

ミャンマー連邦共和国  
建設省橋梁局

ミャンマー国  
バゴー橋建設事業詳細設計調査

技術移転完了報告書  
付録

平成 30 年 3 月  
(2018 年)

独立行政法人  
国際協力機構 (JICA)

日本工営株式会社  
株式会社オリエンタルコンサルタンツグローバル  
首都高速道路株式会社  
株式会社長大  
大日本コンサルタント株式会社

基盤
JR
18-041



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付録リスト

## 付録 A 講義及びセミナー資料

- A-1 橋梁設計特別講義
- A-2 構造解析概論
- A-3 上部工の設計概論（コンクリート橋）
- A-4 上部工の設計概論（鋼橋）
- A-5 基礎工・下部工の設計概論
- A-6 風洞実験セミナー
- A-7 積算セミナー
- A-8 鋼斜張橋の設計演習
- A-9 鋼箱桁橋の設計演習
- A-10 PC 箱桁橋の設計演習
- A-11 下部工・基礎工の設計演習

## 付録 B 理解度測定試験

- B-1 特別講義のプレテスト
- B-2 上部工の設計概論（コンクリート橋）のプレテスト
- B-3 基礎工・下部工の設計概論のプレテスト
- B-4 特別講義の中間試験
- B-5 上部工の設計概論（コンクリート橋）の中間試験
- B-6 上部工の設計概論（鋼橋）の中間試験
- B-7 基礎工・下部工の設計概論の中間試験



付録 C 写真

C-1 講義及びセミナー写真

C-2 認定書授与式の写真

付録 D 出席表



付録 A 講義及びセミナー資料

A-1 橋梁設計特別講義



[10-1-1]

## Opening

Emeritus Prof. of  
Nagaoka University of Technology  
Masatsugu NAGAI

[10-1-1] Opening  
Cable-stayed bridges in Japan involved in  
Visited cable-stayed bridges  
Overseas bridges constructed by Japanese companies

[10-1-2] History of Bridges & Big Accidents

[10-1-3,4] Bridge General

Name of bridges, Bridge type, Selection of bridge type  
Material, Erection, DVD (Yokohama-Bay Br.)

[10-2-1] Slabs & Girder Bridges

[10-2-2] Truss & Arch Bridges

[10-2-3] Cable-stayed Bridges & Suspension Bridges

[10-2-4] DVD (Akashi kaikyo Br., Shinminato Br.)

## Contents of October Lecture

[10-3-1] Structural Mechanics

[10-3-2] Stress & Strain

[10-3-3] Cross-sectional Properties

[10-3-4] Practice

[10-4-1,2] Bending & Shear of Beam (Girder)

[10-4-3] Girder Stress

[10-4-4] Practice

[10-5-1,2] Axial Force of Truss Structures

[10-5-3] Practice

[10-5-4] DVD (steel manufacturing)

## Contents of November Lecture

[1] Maintenance, Monitoring

[2] Design method

[3] Buckling and its strength design, DVD

[4] Connection (bolt and welding design),  
DVD & Fatigue design

[5] (Slab, Girder, Truss) Bridge Design

## Cable-stayed bridge PJ in Japan involved in\*

\*[detailed design,  
chair or member of technical committee etc.]

## Contents of December Lecture

[1] Vibration DVD and basic theory

[2~5] Cable-stayed bridges

History

Name of members and structure

Design parameters and selection

Estimation of stress resultants

Design of Girder, Tower and Cable

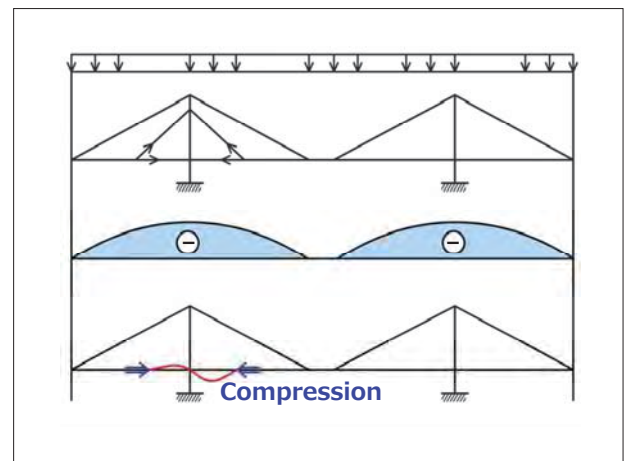
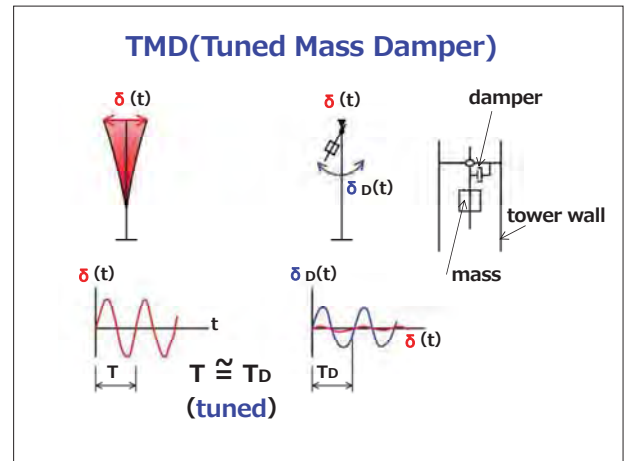
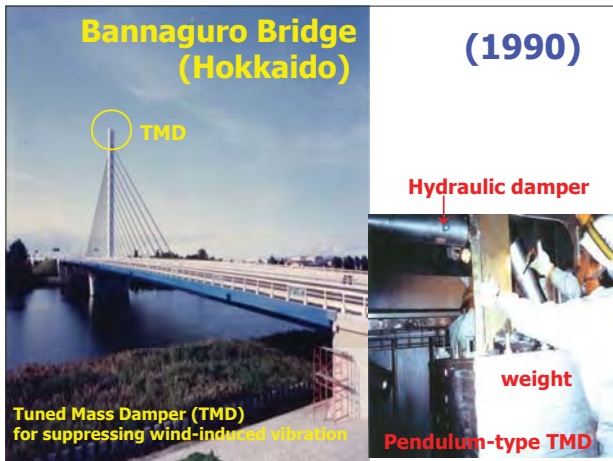
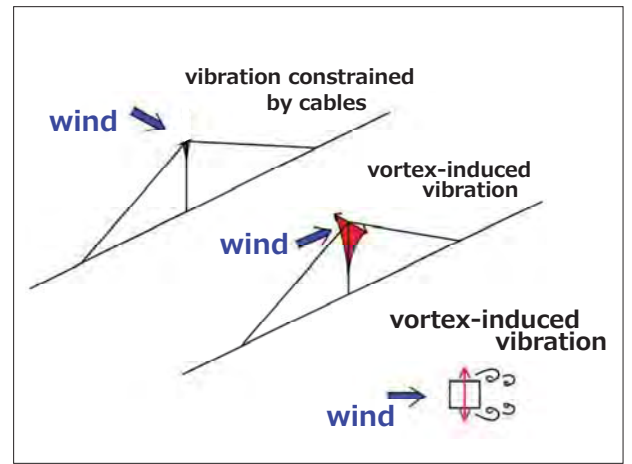
Erection, DVD

Wind-resistance design

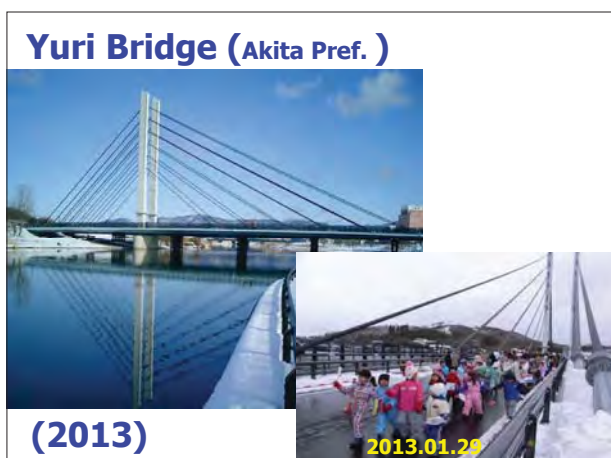
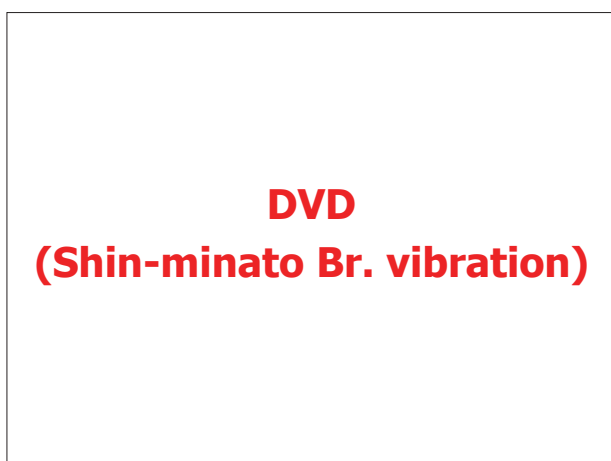
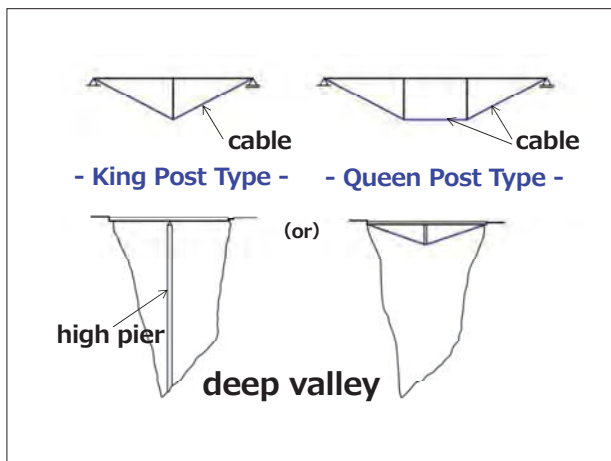
Super long-span bridges



