + \beta_6 x + \beta_7 y + \beta_8 xy$$

where u and v are displacements for x and y axes directions, respectively.

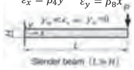
β_i $(i = 1 \cdots 8)$ are represented by nodal displacements.

In plane problem, strains are

$$\varepsilon_x = \frac{\partial u}{\partial x} = \beta_2 + \beta_4 y$$
$$\varepsilon_y = \frac{\partial v}{\partial y} = \beta_7 + \beta_8 x$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = (\beta_3 + \beta_6) + \beta_4 x + \beta_8 y$$



$\varepsilon_x = \beta_4 y \qquad \varepsilon_y = \beta_8 x$



In slender beam, shear strain γ_{xy} is much smaller than axial strain ε_x or ε_y. We can be regarded $\gamma_{xy} \approx 0$.

$\gamma_{xy} \approx 0 \Rightarrow \beta_3 + \beta_6 \approx 0 \quad \beta_4 \approx 0 \quad \beta_8 \approx 0$ Any position in the element

$\Rightarrow \varepsilon_x \approx \beta_2 \quad \varepsilon_y \approx \beta_7$ Accuracy will decrease. Called *shear locking*

In many commercial software Q4 element are revised to avoidance this shear locking.

Ex.2 Cantilever beam problem

Plane elements of different types are compared by using cantilever beam problem.

- Types of element : T3, T6, Q4 and Q8 elements
- The number of Nodes : 11x3, 21x5, 41x7, 61x9

(1) T3, 11x3 

(2) T3, 21x5 

(3) T3, 41x7 

(4) T3, 61x9 

2018/1/17

Ex.2 Cantilever beam problem

Plane elements of different types are compared by using cantilever beam problem.

- Types of element : T3, T6, Q4 and Q8 elements
- The number of Nodes : 11x3, 21x5, 41x7, 61x9

(5) T6, 11x3 

(6) T6, 21x5 

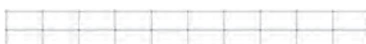
(7) T6, 41x7 

(8) T6, 61x9 

Ex.2 Cantilever beam problem

Plane elements of different types are compared by using cantilever beam problem.

- Types of element : T3, T6, Q4 and Q8 elements
- The number of Nodes : 11x3, 21x5, 41x7, 61x9

(9) Q4, 11x3 

(10) Q4, 21x5 

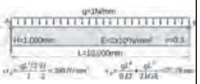
(11) Q4, 41x7 

(12) Q4, 61x9 

Ex.2 Cantilever beam problem

Plane elements of different types are compared by using cantilever beam problem.

- Types of element : T3, T6, Q4 and Q8 elements
- The number of Nodes : 11x3, 21x5, 41x7, 61x9 excluding the number of Q8 elements

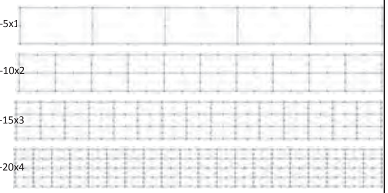


(13) Q8, 11x3-5x1

(14) Q8, 21x5-10x2

(15) Q8, 41x7-15x3

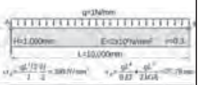
(16) Q8, 61x9-20x4

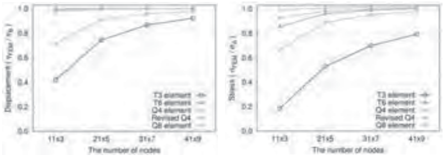


Ex.2 Cantilever beam problem

Plane elements of different types are compared by using cantilever beam problem.

- Types of element : T3, T6, Q4 and Q8 elements
- The number of Nodes : 11x3, 21x5, 41x7, 61x9





- The accuracy of the stress is no so good compared with that of displacement.
- The accuracy of results will improve when many nodes and/or elements are used.
- Performance of T3 element is not good,
  ...but this element is robust for more complicated problems such as material nonlinearity.

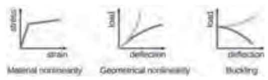
Ex.3 Modeling near the connection of members

3. Nonlinear problems

3.1 Type of nonlinear problem

In structural mechanics, types of nonlinearity include the following

- Material nonlinearity, in which material properties are functions of the state of stress or strain, such as nonlinear elasticity, plasticity etc.
- Geometrical nonlinearity, in which displacement is large enough that equilibrium equations must be written with respect to the deformed structural geometry.
- Buckling phenomena, in which a mode of deformation suddenly change to other mode as the load increases.

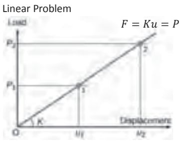


In these problems, equilibrium equation that is the relation between nodal displacements and nodal forces are influenced with values of nodal displacements.

Therefore, to solve this equation needs Newton-Raphson iteration method.

3.2 Solution methods of nonlinear problem

Linear Problem

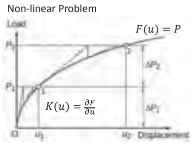


Solving the simultaneous equations once gives an answer.

$$\int_V \delta\boldsymbol{\varepsilon}^T \boldsymbol{\sigma} dV = \delta \boldsymbol{d}^T \boldsymbol{p}$$

$$\boldsymbol{k}\boldsymbol{d} = \boldsymbol{p}$$

Non-linear Problem



Repeatedly solving the simultaneous equations until the answer converges gives an answer.

$$\int_V \delta\Delta\boldsymbol{\varepsilon}^T (\boldsymbol{\sigma} + \Delta\boldsymbol{\sigma}) dV = \delta\Delta\boldsymbol{d}^T (\boldsymbol{p} + \Delta\boldsymbol{p})$$

$$\boldsymbol{k}\Delta\boldsymbol{d} + \boldsymbol{f} = \boldsymbol{p} + \Delta\boldsymbol{p}$$

$$\boldsymbol{k} = \int_V \boldsymbol{B}^T \boldsymbol{E} \boldsymbol{B} dV \qquad \boldsymbol{f} = \int_V \boldsymbol{B}^T \boldsymbol{\sigma} dV$$

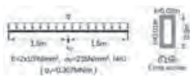
3.2 Solution methods of nonlinear problem

Three kinds of iterative Newton-Raphson methods

- Load incremental method
  give $\Delta\boldsymbol{p}$, solve $\boldsymbol{k}\Delta\boldsymbol{d} = \Delta\boldsymbol{p} + \boldsymbol{p} - \boldsymbol{f}$ for $\Delta\boldsymbol{d}$
- Displacement incremental method
  give specific component of $\Delta\boldsymbol{d}$, solve $\boldsymbol{k}\Delta\boldsymbol{d} = \Delta\boldsymbol{p} + \boldsymbol{p} - \boldsymbol{f}$ for $\Delta\boldsymbol{d}$ and $\Delta\boldsymbol{p}$
- Arch length incremental method
  give arc length ΔS , solve $\boldsymbol{k}\Delta\boldsymbol{d} = \Delta\boldsymbol{p} + \boldsymbol{p} - \boldsymbol{f}$ for $\Delta\boldsymbol{d}$ and $\Delta\boldsymbol{p}$

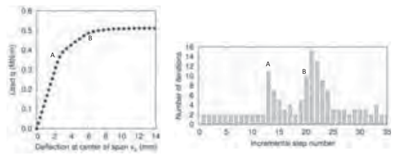
Solving the nonlinear equilibrium equation corresponds to solving the equilibrium curves in load-displacement space.

Ex.2 Material nonlinear problem

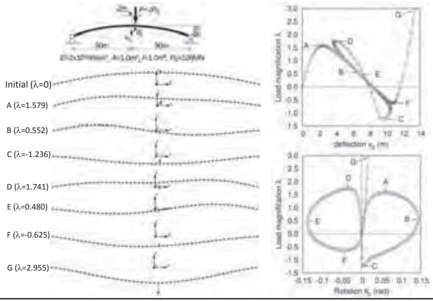


A : 13-th incremental step number

B : 20-th incremental step number

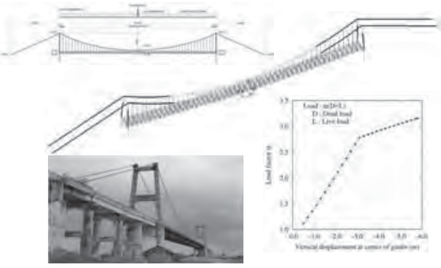


Ex.3 Geometrical nonlinear problem



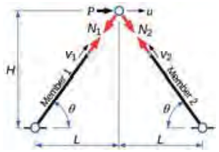
Ex.4 Suspension bridge

Confirmation of ultimate state by nonlinear analysis of suspension bridge



A1. Solve axial forces and displacement

Solve the axial forces of each member and displacement at top point of the truss structure on the right figure.