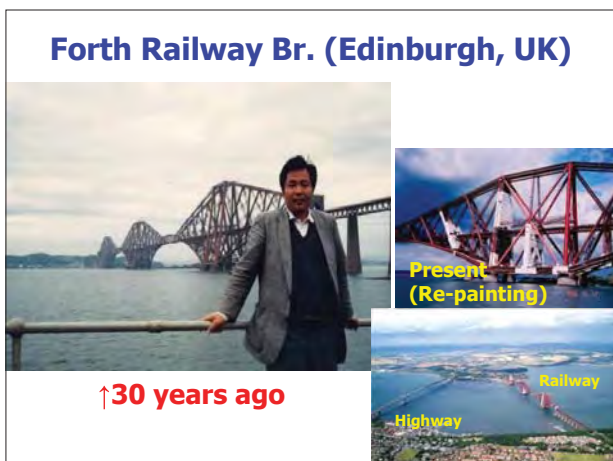
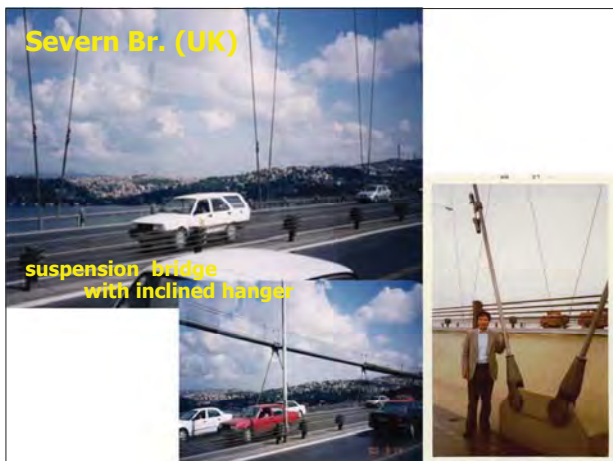
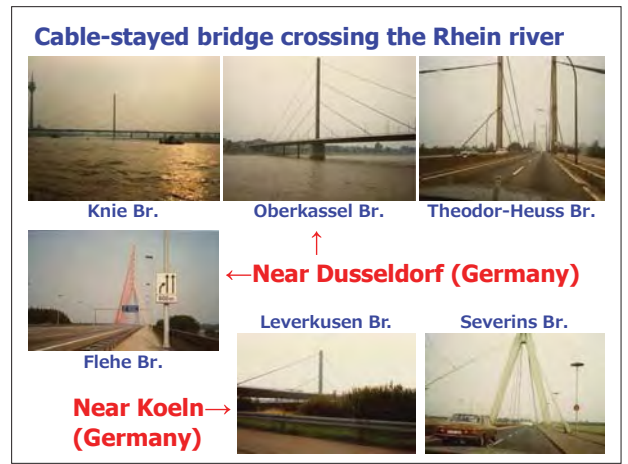














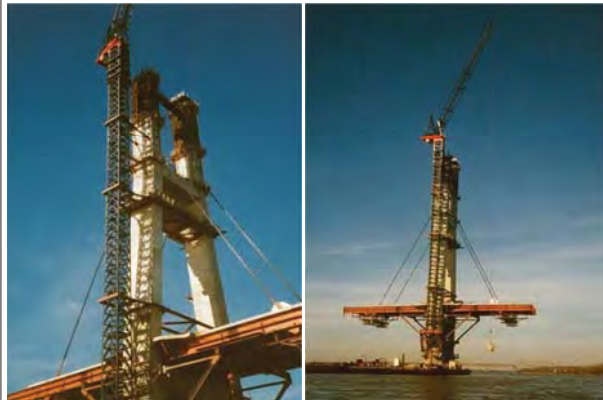
**Alex-Fraser Br. (Vancouver, Canada, 1986)**



**Rio-Antirio Br. (Greece, 2004)**



**Quincy Br. (St. Louis, USA, 1987)**



**Millau Viaduct Br. (Millau, France, 2004)**



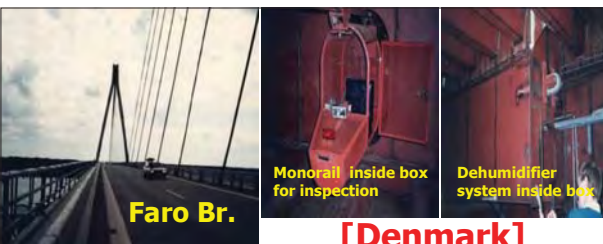
**Golden Ears Br. (Vancouver, Canada, 2010)**



**[Norway]**



**Faro Br.**

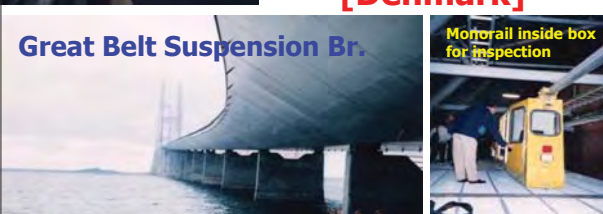


Monorail inside box for inspection

Dehumidifier system inside box

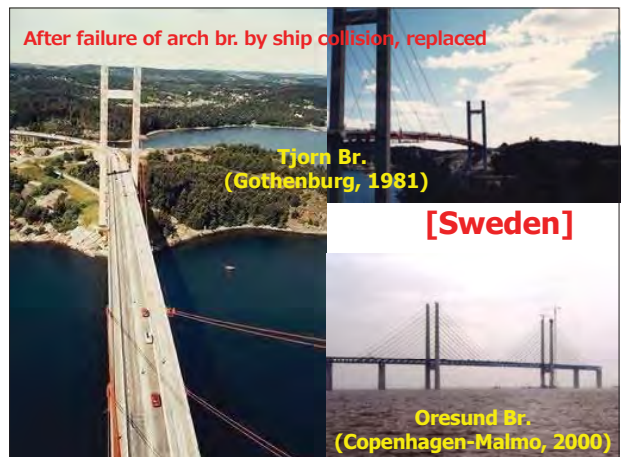
**[Denmark]**

**Great Belt Suspension Br.**

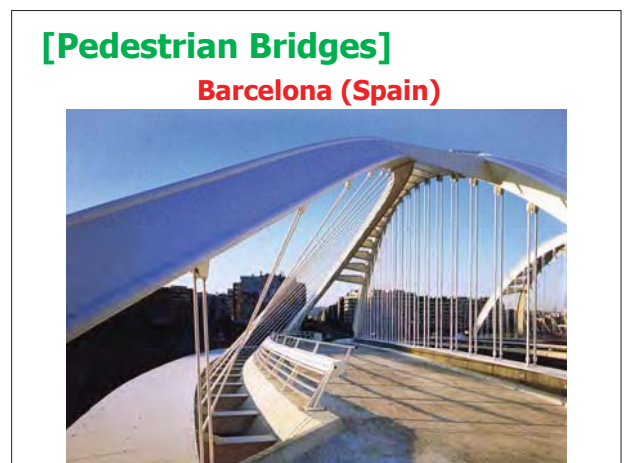


Monorail inside box for inspection

After failure of arch br. by ship collision, replaced









**Pedestrian bridges in London**



**Millennium Br. crossing the Thames river**



## **Oversea bridges constructed by Japanese companies**

**DVD  
(millennium bridge vibration)**

**Rama IX Bridge (Bangkok, 1987)**



**Karnali river Br. (Nepal, 1993)**



**Binh Br. (Vietnam, 2005)**

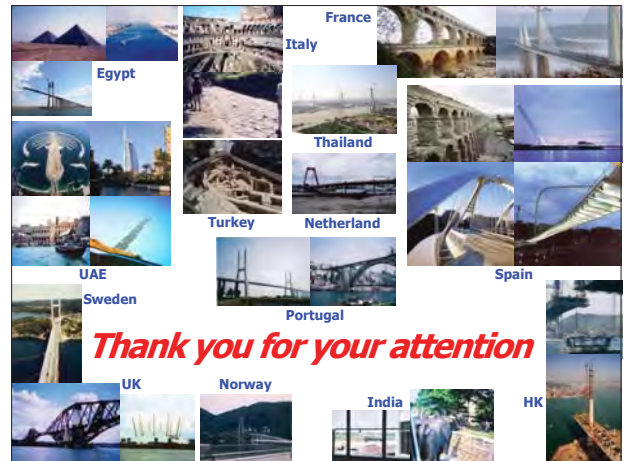
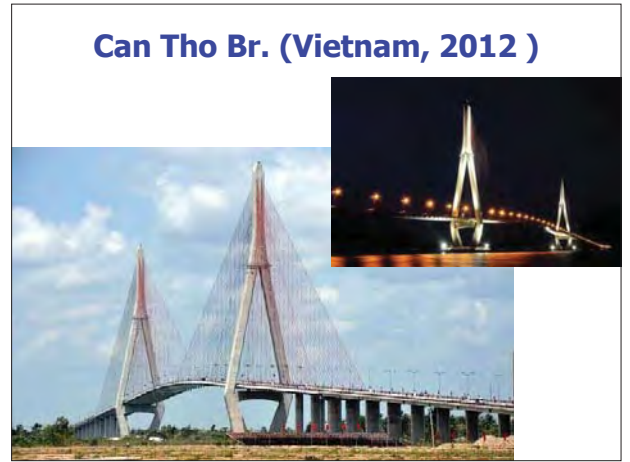
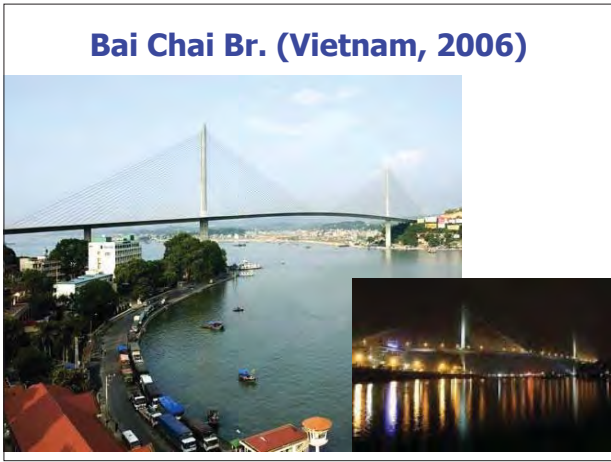
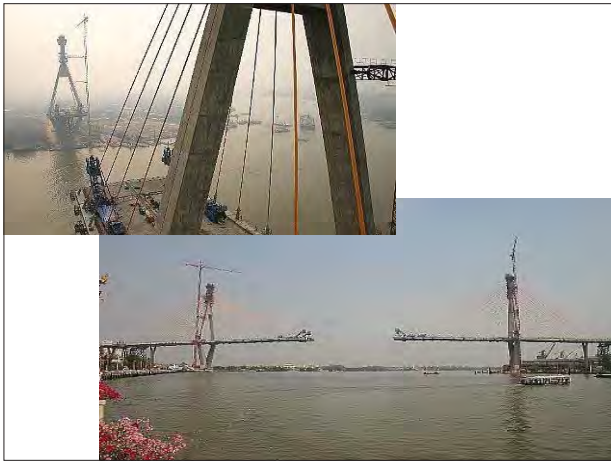


**Kap Shui Mun Br. (HK, China, 1997)**

**Ring Road No.1 & 2 Br. (Bangkok, 2006)**









[10-1-2]

## History of Bridges

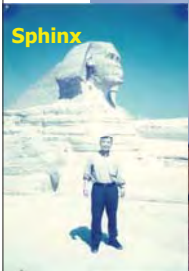


### Suez canal (Egypt)



### Pyramid

Sphinx



2600~2500 BC

Egyptian-Japanese  
Friendship Br.  
crossing Suez canal  
(2001)

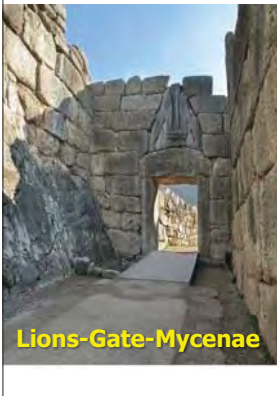


cable-stayed bridge



### Ancient Mycenae Era (1450-1150 BC)

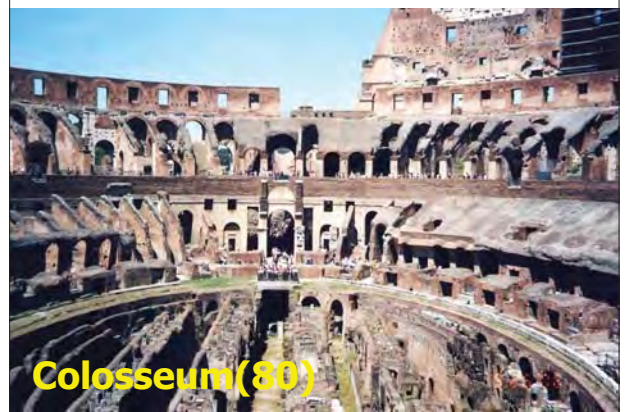
[before ancient Greece era]



Lions-Gate-Mycenae



Arkadiko Bridge  
1300-1190 BC



Colosseum(80)

### Ancient Rome Era

Ponte Fabricio  
(BC62)



Ponte Sant'Angelo  
(BC136)



colosseum

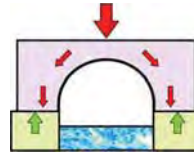
Aqua Claudia  
[total length 69km, 52]

コロッセオ(右上方)とクラウディア水道橋(復元模型)



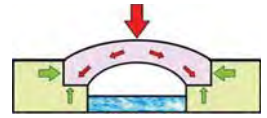


**Pon du Gurd  
(Niems, France)**



円形アーチ

**Circular arch**



扁平アーチ

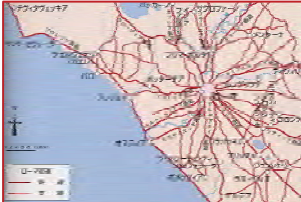
**Flat arch**



**Segovia Aqueduct  
(Segovia, Spain)  
「Devil bridge」**

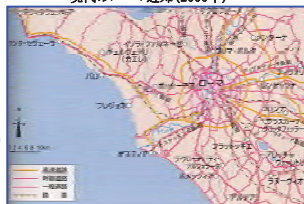


ローマ時代におけるローマ近郊の街道網(帝政期)



**Route network near Rome  
in Roman times  
( Imperial period )**

現代のローマ近郊(2000年)



**Modern Rome  
( the year of 2000 )**

**House built on Bridge**



**Rialto Br. (Venice, 1591)**

**Appian Way (Queen of way)**



**All roads lead to Rome**



**Porta San Sebastiano  
(starting point)**



**Vecchio Br.  
(Florence, Italy, rebuilt in1934)**



## Chapel Br. (Luzern, Switzerland, 1367) oldest timber bridge in Europe



**Partly burnt out, rebuild in 1994**

## Iron Bridge



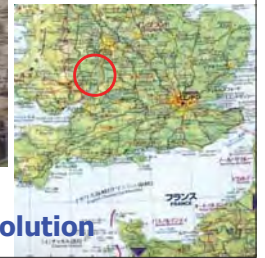
**(1779)**

**Symbol of Industrial Revolution**

**Cast iron**

→ **Wrought Iron**

→ **Steel**



**Menai Br.  
(1826, UK)**

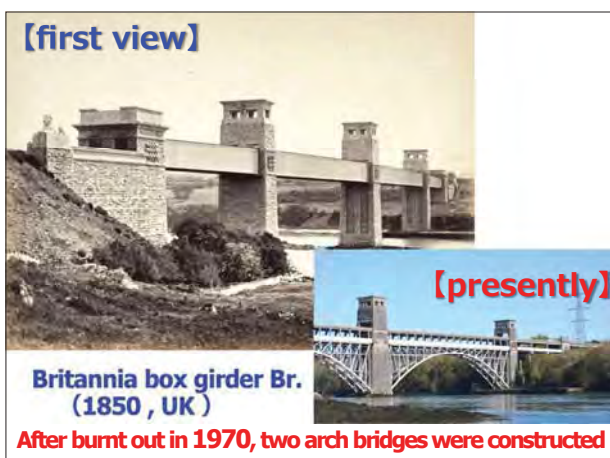


**Clifton Br.  
(1850, UK)**



**Brooklyn Br. (NY, 1883)**

**Construction of RC bridges started at around this time  
(end of 19<sup>th</sup> century)**



**[first view]**

**[presently]**

**Britannia box girder Br.  
(1850, UK)**

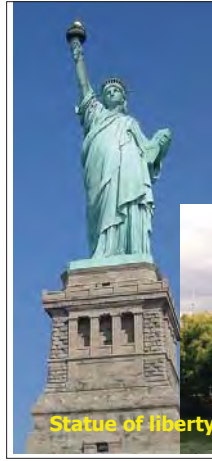
**After burnt out in 1970, two arch bridges were constructed**