E : Elasticity modulus
A : Cross-sectional area

Let N_1 and N_2 be the axial forces of each member.

By the equilibrium condition at top point,

$$(-N_1 + N_2)\cos\theta + P = 0$$
$$(N_1 + N_2)\sin\theta = 0$$

where $\sin\theta = \dfrac{H}{\sqrt{L^2+H^2}}$ $\quad \cos\theta = \dfrac{L}{\sqrt{L^2+H^2}}$

Solving the equilibrium equation,

$$N_1 = \frac{P}{2\cos\theta} \qquad N_2 = -\frac{P}{2\cos\theta}$$

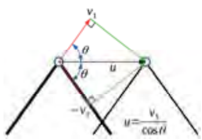
A1. Solve axial forces and displacement

The elongation of each members by axial force are

$$v = \frac{FL}{EA}$$

$$v_1 = \frac{N_1\sqrt{L^2+H^2}}{EA} = \frac{P\sqrt{L^2+H^2}}{2EA\cos\theta}$$

$$v_2 = \frac{N_2\sqrt{L^2+H^2}}{EA} = -\frac{N_1\sqrt{L^2+H^2}}{EA} = -v_1$$



Each member is connected at a point. In other word, by the condition of compatibility,

$$u = \frac{v_1}{\cos\theta} = \frac{P\sqrt{L^2+H^2}}{2EA\cos^2\theta}$$

The displacement u can also be obtained from the axial force by the theorem of Castigliano.

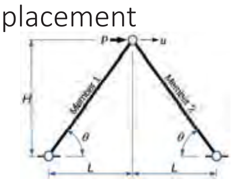
$$u = \frac{\partial U}{\partial P} = \frac{N_1\sqrt{L^2+H^2}}{EA}\frac{\partial N_1}{\partial P} + \frac{N_2\sqrt{L^2+H^2}}{EA}\frac{\partial N_2}{\partial P} = \frac{P\sqrt{L^2+H^2}}{2EA\cos^2\theta}$$

Where U is called strain energy that defined by following

$$U = \sum_i \frac{N_i^2 L_i}{2EA}$$

A1. Solve axial forces and displacement

In the previous solution, the axial forces were used as unknown variable. Here, we use displacement as a variable.

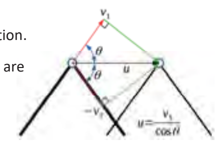


Let u is displacement at top point.

The elongations of each members are

$$v_1 = u\cos\theta$$
$$v_2 = -u\cos\theta$$

The above elongations are satisfied the compatibility condition.



By the above elongations, the axial forces of each members are

$$N_1 = EA\frac{v_1}{\sqrt{L^2+H^2}} = EA\frac{u\cos\theta}{\sqrt{L^2+H^2}}$$

$$N_2 = EA\frac{v_2}{\sqrt{L^2+H^2}} = -EA\frac{u\cos\theta}{\sqrt{L^2+H^2}}$$

When the above axial forces are applied to equation of equilibrium condition, displacement u is obtained.
Alternatively, it can be obtained by using the theorem of Castigliano.

$$P = \frac{\partial U}{\partial u} \qquad \text{where} \quad U = \sum EA\varepsilon_i^2 L_i$$

A2. Derive truss element stiffness matrix by the principle of virtual work

The element stiffness matrix $\boldsymbol{k}$ is defined by following equation.

$$\boldsymbol{k} = \int_V \boldsymbol{B}^T E \boldsymbol{B} dV = \int_0^L \int_A \boldsymbol{B}^T E \boldsymbol{B} dA\, dx \qquad \text{where} \qquad \boldsymbol{B} = \boldsymbol{N}_t' = \frac{d\boldsymbol{N}_t}{dx}$$

In truss element, $\boldsymbol{N}_t$ is

$$\boldsymbol{N}_t = \{N_1 \quad N_2\} \qquad N_1 = 1 - \frac{x}{L} \qquad N_2 = \frac{x}{L}$$

Derive stiffness matrix $\boldsymbol{k}$ in truss element.

$$\boldsymbol{B} = \boldsymbol{N}_t' = \frac{d\boldsymbol{N}_t}{dx} = \left\{-\frac{1}{L} \quad \frac{1}{L}\right\}$$

$$\boldsymbol{k} = \int_0^L \int_A \boldsymbol{B}^T E \boldsymbol{B} dA\, dx = \int_0^L \int_A \begin{Bmatrix} -1/L \\ 1/L \end{Bmatrix} E \left\{-\frac{1}{L} \quad \frac{1}{L}\right\} dA\, dx = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

A3. Derive truss element stiffness matrix on the global structural coordinate

The truss element stiffness matrix is written as follows on the element coordinates system

$$\boldsymbol{k} = \frac{EA}{L}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

Transformation matrix is

$$\boldsymbol{T} = \begin{pmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{pmatrix}$$

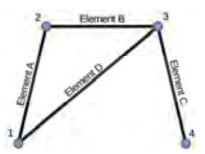
Derive truss element matrix on the global structural coordinate system.

$$\boldsymbol{K} = \boldsymbol{T}^T \boldsymbol{k} \boldsymbol{T} = \frac{EA}{L}\begin{pmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{pmatrix}$$

$$= \frac{EA}{L}\begin{pmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{pmatrix}\begin{pmatrix} c & s & -c & -s \\ -c & -s & c & s \end{pmatrix} = \frac{EA}{L}\begin{pmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{pmatrix}$$

A4. Derive global equation

Derive global equation for structure that consists four elements.



$$\begin{pmatrix} \boldsymbol{K}_{11}^A + \boldsymbol{K}_{11}^D & \boldsymbol{K}_{12}^A & \boldsymbol{K}_{13}^D & \boldsymbol{0} \\ \boldsymbol{K}_{21}^A & \boldsymbol{K}_{22}^A + \boldsymbol{K}_{22}^B & \boldsymbol{K}_{23}^B & \boldsymbol{0} \\ \boldsymbol{K}_{31}^D & \boldsymbol{K}_{32}^B & \boldsymbol{K}_{33}^B + \boldsymbol{K}_{33}^D & \boldsymbol{K}_{34}^C \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{K}_{43}^C & \boldsymbol{K}_{44}^C \end{pmatrix}\begin{Bmatrix} \boldsymbol{D}_1 \\ \boldsymbol{D}_2 \\ \boldsymbol{D}_3 \\ \boldsymbol{D}_4 \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_1 \\ \boldsymbol{F}_2 \\ \boldsymbol{F}_3 \\ \boldsymbol{F}_4 \end{Bmatrix}$$

A5. Confirm the singularity of stiffness matrix

From $v_1 = v_2 = 0$,

Both sides roller

$v_1 = 0 \qquad v_2 = 0$

Q_1 and Q_2 are reaction forces

$$\begin{pmatrix} \frac{EA}{L} & 0 & -\frac{EA}{L} & 0 \\ 0 & \frac{6EI}{L^2} & 0 & \frac{6EI}{L^2} \\ 0 & \frac{4EI}{L} & 0 & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & \frac{EA}{L} & 0 \\ 0 & -\frac{6EI}{L^2} & 0 & -\frac{6EI}{L^2} \\ 0 & \frac{2EI}{L} & 0 & \frac{4EI}{L} \end{pmatrix}\begin{Bmatrix} u_1 \\ \theta_1 \\ u_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ Q_1 \\ M_1 \\ F_2 \\ Q_2 \\ M_2 \end{Bmatrix}$$

Next, the previous equation is separated into two parts.

$$\begin{pmatrix} \frac{EA}{L} & 0 & -\frac{EA}{L} & 0 \\ 0 & \frac{4EI}{L} & 0 & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & \frac{EA}{L} & 0 \\ 0 & \frac{2EI}{L} & 0 & \frac{4EI}{L} \end{pmatrix}\begin{Bmatrix} u_1 \\ \theta_1 \\ u_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix} \qquad \begin{pmatrix} 0 & \frac{6EI}{L^2} & 0 & \frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & 0 & -\frac{6EI}{L^2} \end{pmatrix}\begin{Bmatrix} u_1 \\ \theta_1 \\ u_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix}$$

From equation of left side, we understand that the stiffness matrix is singular.

Displacement can not be solved from this equation.