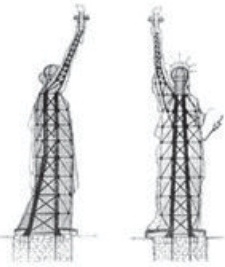
**Eiffel tower  
(Paris, 1889)**



**Statue of liberty (NY, 1886)**



**(Alexandre Gustave Eiffel)  
(1832~1923)**



**Maria Pia Bridge Porto  
(Portugal, 1877)**

**Eiffel's Design  
railway bridges**



**Garabit viaduct  
(France, 1884)**



**Long Bien Bridge  
(Hanoi, Vietnam 1903)**



**George Washington Br. (NY, 1931)**



**Tower Br. (London, 1894 )  
Movable br. crossing the Thames**

**steam engine**



**Golden Gate Br. (SF, USA, 1937 )**

**Construction of PC bridges started at around 1940**



**Tacoma Narrows Br.**  
(Washington , USA, 1940)



**Flutter**(self-excited vibration)



**Severn Bridge (1981, UK)**

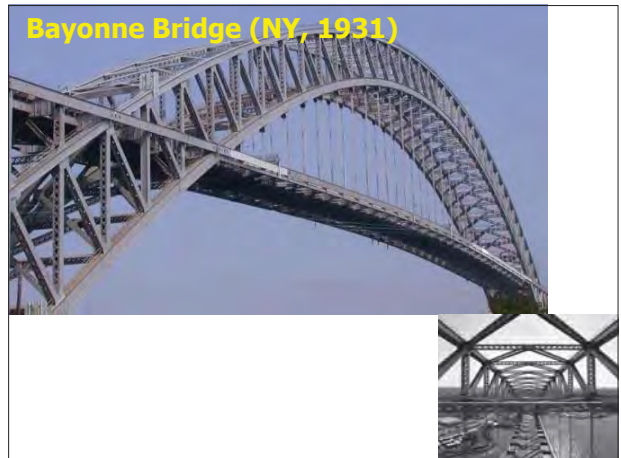
**Flutter (DVD)**

**Innovation**  
[Streamlined-shape box ]  
[Inclined hangers]



**Forth railway bridge**  
(Edinburgh, UK, 1890)

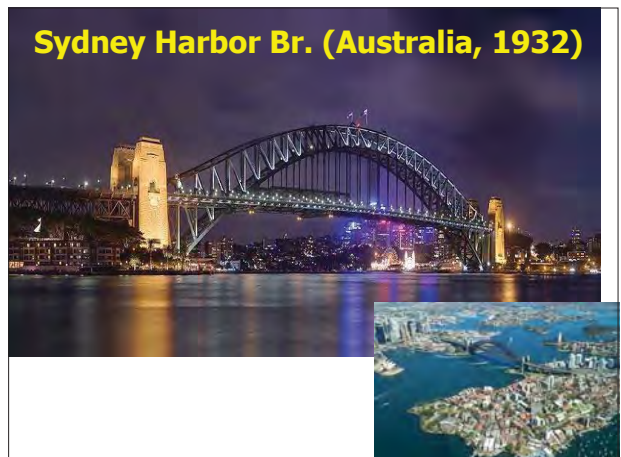
**Bayonne Bridge (NY, 1931)**



**Quebec Br. (Canada, 1919)**

**Collapse, 1907**

**Sydney Harbor Br. (Australia, 1932)**





# Introduction of unusual bridges and stone bridges in Japan



Kintai arch Br.



Saru Br.



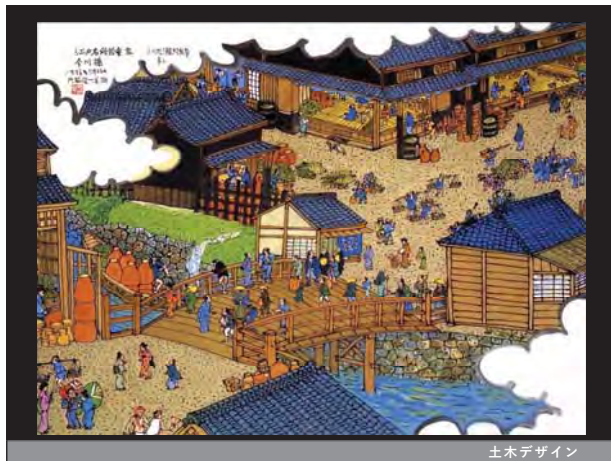
Kiso Sanbashi



Kazura Br.



Aimoto Br.



Kintai Bridge (timber arch br.)



Saru-hash (Saru Br.)



Old Aimoto Br.



Piled Jetty at Kiso district



Kazura Br.

cable is  
from climbing plant





## Stone Bridges in Japan



**Eye Glass Bridge (Nagasaki)**



**Tsunjun Bridge**



**Isahaya arch bridge**

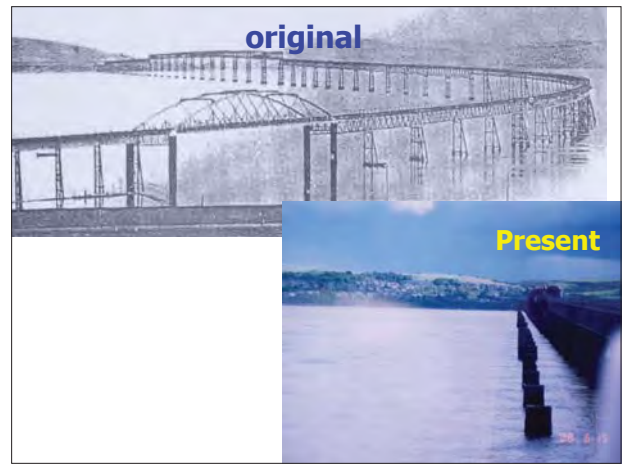
## Big Bridge Accident (around every 30 years)

## Explanation of lateral torsional buckling

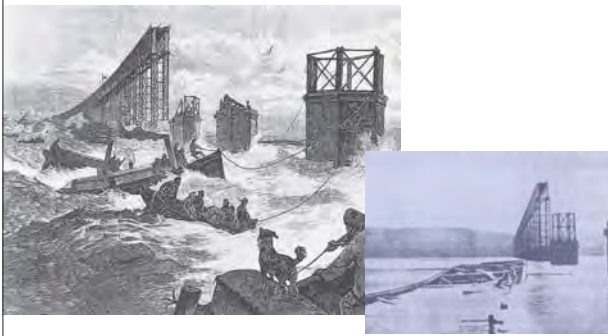
**1847, Dee Br. (Chester, UK)**  
**lateral torsional buckling**