Bago Br. TTP [2018.02.22]

Review (look back) of Design method

【Design Method】

・Performance-based Design Method

・Limit State Design Method

Required performance and its level for structures are defined.

[Required performance]

・Safety
・Serviceability
・Constructability
・・・・・・・・

[Limit State]

・Safety (Ultimate, Strength)Limit
・Serviceability Limit
・Fatigue Limit
・・・・・・・・

whether or not the required performance level is satisfied

【Check Method】

・Load Resistance Factor Design Method (LRFD)
・Partial Factor Design Method (PFD)
・Allowable Stress Design Method (ASD)

4

Basis

$S \leqq R$ (Safety check)

S : Action (N^*, M^*, Q^*)
R : Resistance ($N_{ult.}^*$, $M_{ult.}^*$, $Q_{ult.}^*$)
$*$: factored value

Ex.
$M_{1.3D + 1.75[L + I]} \leqq \Phi_u \cdot M_{ult.}$
$\sigma_{1.0D + 1.30[L + I]} \leqq \Phi_s \cdot \sigma_y$

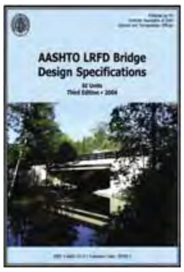
Design Level

・Level-Ⅰ «Standard»
Partial factor is used
«$S^* \leqq R^*$» (S^*,R^*) : factored action & resistance

・Level-Ⅱ
Safety index(β) is used
«$\beta \geqq \beta_{target}$»

・Level-Ⅲ
Failure probability(P_f) is used
«$P_f \leqq P_{f,\,target}$»

AASHTO LRFD



7

Limit State to be checked

・Strength Limit State Ⅰ～Ⅴ
Ex. Ⅰ:vehicle running(no wind)
Ⅱ:allowed special type of vehicle (no wind)

・Extreme Limit State Ⅰ, Ⅱ
Ⅰ:erathquake load
Ⅱ:collisoion (ship, ice)

・Serviceabilty Limit State Ⅰ～Ⅳ
Ex. Ⅱ:Yielding of material

・Fatigue Limit State Ⅰ(forever),Ⅱ(limited duration)

[totally 13 cases are checked] 8

Check Format(LRFD)

$\underline{\Sigma\eta_i\gamma_i Q_i \leqq \Phi R_n = R_r}$

γ_i : load factor*
Φ: resistance factor*
η_i :modification factorfor load (= $\eta_D\eta_R\eta_I$)
[η_D:ductility, η_R: redandancy, η_I: importance]
Q : load effect
R_n : (Nominal) resistance
R_r : factored resistance

*γ_i, Φ are based on reliability theory (safety index β)

Strength Limit State - Ⅰ

$\underline{S_{1.25DC + 1.50DW + 1.75[LL + IM]} \leqq S_{ult.}}$

S : Stress resultants
Sult. : Ultimate strength(= ΦRn {Rn : nominal strength})
DC : Dead load excluding (DW)
DW : Wearing surface [concrete pavement in USA]
LL + IM : Livre load (LL) including inpac (IM)

Serviceability Limit State - Ⅱ

$\underline{f_{1.00D + 1.30[LL + IM]} \leqq 0.95 f_y}$
↑ overload (heavy vehicle)

f : stress
f_y : yield stress

10

Ex. of check of outer girder

[Strength Limit State Ⅰ (flexure)]

M = 1.25 x (2,119 + 302.5) + 1.50 x (388.9)
↑DC ↑parapet ↑DW
+ 1.75 x (2,961)
↑live load
= 8,792kft ≦ Φ_f · M_n = 10,973kft
↑=1.0 ↑plastic strength
【action/ resistance = 0.80】

DC : dead load excluding wearing surface load (DW)

11

[Serviceability Limit State Ⅱ
(Lower flange)]

f = 1.00 x (17.73 + 1.91)
+ 1.00 x (2.46) + 1.30 x (13.3)
= 44.3kf/in^2
≦ Φ_b·F_y = 0.95 x 50 = 47.5kf/in^2

SI Unit {305N/mm^2 ≦ 327N/mm^2}

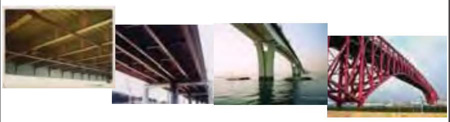
【action/resistance = 0.93】← controlled

12

Bago Br. TTP[22.02.2018]

Review (look back) of design of

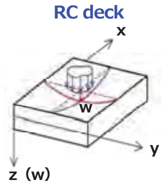
Slab,
girder bridges
and truss bridges



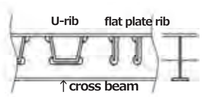
Design of slab

Concrete and steel decks

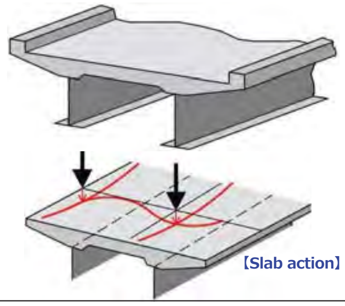
RC deck

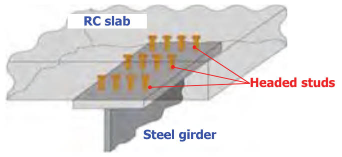


Steel deck

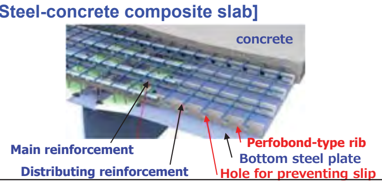


RC Slab

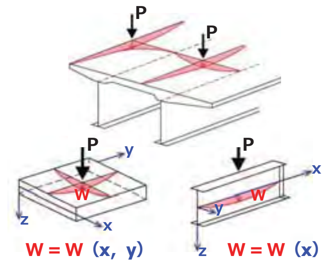




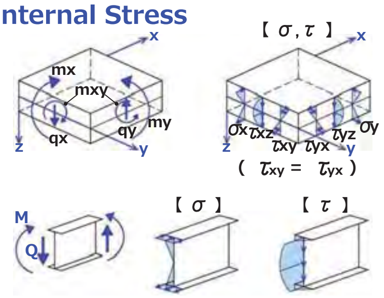
[Steel-concrete composite slab]



Load and Displacement



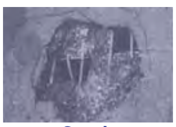
Internal Stress



Design of RC deck

Damage of slabs

Crack in pavement surface



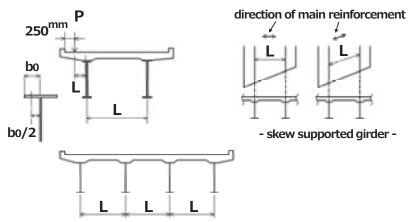
Cave in

Separation of mortar and aggregate

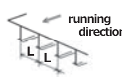
Crack net

[upperpart of slab] [underneath of slab]

Definition of slab Span (L)



Minimum slab thickness (d) required

	d_0 (mm)		L : span (m)
	 running direction		 running direction
simple slab	40 L + 110		65 L + 130
continuous slab	30 L + 110		40 L + 130
cantilever slab	0 < L < 0.25	280L + 160	240 L + 130
	0.25 < L	80 L + 210	

$d\ (> d_0) = k_1 k_2 d_0$

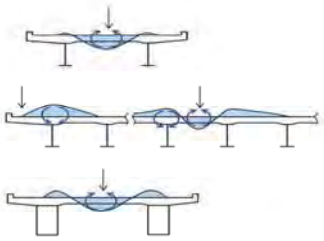
Coefficient K_1 and K_2

k_1 : effect of large-size truck volume

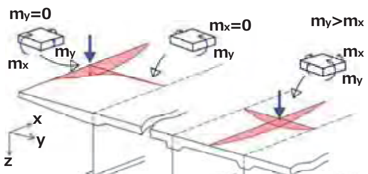
N : Number of truck / day	k_1
N < 500	1.10
500 ≦ N < 1,000	1.15
1,000 ≦ N < 2,000	1.20
2,000 ≦ N	1.25

k_2 $(=0.9\sqrt{M/M_0} > 1.0)$:
: effect of differential settlement
M_0 : design moment
M : $M_0 + \Delta M\ (1 + i)$
ΔM : additional moment
i : impact coefficient