**1879, Tay Br. (Scotland, UK)**  
**[Mistake of evaluation of wind loading]**



**Forth railway bridge (Edinburgh, UK, 1890)**



Higher safety factor was set affected by Tay railway bridge failure

**Dee Br., Tay Br. & Forth railway Br. (UK)**

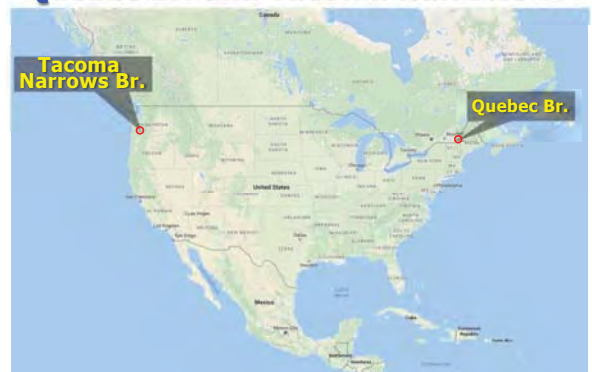


**1940, Tacoma Narrows Br. (Washington, USA)**

**Flutter**



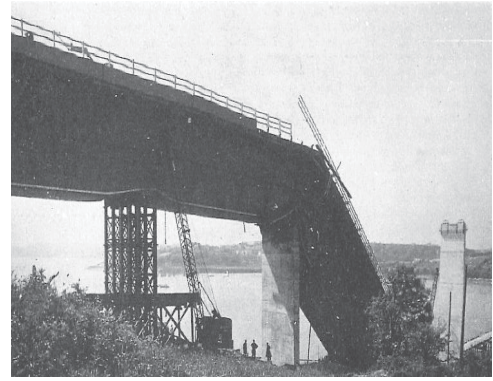
**Quebec Br. and Tacoma Narrows Br.**



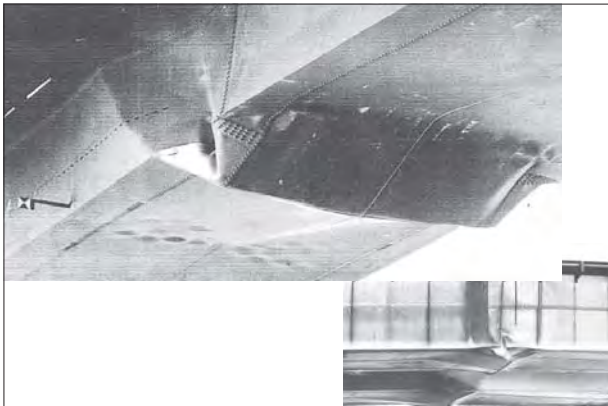


## Collapse of 4 box girder bridges at around 1970

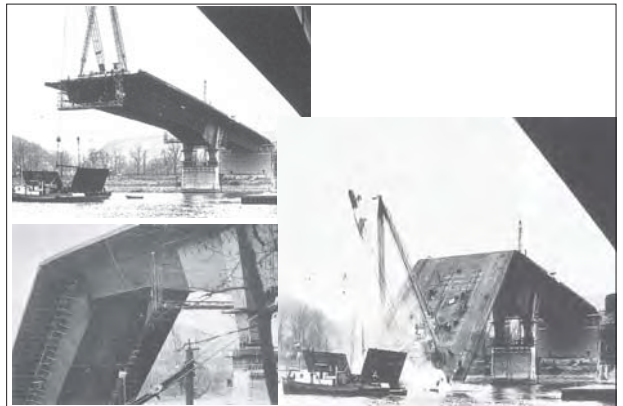
- 4<sup>th</sup> Danube Br. at Vienna (Austria, 1969)
- Milford Haven Br. (Wales, UK, 1970)
- Rhine Br. at Koblenz (Germany, 1971)
- Bridge at Zeulenroda (Germany, 1973)



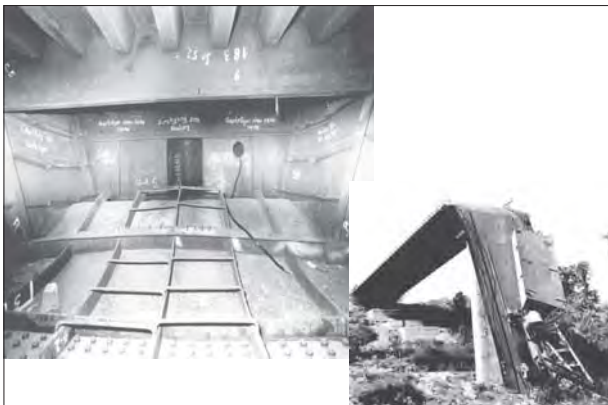
**Cleddau Bridge (Milford Haven, UK, 1970)**



**4th Danube Bridge (Vienna, 1969)**



**Rhine Bridge at Koblenz (1971)**



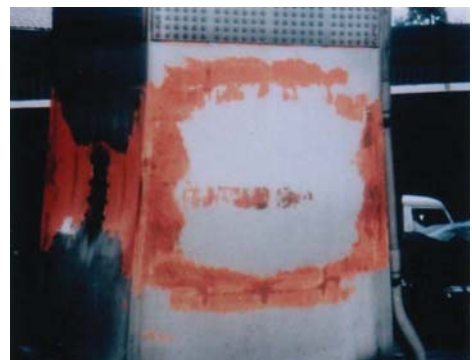
**Bridge at Zeulenroda (Germany, 1973)**

## Buckling of longitudinal ribs



## Explanation of buckling of stiffened plates

## Global buckling of stiffened plate





## Local buckling of plates



## Big accident after completion

### Major causes

#### «buckling»

- lateral torsional buckling(1847)
  - column buckling(1907)
    - stiffened plate buckling(around 1970)

#### «wind action »

- Evaluation of load(1879)
  - Dynamic action[self-excited vibration] (1940)

### Silver Br. (USA, 1967, 39-year old)



### Mainus Br. (USA, 1983, 28-year old)



### Seongsu Br. (Seoul, 1994, 15-year old)



### I-35W Br. (Minnesota, USA, 2007, 40-year old)



### 40-year old, USA



### 35-year old, Canada



**corrosion**



**Thank you for your kind attention**

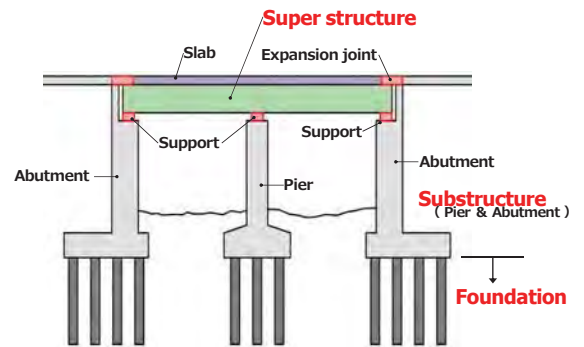




[10-1-3,4]

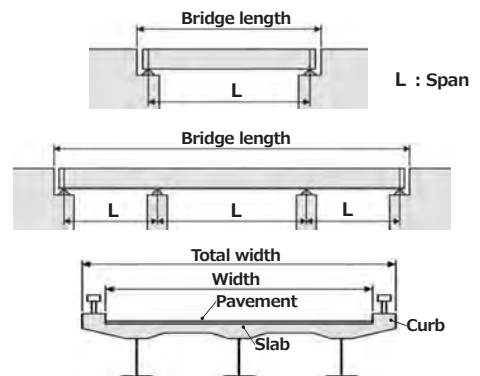
## Bridge General

### Name of each member

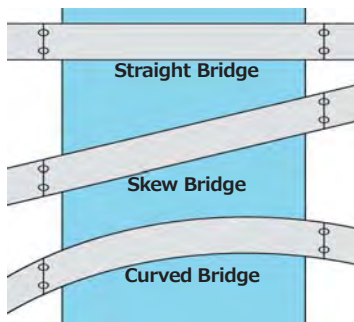


## Name of Bridges

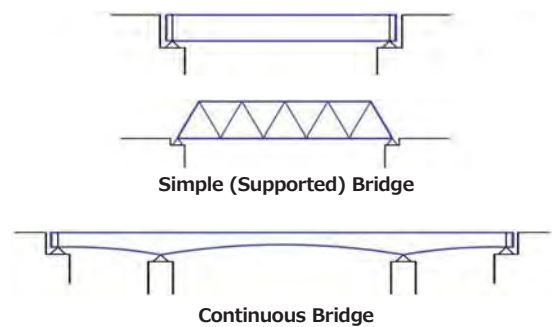
### Bridge Length and Width



### Skew and Curved Bridge



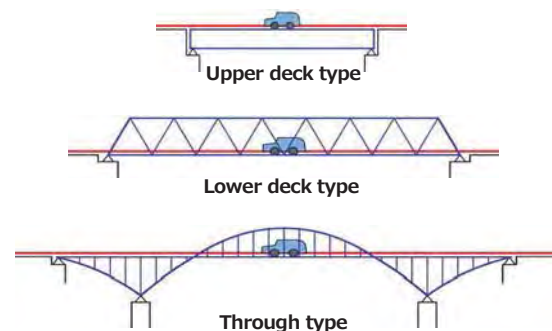
### Simple and Continuous Bridges



### Curved Bridges



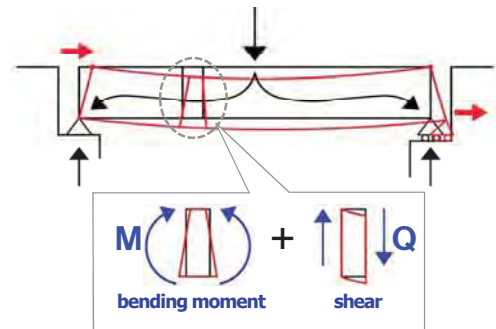
### Vehicle Traveling Position





# Bridge Types

## Load, Deformation and Stress



## Plate girder bridges

I-shaped(open section)  
&  
box(closed section) girder bridges

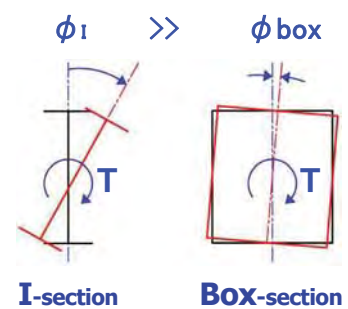
## Plate Girder Bridge (I-girder Bridge)