Slab moment



Deformation of RC slab



Design moment per unit length (1m) by T-load for RC slab

		simple slab (0 < L ≦ 4m)	continuous slab (0 < L ≦ 4m)			cantilevered slab (0 < L ≦ 1.5m)	
		at span center	at span center	at span center (end span)	at intermediate support	at support	at tip
dead load(*) (w)		$\frac{wL^2}{8}$	$\frac{wL^2}{14}$	$\frac{wL^2}{10}$	2-span $-\frac{wL^2}{8}$ 3-span more $-\frac{wL^2}{10}$	$-\frac{wL^2}{2}$	—
T-load	main reinforcement	(0.12L +0.07) p Ⓐ	0.8×Ⓐ	0.8×Ⓐ	−0.8×Ⓐ	$-\frac{PL}{1.30L+0.25}$	—
	distributing reinforcement	(0.10L +0.04) p Ⓑ	0.8×Ⓑ	0.8×Ⓑ	—	—	(0.15L +0.13)p

L : slab span
$p = 100^{kN}$
(*) : distributing direction (M=0)

Additional (increase) rate for simple and continuous slab

L (m)	L ≦ 2.5	2.5 < L ≦ 4.0
coefficient	1.0	1.0 + (L-2.5) / 12

(direction of main reinforcement)

Allowable stress of reinforcement

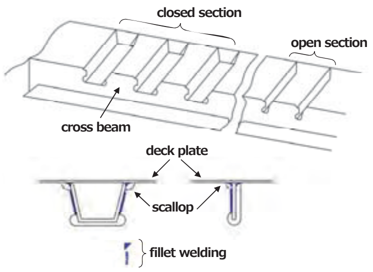
(N/mm^2)

	SD345
tension	140
compression	200

Design of steel deck plates

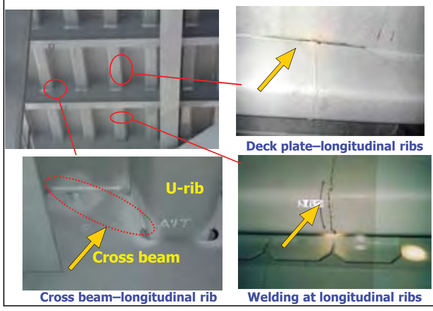
Check of stress of concrete (compression) and reinforcement (tension)

Steel deck plate



Steel deck

Steel deck fatigue



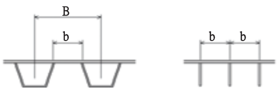
Deck plate–longitudinal ribs

Cross beam–longitudinal rib

Welding at longitudinal ribs

Deck plate thickness and rib arrangement

t = 0.035 b　(≧ 12^{mm})



B = 620 ～ 660^{mm}　b = 300 ～ 340^{mm}

b = 300 ～ 340^{mm}

Steel deck fatigue

Penetrating to deck surface

Recent topics (due to fatigue problem)

Mostly , 12^{mm} thickness has been used so far.

Due to severe fatigue damage,

⇨

16mm thickness is recommended

(in case of trough [U-shaped] ribs)

Impact (i) for the design

longitudinal ribs　$i = 0.4$

cross beams　$i = \dfrac{20}{50 + L}$

L : span of cross beams

Additional increase rate (k) for cross beams

$k = k_0$　(L ≦ 4)

$k = k_0 - (k_0 - 1) \times (L - 4) / 6$　(4 < L ≦ 10)

$k = 1.0$　(10 < L)

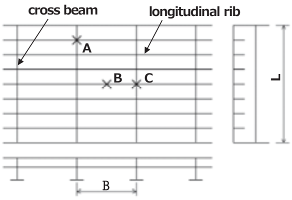
$k_0 = 1.0$　(B ≦ 2)

$k_0 = 1.0 + 0.2 \times (B - 2)$　(2 < B ≦ 3)

$k_0 = 1.2$　(3 <)

B : distance of cross beams

Calculation of stress resultants (grid model)

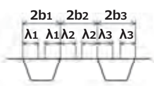


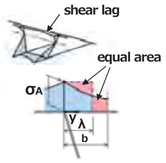
Pts. A,B,C : Observation points

Pts. A,B : for designing longitudinal ribs

Pt. C : for designing cross beams

Effective Width





$\lambda = b$　($b / L_e \leqq 0.02$)

$\lambda = \left\{1.06 - 3.2\left(\dfrac{b}{L_e}\right) + 4.5\left(\dfrac{b}{L_e}\right)^2\right\} b$

($0.02 < b / L_e < 0.30$)

$\lambda = 0.15 L_e$　($0.30 \leqq b / L_e$)

Le : equivalent span length

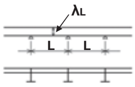
effective width (λ) is introduced to catch (σA)

$\lambda \sigma_A = \int_0^b \sigma(y)\, dy$

Equivalent span length（Le）

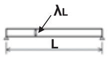
longitudinal ribs

λL （Le = 0.6 L）

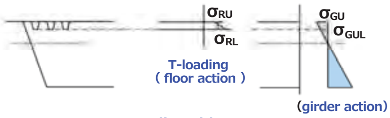


cross beams

λL （Le = L）



Stress evaluation

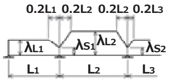


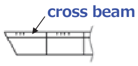
allowable stress

（N/mm²）

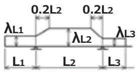
	SM400 SMA400W	SM490	SM490Y SM520 SMA490W	SM570 SMA570W
t ≦ 40	195	260	295	355
40 < t ≦ 75	175	245	275	345
75 < t ≦ 100			265	335

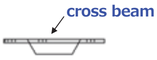
（↑ in case of combined stress check ）





λL1 （Le = 0.8 L1 ）
λS1 （Le = 0.2 （L1+L2））
λL2 （Le = 0.6 L2 ）
λS2 （Le = 0.2 （L2+L3））

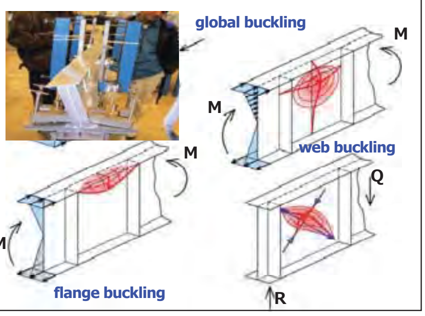




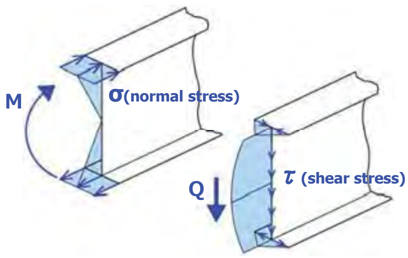
λL1 （Le = 2 L1 ）
λL2 （Le = 0.2 （L1+L2））
λL3 （Le = 2 L3 ）

Design of girder bridges

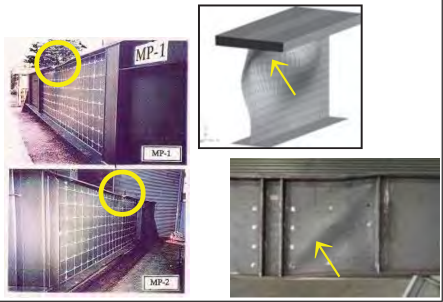
Buckling of girder and plates



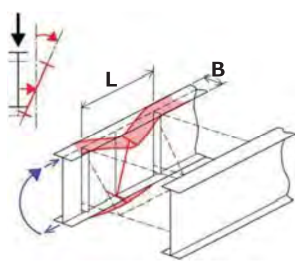
Internal Stress of I-girder

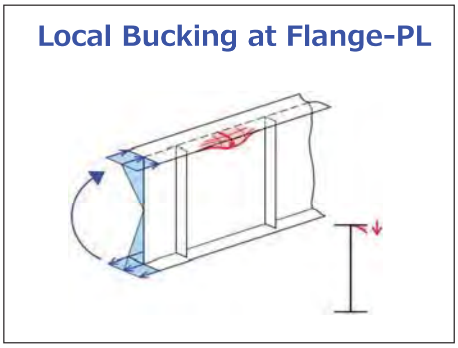


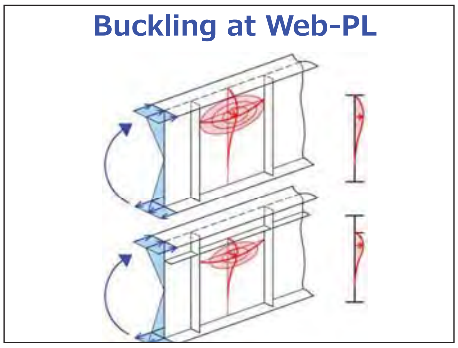
Local buckling of compressed plate and web