## Plate Girder Bridge (Box-girder Bridge)



## Torsional Deformation



Composite girder  
with triangular cross section

PC box girder  
with steel corrugated web

## Rigid Frame Type Bridges



## Π-shaped Rigid Frame Bridge



## Vierendeel\* Bridge



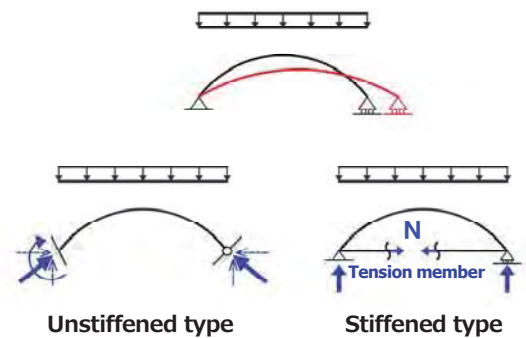
## V-leg Rigid Frame Bridge



## Truss Bridges

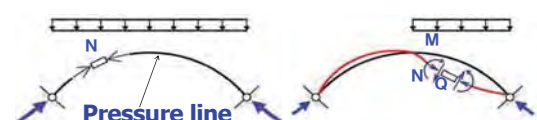


## Load and Deformation of Arch



## Arch Bridges

## Arch Member Shape



for short span : Circular  
for long span : Parabolic



The figure consists of four photographs arranged in a 2x2 grid, each showing a different bridge design:

- Top Left:** A large steel arch bridge with a prominent central arch and multiple smaller arches, spanning a body of water. A large ship is visible in the foreground.
- Top Right:** A concrete arch bridge with a single large arch, supported by multiple concrete piers, spanning a body of water.
- Bottom Left:** A red steel arch bridge with two distinct arches, spanning a body of water. A large ship is visible in the foreground.
- Bottom Right:** A concrete arch bridge with a single large arch, supported by a large yellow concrete pier, spanning a body of water.

[illegible]

# Cable-stayed Bridges

The diagram illustrates the internal forces in a continuous beam with two equal spans of length  $L$ . The top part shows the deflection curve, which is symmetric about the center of each span. The middle part shows the shear force diagram, which is a constant negative value of  $-wL/2$  in each span. The bottom part shows the bending moment diagram, which is a parabolic shape, reaching a maximum of  $-wL^2/8$  at the supports and zero at the free ends. The labels "Bending moment" and "axial force" are present, indicating the nature of the internal forces.



## World Longest Cable-stayed Bridges (1,104m)



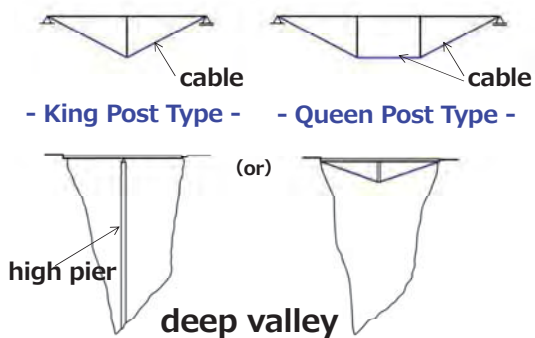
## Curved Cable-stayed Bridges



## Continuous Cable-stayed Bridges



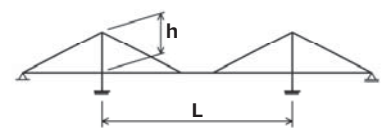
## Tower (post) underneath girder



## Extradosed Bridge

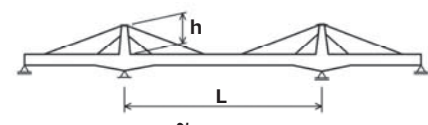


## Cable-stayed bridges



$$L / h \approx 5.0 \text{ (economical)}$$

## Extradosed type



$$L / h \approx 10.0$$

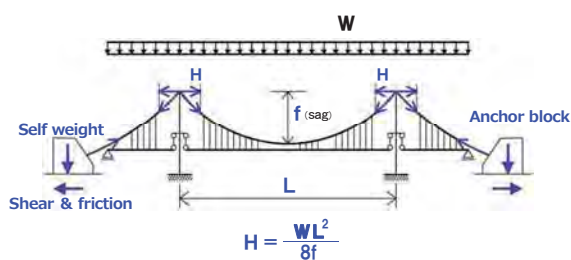


## Suspension Bridges

### Suspension bridges



### Load and Stress Resultant



### How to decide bridge types?

**Economical solution depends on the span length**

### Applied span of girder bridges [from 10 to 150m, and max. span 300m]

Bridge Type	Span Length (m)	Typical Cross Section	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
Single Composite I-Girder																	
Single Non-composite I-Girder																	
Single Composite I-Girder																	
Single Non-composite Box Girder																	
Single Composite Box Girder																	
Continuous Non-composite I-Girder																	
Continuous Non-composite Box Girder																	
Steel Plate Deck I-Girder																	
Steel Plate Deck Box Girder																	
Single I-Girder (PC Composite Slab)																	
Continuous I-Girder (PC Composite Slab)																	
Narrow Box Girder (PC Composite Slab)																	
Open Box Girder																	

**Normally**, the short span length leads to the cheap cost.  
**Because**, the cost of substructures is cheaper than that of superstructures.

**However**, if the soil condition at construction site is very bad, cost evaluation will change.

### Truss bridges [applied from 50 to 500m]



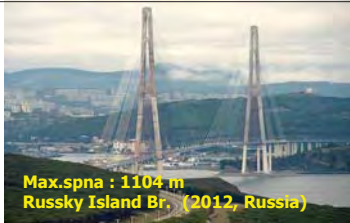
Max. span : 549 m  
Chaotianman Br. (2009, China)



### Arch bridges [applied from 80 to 500m]



**Cable-stayed bridges**  
[applied from  
**100 to 1100m]**



Max. span : 1104 m  
Russky Island Br. (2012, Russia)



Max. span : 1991 m  
Akashi Strait Br. (1998, Japan)

**Suspension bridges**  
[applied from  
**300 to 2000m]**

**Cable-stayed Type**



**Pedestrian Bridges**

**Floating Type**



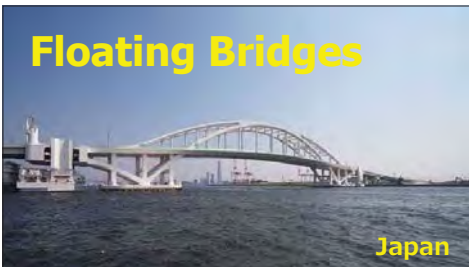
**Arch type**



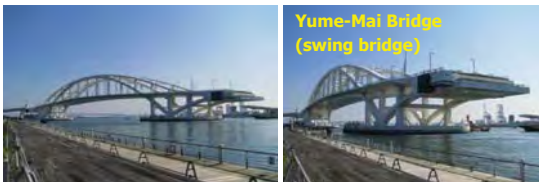
**Norway**



**Floating Bridges**



**Japan**



Yume-Mai Bridge  
(swing bridge)

**Materials**



- Steel Bridges
- Concrete Bridges
- Steel-concrete Hybrid Bridges
- Other materials such as CFRP, Stainless and so on

## RC Bridges (Japan)



No. 11 Bridge  
Japan's First  
Western-style RC Bridge



Hijiri Bridge  
RC Arch Bridge



Tenshou Bridge  
The longest Span & highest  
under clearance (h=143m)  
RC Arch Bridge in Japan

## Steel Bridges

Bridges of steel such as,



Japan

## PC Bridges (Japan)



Sugitani Bridge  
PC 6-Span Rigid Frame  
Box-girder with Corrugate  
Steel Web

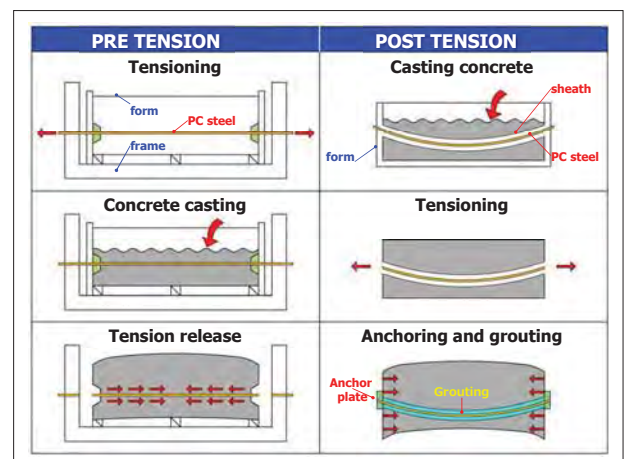
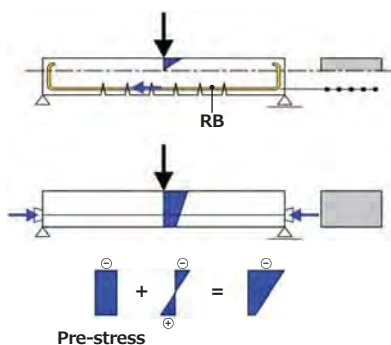


The longest Span PC  
Extra-doused Bridge  
in Japan



PC Bridge of SHIN-TOMEI  
Expressway

## RC Beam and PC Beam

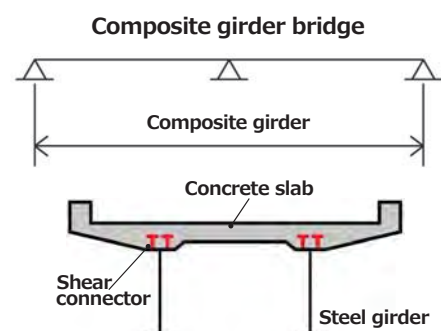


## Pre-stress introduction method

Pre-tensioning system

Post-tensioning system

## Hybrid bridge (Composite type)





## Composite Girder Bridge 2-I-Girder type



## Sear Connectors for composite action

Headed studs

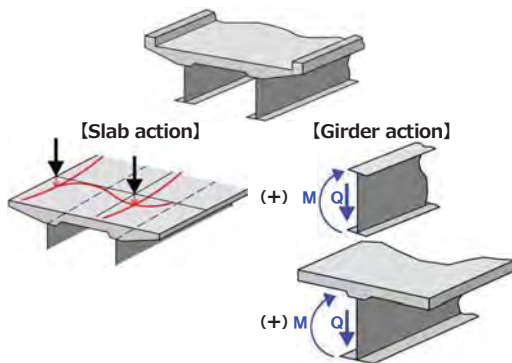
Perforated steel plates

## In case of non composite design

slab anchor [bent reinforcement]

↑ Preventing separation between slab and steel girder

## Slab and Girder action



## [composite design] Shear connectors

Headed stud

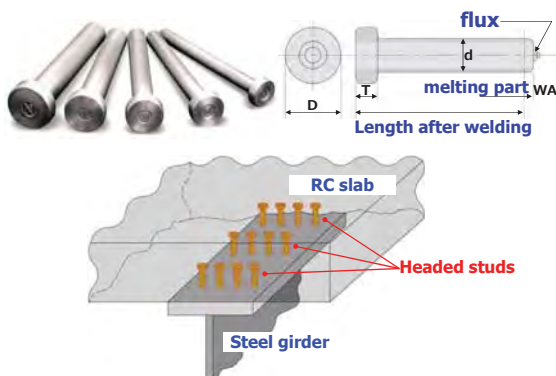
Perfobond strip



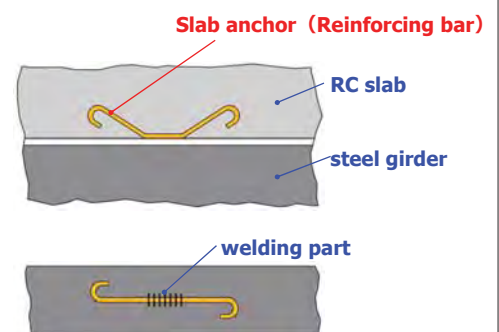
## [non composite design]



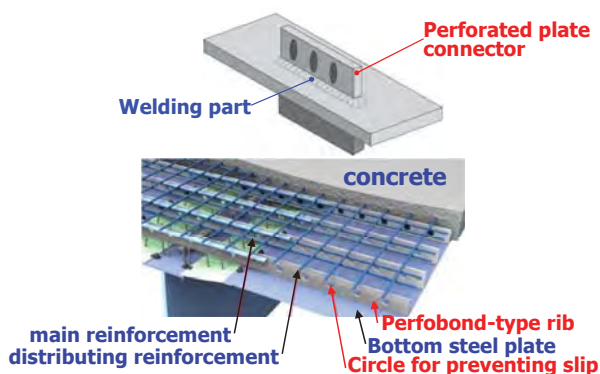
## Headed studs



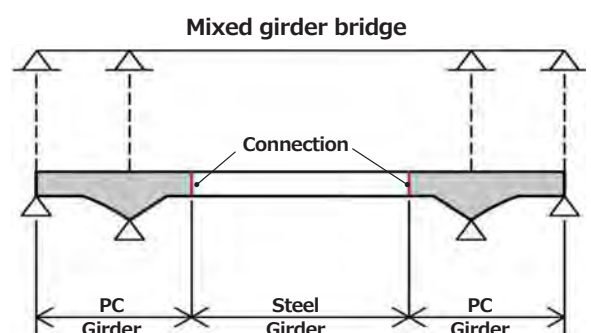
## Non-composite design(bent of RB)



## Perforated steel Plate Connector



## Hybrid bridge (Mixed type)

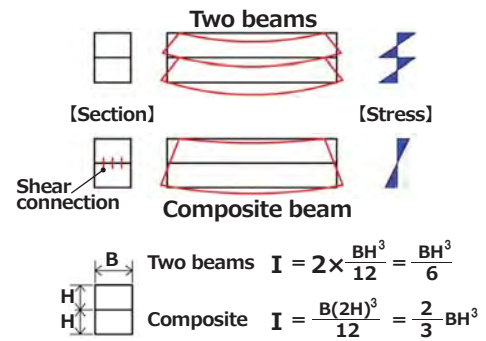




## Shinkawa Bridge (Japan)



## Two beams and Composite beam



## Basic concept of Hybrid bridges

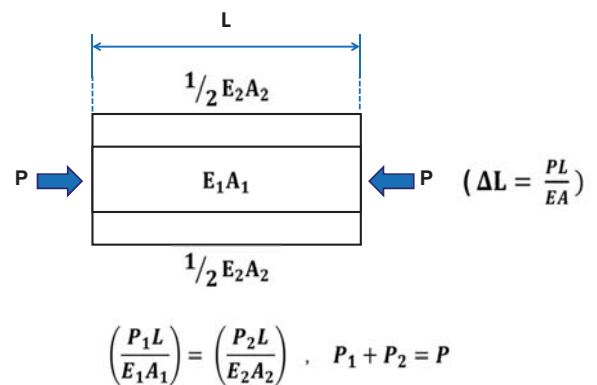
- utilize or combination of both merits -

### Steel

- High strength [400~700MPa]  
→ (designed structures is light)
- prone to buckle since thin plates

### Concrete

- Tough against Compression  
[strength is 30~100MPa]  
→ less possibility of buckling  
→ heavy structures
- However, cheap structures is designed



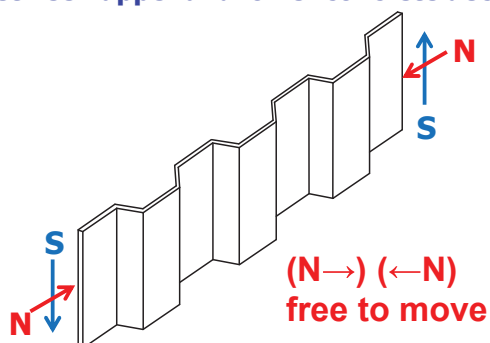
## PC Box Girder with Steel Corrugated Webs



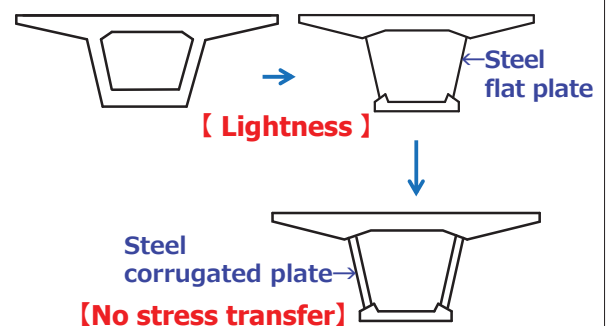
## Basic concept (aim of steel web)

- (1) Lightness by introducing steel web
- (2) Effective introduction of Pre-stress by PC tendon
- (3) Reduction of stress transfer between concrete decks (shrinkage and creep) and steel webs