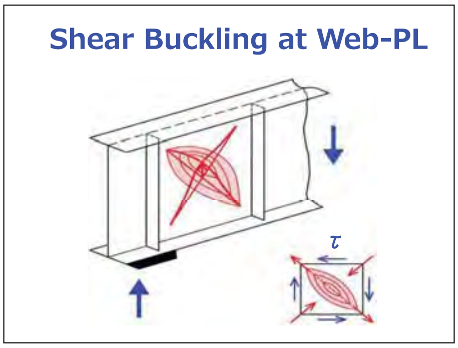


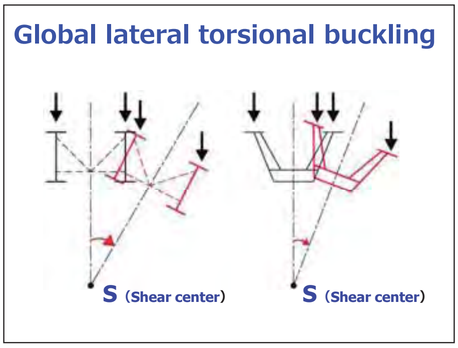
Lateral torsional buckling

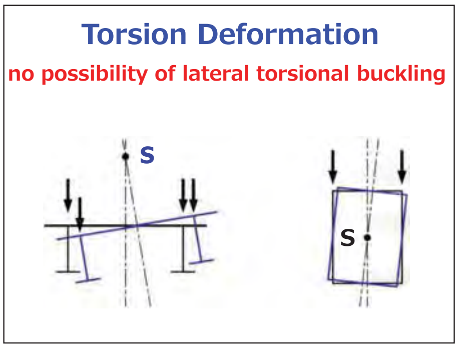


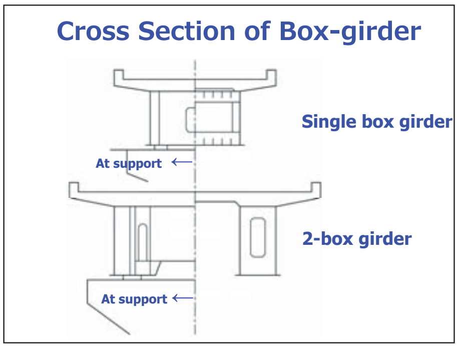

Local Bucking at Flange-PL

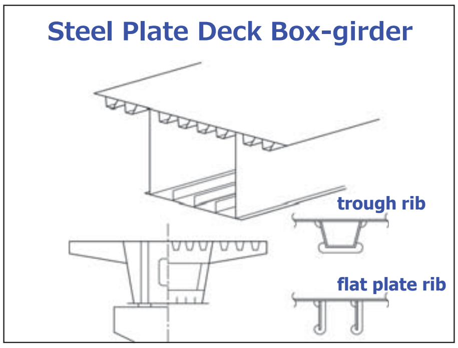

Buckling at Web-PL

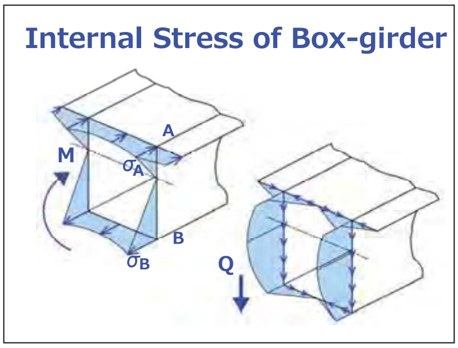

Shear Buckling at Web-PL
τ


Global lateral torsional buckling
S (Shear center)
S (Shear center)

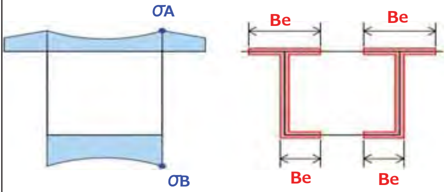

Torsion Deformation
no possibility of lateral torsional buckling
S
S


Cross Section of Box-girder
Single box girder
At support
2-box girder
At support


Steel Plate Deck Box-girder
trough rib
flat plate rib


Internal Stress of Box-girder
A
M
σA
B
σB
Q

Effective Width by Bending Stress

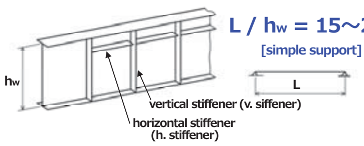


Be = Effective width

Minimum web thickness

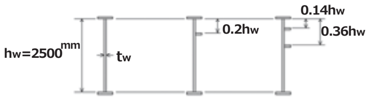
L / h_w = 15~20

[simple support]



	SS400 SM400 SMA400W	SM490	SM490Y SM520 SMA490YW	SM570 SMA570W
no h.stiffener	hw/152	hw/130	hw/123	hw/110
one h.stiffener	hw/256	hw/220	hw/209	hw/188
two h.stiffener	hw/310	hw/310	hw/294	hw/262

Ex. (SM490Y)



$t_{w,\,min.} = \frac{h_w}{123} = \frac{2{,}500}{123} = 20.3^{mm} \rightarrow \underline{21^{mm}}$

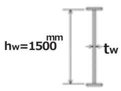
$t_{w,\,min.} = \frac{h_w}{209} = 11.97^{mm} \rightarrow \underline{12^{mm}}$

$t_{w,\,min.} = \frac{h_w}{294} = 8.5^{mm} \rightarrow \underline{9^{mm}}$

h_w without vertical stiffener

	SS400 SM400 SMA400W	SM490	SM490Y SM520 SMA490W	SM570 SMA570W
min.hw	70tw	60tw	57tw	50tw

ex. (SM400)



$t_w > \frac{h_w}{70} = 21.4^{mm}$

$\rightarrow \underline{22^{mm}}$

more than 22^{mm},
no v.stiffener allowed

(SM490Y)

$t_w > \frac{h_w}{57} = 26.3^{mm}$

$\rightarrow \underline{27^{mm}}$

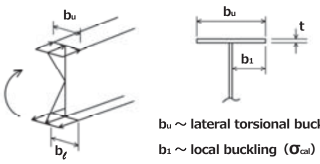
more than 27^{mm},
no v.stiffener allowed

When span (L) becomes longer,
web depth (H_w) becomes higher.
(L / H_w : 15~20 {simple span})

Thickness of web without stiffeners
becomes considerably large.

→ To avoid thick web,
stiffeners (H & V) are employed
to prevent buckling.

Flange in compression



b_u ~ lateral torsional buckling (σ_{ba})

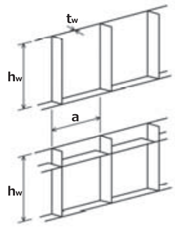
b_1 ~ local buckling (σ_{cal})

$\underline{\sigma_a = \min.\ \{\sigma_{ba},\ \sigma_{cal}\}}$
strength of beams

In tension

$t > \frac{b_1}{16}$

Design of web



(1) Horizontal stiffeners

See PPT No. 5

(2) Vertical stiffeners

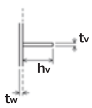
$$\left(\frac{\sigma}{\sigma_E}\right)^2 + \left(\frac{\tau}{\tau_E}\right)^2 \leqq \frac{1}{\gamma^2}$$

$\sigma_E = k\sigma(=23.9) \cdot \sigma_{E0}$

$\tau_E = k_\tau \cdot \sigma_{E0}$

$\gamma = 1.25$

verification formula



$I_v = \frac{t_v h_v^3}{3}$

$I_v > \frac{h_w t_w^3}{11} \cdot \gamma_{v,req.}$

$\gamma_{v,req.} = 8.0\ \left(\frac{h_w}{a}\right)^2$

Check of shear strength of web

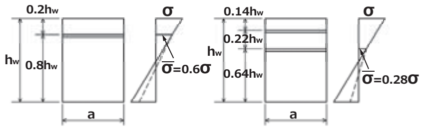
[ex. In case of no horizontal stiffener]

$$\left(\frac{h_w}{100t_w}\right)^4 \left[\left(\frac{\sigma}{345}\right)^2 + \left\{\frac{\tau}{77+58(h_w/a)^2}\right\}^2\right] \leqq 1.0 \quad (a/h_w > 1.0)$$

$$\left(\frac{h_w}{100t_w}\right)^4 \left[\left(\frac{\sigma}{345}\right)^2 + \left\{\frac{\tau}{58+77(h_w/a)^2}\right\}^2\right] \leqq 1.0 \quad (a/h_w \leqq 1.0)$$

must be satisfied

In case of the web with stiffener



one h. stiffener

$B = 0.8h_w$
$\bar{\sigma} = 0.6\sigma$

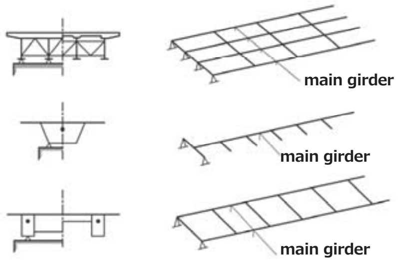
two h. stiffener

$B = 0.64h_w$
$\bar{\sigma} = 0.28\sigma$

$\left.\begin{array}{l} B \rightarrow h_w \\ \bar{\sigma} \rightarrow \sigma \end{array}\right\} \rightarrow$ formula without stiffener

Stress resultants (M, Q)

are calculated using following model



Safety check

(1a) normal stress (σ_b) of I-gider

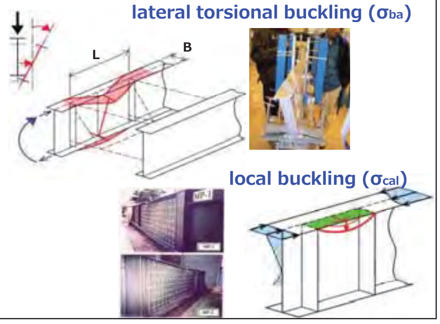
$$\sigma_b = (M/I) \cdot y \leqq \sigma_a$$

$$\sigma_a = \min.\{\sigma_{ba}, \sigma_{cal}\}$$

M : bending moment
I : geometrical moment of inertia
σ_{ba} : allowable flexural compressive stress
σ_{cal} : allowable local buckling stress

lateral torsional buckling (σ_{ba})

local buckling (σ_{cal})



(1b) normal stress (σ_b) of box girder

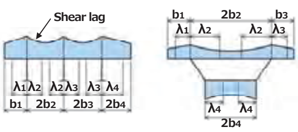
$$\sigma_b = (M/I) \cdot y \leqq \sigma_{ba}$$

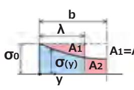
$$\sigma_{ba} = \underline{\sigma_y/1.7}$$

↑ no lateral torsional buckling

I : geometrical moment of inertia
(calculate using effective width λ)

Effective width (λ)





$\sigma_0\lambda = \int_0^b \sigma(y)\,dy$

$\lambda = \int_0^b \sigma(y)\,dy \,/\, \sigma_0$

λ : for evaluating peak stress (σ_0)

$$\left.\begin{array}{lr} \lambda = b & \left(\frac{b}{L_e} \leqq 0.05\right) \\ \lambda = \left\{1.1 - 2\left(\frac{b}{L_e}\right)\right\} b & \left(0.05 < \frac{b}{L_e} < 0.30\right) \\ \lambda = 0.15L_e & \left(0.30 \leqq \frac{b}{L_e}\right) \end{array}\right\} - (1)$$

- parabolic -

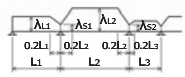
$$\left.\begin{array}{lr} \lambda = b & \left(\frac{b}{L_e} \leqq 0.02\right) \\ \lambda = \left\{1.06 - 3.2\left(\frac{b}{L_e}\right) + 4.5\left(\frac{b}{L_e}\right)^2\right\} b & \left(0.02 < \frac{b}{L_e} < 0.30\right) \\ \lambda = 0.15L_e & \left(0.30 \leqq \frac{b}{L_e}\right) \end{array}\right\} - (2)$$

- straight -

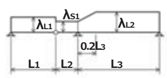
⇑
moment distribution

L_e : equivalent span length

λ_L ($L_e = L$) by eq. (1)

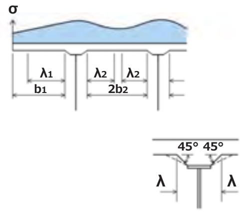


λ_{L1} ($L_e = 0.8L$) by eq. (1)
λ_{L2} ($L_e = 0.6L$) 〃
λ_{S1} ($L_e = 0.2(L_1+L_2)$) by eq. (2)
λ_{S2} ($L_e = 0.2(L_2+L_3)$) 〃



λ_{L1} ($L_e = L_1$) by eq. (1)
λ_{L2} ($L_e = 0.8L_3$) by eq. (1)
λ_{L3} ($L_e = 2L_2$) by eq. (2)

Effective width of concrete slab



(2) shear stress (τ_b) in flexure

$$\tau_b = \frac{Q}{A_w} < \tau_a \; (= \tau_y / 1.7)$$

Q : shear force
A_w : cross sectional area of webs
τ_a : allowable shear stress
τ_y : shear yield stress ($= \sigma_y / \sqrt{3}$)

※ in case of checking flange, shear stress based on shear flow theory is recommended

(3) normal and shear stresses (σ_w, τ_s, τ_w) in torsion

in case of I-section, (σ_w, τ_s, τ_w) can be neglected.
in case of box-section, (σ_w, τ_w) can be neglected.

σ_w : warping stress
τ_s : St.Venante shear stress (pure torsion)
τ_w : shear stress due to warping torsion

(4) combined stress (σ_b, τ_b) check

$$\left(\frac{\sigma_b}{\sigma_a}\right)^2 + \left(\frac{\tau_b}{\tau_a}\right)^2 < 1.2^*$$

$\sigma_b < \sigma_a$
$\tau_b < \tau_a$

(5) with torsional moment

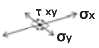
$$\left(\frac{\sigma}{\sigma_a}\right)^2 + \left(\frac{\tau}{\tau_a}\right)^2 < 1.2^*$$

$\sigma < \sigma_a$
$\tau < \tau_a$
$\sigma = \sigma_b + \sigma_w$
$\tau = \tau_b + \tau_s + \tau_w$

* take into account that loading conditions for $\sigma_{max.}$, τ_{max} are different

(6) bi-axial stress (σ_x, σ_y, τ_{xy}) check

$$\left(\frac{\sigma_x}{\sigma_a}\right)^2 - \left(\frac{\sigma_x}{\sigma_a}\right)\left(\frac{\sigma_y}{\sigma_a}\right) + \left(\frac{\sigma_y}{\sigma_a}\right)^2 + \left(\frac{\tau_{xy}}{\tau_a}\right)^2 < 1.2$$



Mises stress (σ_e)

$$\sigma_e = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2} < \underline{1.1\sigma_a} \quad \text{10\%up}$$

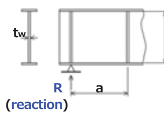
$$\left(\frac{\sigma_x}{\sigma_a}\right)^2 - \left(\frac{\sigma_x}{\sigma_a}\right)\left(\frac{\sigma_y}{\sigma_a}\right) + \left(\frac{\sigma_y}{\sigma_a}\right)^2 + 3\left(\frac{\tau_{xy}}{\sigma_a}\right)^2 < 1.2$$

$$\tau_y = \tilde{\sigma}_y / \sqrt{3} \rightarrow \sigma_a = \sqrt{3}\tau_a \quad (\tilde{\sigma}_y : \text{yield stress})$$

$$\left(\frac{\sigma_x}{\sigma_a}\right)^2 - \left(\frac{\sigma_x}{\sigma_a}\right)\left(\frac{\sigma_y}{\sigma_a}\right) + \left(\frac{\sigma_y}{\sigma_a}\right)^2 + \left(\frac{\tau}{\tau_a}\right)^2 < 1.2$$

Design of stiffeners

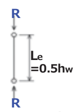
Vertical stiffener at support

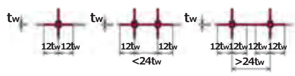


$$\sigma = \frac{R}{A_{eff}} < \sigma_{ca}$$

$$\sigma_{ca} = \sigma_{cr}/1.7$$

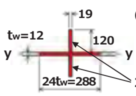
(σ_{cr} : column buckling strength)





Effective area ($A_{eff} < 1.7A_{stiffener}$)

Example

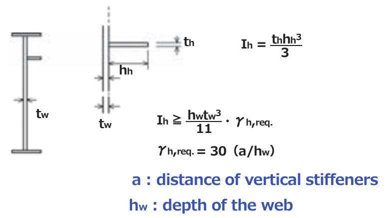


(material grade : SM400)

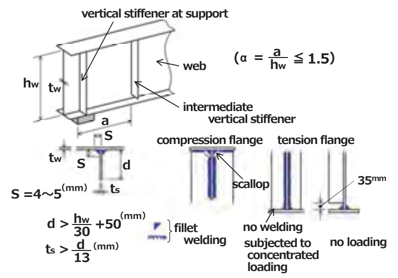
$A_{web} = 288 \times 12 \quad = 34.^{56}$
$A_{rib} = 2 \times 120 \times 19 = 45.^{60}$
$\qquad 80.16^{(cm^2)} < 1.7A_{rib} = 82.08^{(cm^2)}$
$I_y = 2{,}534^{cm^4}$, $r_y = 5.62^{cm}$
$L_e/r_y = 0.5 \times 160^{(cm)} / 5.62 = 14.2 \quad (h_w : 1600^{mm})$
$R = 879.4$ (kN)
$\sigma_c = \dfrac{879.4 \times 10^3}{80.16 \times 10^2} = 109.0^{(N/mm^2)} < \sigma_{ca}$ (ok)

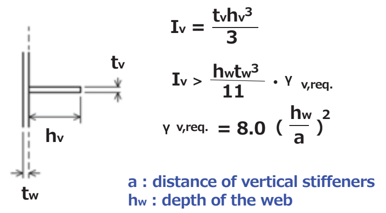
($L_e/r_y < 18 \rightarrow \sigma_{ca} = 140$ N/mm^2)

Horizontal stiffeners



Intermediate vertical stiffeners

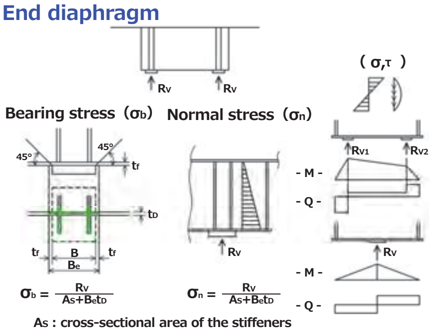




Design of support diaphragms

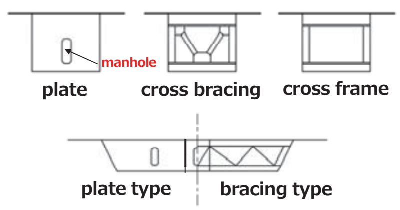
& intermediate diaphragms

End diaphragm

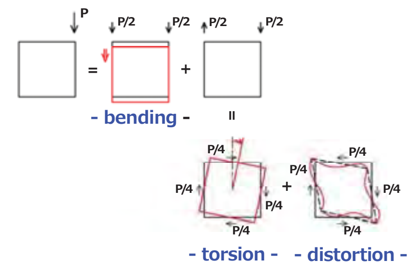


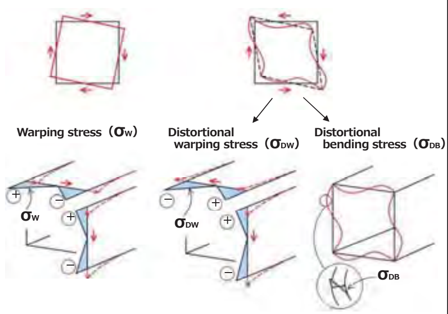
Intermediate diaphragms

(to prevent distortion)

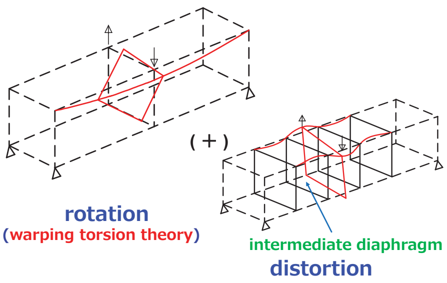


Torsion and distortion under eccentric loading

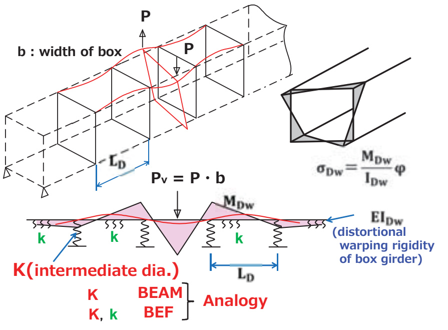




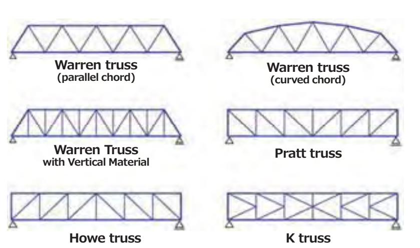
Torsion and distortion



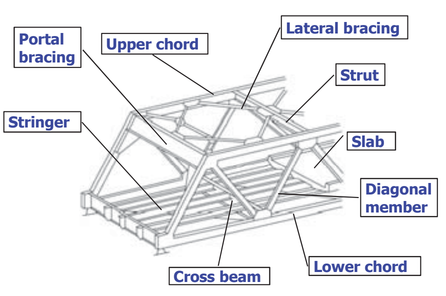
Design of truss bridges



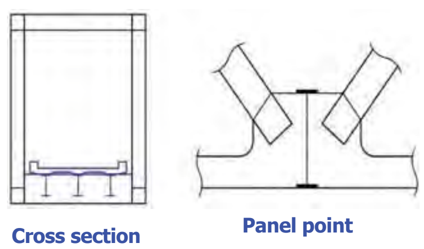
Types of typical Truss Bridge



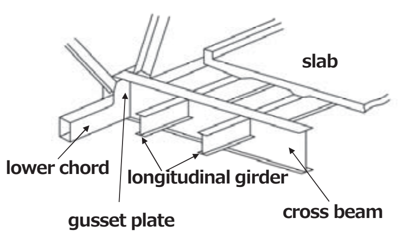
Name of Truss Members

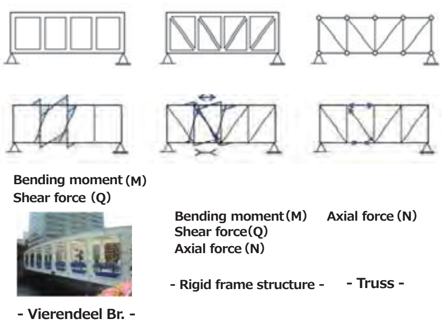


Cross section and Panel point

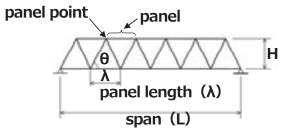


Members





Preliminary design



$H/L = 1/6 \sim 1/8$

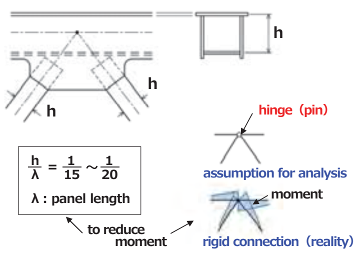
$\theta = 40 \sim 60°$

$\lambda = 6 \sim 10^{m}$

$\delta^* < L / 600$

＊deflection by live load（excluding impact）

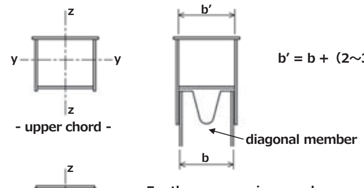
h/λ for pin structure



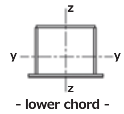
$\frac{h}{\lambda} = \frac{1}{15} \sim \frac{1}{20}$

λ : panel length

If (h/λ) is large, rigid frame model is used.



$b' = b + (2\sim3)^{mm}$



・For the compressive member, box section is used and the condition $\lambda_z\ (=L_e/r_z) < \lambda_y\ (=L_e/r_y)$ must be satisfied.

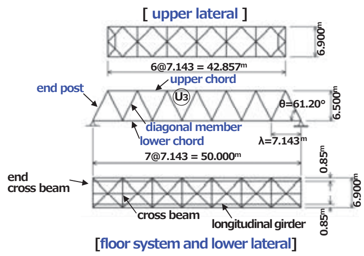
λ : slenderness ratio

・(A_w) must be greater than ($0.4A_g$).

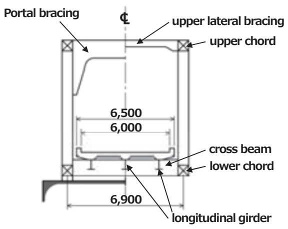
A_w : cross-sectional area of the web

A_g : cross-sectional area of the chord

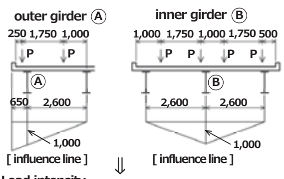
Frame model



Cross section



Design of longitudinal girder



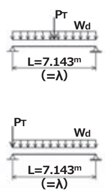
⇓

Load intensity

W_d, Ⓐ
W_d, Ⓑ } dead load

P_T, Ⓐ
P_T, Ⓑ } live load

Maximum bending moment and shear force



$M_d = \frac{W_d L^2}{8}$

$M_{L+i} = \frac{P_T L}{4}(1+i)$

$i = \frac{20}{50+L} = \frac{20}{50+7.143} = 0.35$

$Q_d = \frac{W_d L}{2}$

$Q_{L+i} = P_T(1+i)$

Stress check

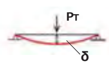
$\sigma = \frac{(M_d+M_{L+i})}{I} y < \sigma_a (= \sigma_y / 1.7)$

since no possibility of buckling

$\tau = \frac{Q_d+Q_{L+i}}{A_w} < \tau_a (= \tau_y / 1.7)$

(Plus design of vertical stiffener)

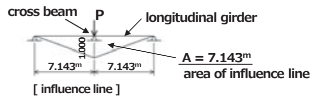
Deflection check



$\delta < \frac{L}{2,000}\ (L < 10^m)$

Design of cross beam

Span of cross beam is assumed to be distance between chord member

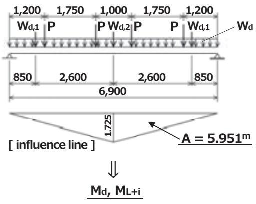


$W_{d,1}$ (outer girder) $= W_{d,Ⓐ} \times A$

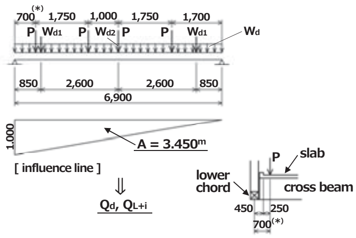
$W_{d,2}$ (inner girder) $= W_{d,Ⓑ} \times A$

W_d : Self weight of cross beam

Maximum bending moment

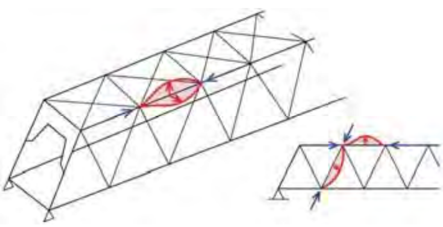


Shear force

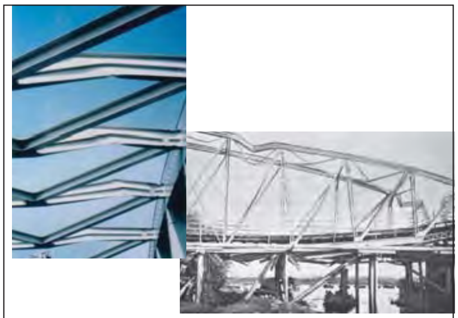


Design of chord and web members

Buckling of Truss Member



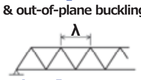
Buckling due to compression



Effective buckling length (Le)

[Chord member]

in-plane & out-of-plane buckling $L_e = \lambda$ (panel length)

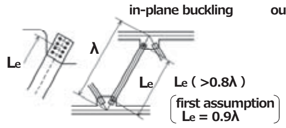


[Web member]

in-plane buckling

out-of-plane buckling

$L_e = \lambda$



L_e ($>0.8\lambda$)

(first assumption $L_e = 0.9\lambda$)

Maximum allowable slenderness ratio*

		L^{**}/r
compression	main member	120
	secondary member***	150
tension	main member	200
	secondary member	240

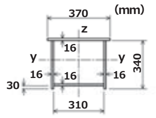
* to ensure bridge global rigidity

** effective buckling length (in compression) panel length (in tension)

*** members in cross or lateral bracing

Ex. Design of upper chord

(ex.) Upper chord U_3 (Axial force = -2370.1^{kN})



$A = 217.6^{cm^2}$
$I_y = 37{,}151^{cm^4}$
$I_z = 39{,}633^{cm^4} > I_y$
(SM400)

local buckling of plate b/t = 31/1.6 = 19.4 < 38.7 (ok)

global buckling of member

$L_e/r = \underline{714.3} / \sqrt{37{,}151/217.6} = 54.6$

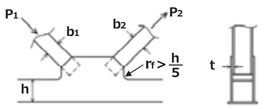
$\sigma_{ca} = 140 - 0.82\,(54.6 - 18) = 110^{N/mm^2}$

$\sigma = \dfrac{2370.1\times10^3}{217.6\times10^2} = 108.9\ ^{N/mm^2} < \sigma_{ca}$ (ok)

Design of gusset plate

t (plate thickness, mm) $> 2 \times \dfrac{P}{b}$

P : maximum force of end post or web member (kN)
b : width of end post or web member (mm)



Design of lateral bracing members

buckling length

$L_e = 0.9\lambda$ (λ : panel length)
(*from conservative viewpoint, $L_e = \lambda$)

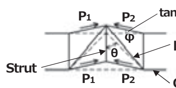
max. allowable slenderness ratio

in compression $L_e/r < 150$
in tension $\lambda/r < 240$

Strut and lateral bracing members attached to chord in compression have to be designed to resist the following loads

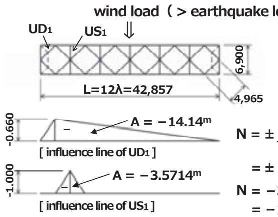
Strut : $\dfrac{P_1 + P_2}{100}$

lateral bracing : $\dfrac{P_1 + P_2}{100}\sec\theta$



Design of upper lateral bracing

wind load (> earthquake load)

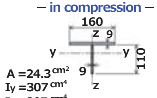


$N = \pm\ \underline{3.80}\times14.14$
↑wind load (kN/m)
$= \pm\ 53.73^{(kN)}$

$N = -3.80\times3.5714$
$= -13.57^{(kN)}$

UD1

– in compression –



$A = 24.3^{cm^2}$
$I_y = 307^{cm^4}$
$I_z = 307^{cm^4}$

check of slenderness ratio

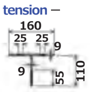
$L_e/r_y = 496.5/\sqrt{307/24.3} = \underline{140 < 150}$

$\sigma_{cay} = \dfrac{1{,}200{,}000}{6{,}700+140^2} = 45.6^{\,(N/mm^2)}$

$\sigma'_{cay} = 45.6\ (0.5+\dfrac{140}{1{,}000}) = 29.1^{(N/mm^2)}$

$\sigma_c = \dfrac{53.73\times10^3}{24.3\times10^2} = 22.1^{(N/mm^2)} < \underline{1.2}\sigma'_{cay} = 34.9^{(N/mm^2)}$
extra coefficient
(ok!!)

– in tension –

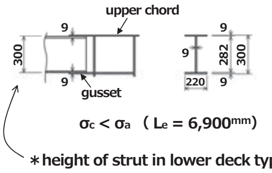


$\sigma_t = \dfrac{53.73\times10^3}{1{,}485} = 36.2^{(N/mm^2)} < 1.2\sigma_{ta}$
$= 1.2\times140 = 168^{(N/mm^2)}$

$A_n = (160-2\times25)\times9 + \dfrac{110}{2}\times9 = 1{,}485^{cm^2}$

A_n : net cross sectional area

US1

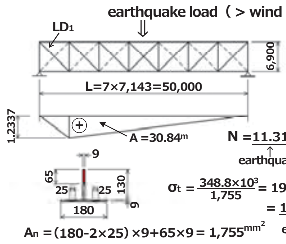


$\sigma_c < \sigma_a$ ($L_e = 6{,}900^{mm}$)

*height of strut in lower deck type bridge shall have the same height of the chord

Design of lower lateral bracing

earthquake load (> wind load)



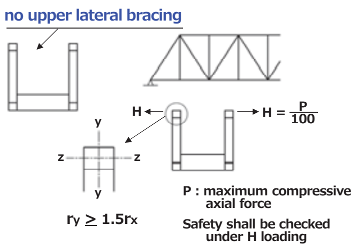
$N = \underline{11.31}\times30.84 = 348.8^{(kN)}$
earthquake load (kN/m)

$\sigma_t = \dfrac{348.8\times10^3}{1{,}755} = 198.8^{(N/mm^2)} < 1.5\sigma_{ta}$
$= \underline{1.5}\times140 = 210^{(N/mm^2)}$
extra coefficient

$A_n = (180-2\times25)\times9 + 65\times9 = 1{,}755^{mm^2}$

Pony Truss Bridges

Design of Pony truss



P : maximum compressive axial force

Safety shall be checked under H loading