## Steel corrugated web between upper and lower concrete decks



## PC girder

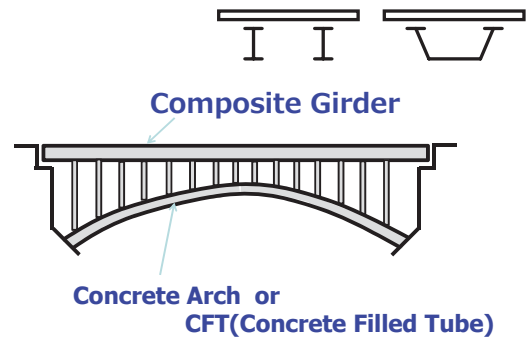




## Butterfly (High Strength)Concrete Web



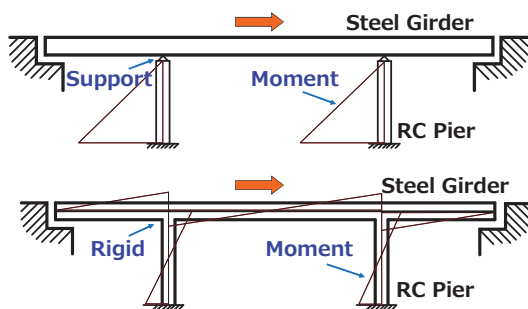
## Hybrid Arch Bridge



## PC Box Girder with steel truss web



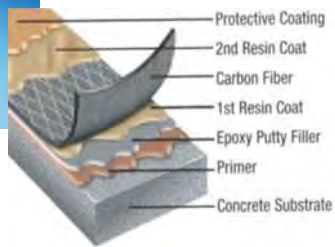
## Hybrid Rigid Frame Bridge



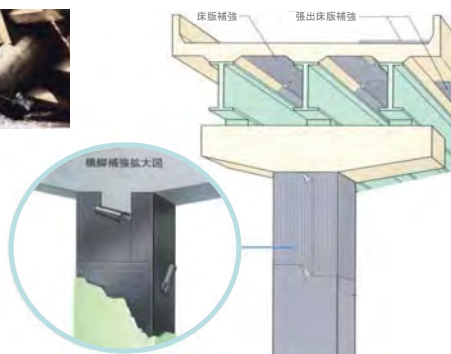
## New Materials



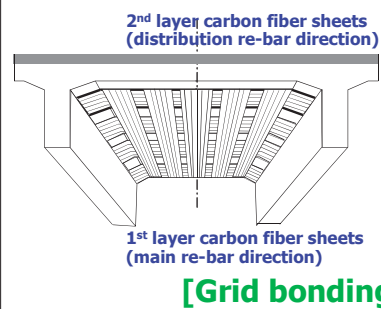
## Carbon Fiber Sheets



## Seismic retrofitting of concrete piers with Carbon Fiber Sheets



## Strengthening of RC Slabs using Carbon Fiber Sheet



## NEW Repairing method for the corroded Steel

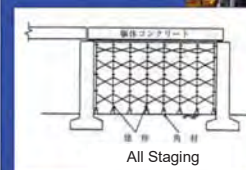


Strand Sheet



No heat, No bolt hole  
Efficient and Economical

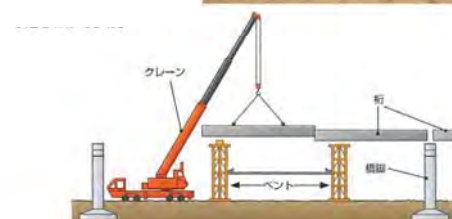
## Erection with All Staging Method



Reference: Japan Bridge Association Inc. HP

## How to erect ?

## Erection with Bents method





## Launching Method



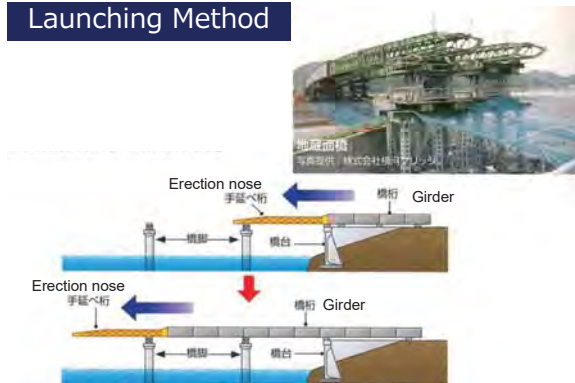
Reference: Japan Bridge Association Inc. HP

## Cantilever Erection Method

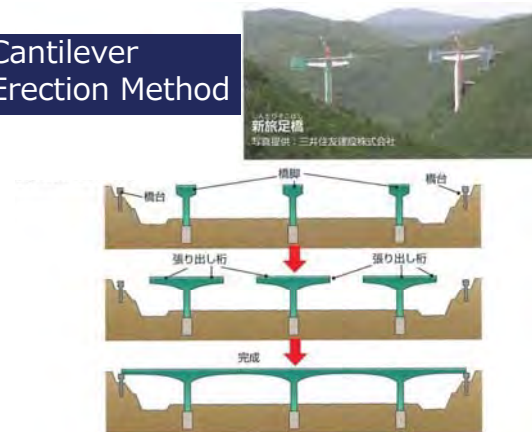


Movable form  
Carrier  
移动式支保工

## Launching Method



## Cantilever Erection Method



## Floating Crane Erection Method



Reference: Japan Bridge Association Inc. HP

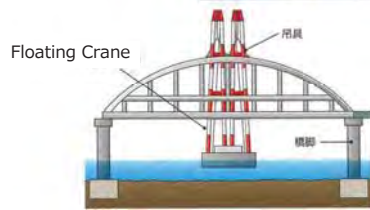


## Floating Crane Erection Method

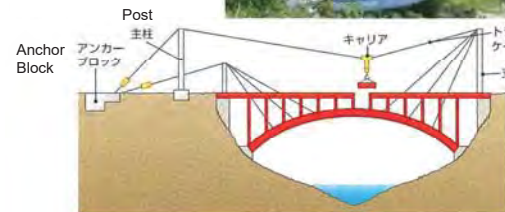




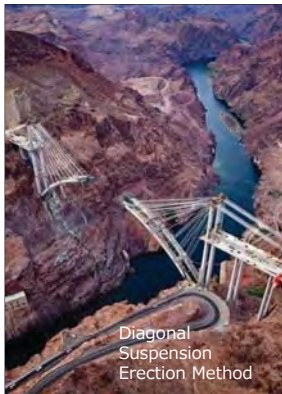
## Large Block Erection Method



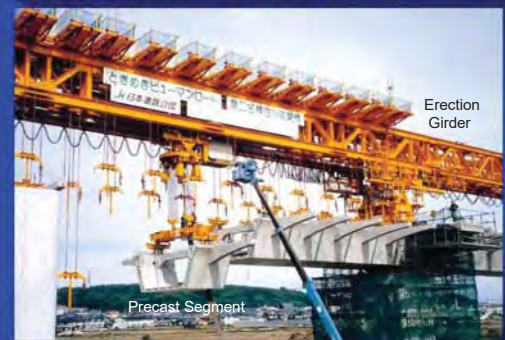
## Cable Erection Method



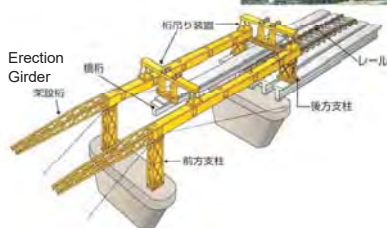
## Erection Method for Arch Bridge



## Precast Segment Construction Method



## Erection by Erection Girder



Thank you for your kind attention



[DVD]

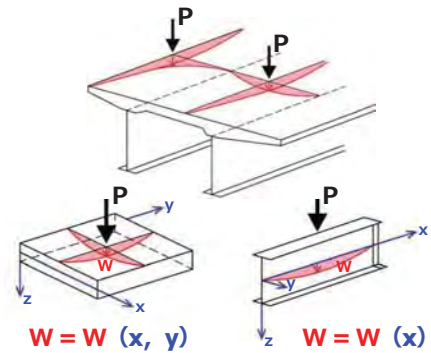
Yokohama-Bay Bridge



[10-2-1]

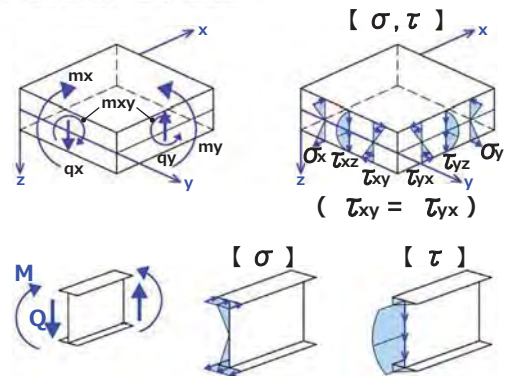
# Slabs and Girder Bridges

## Load and Displacement

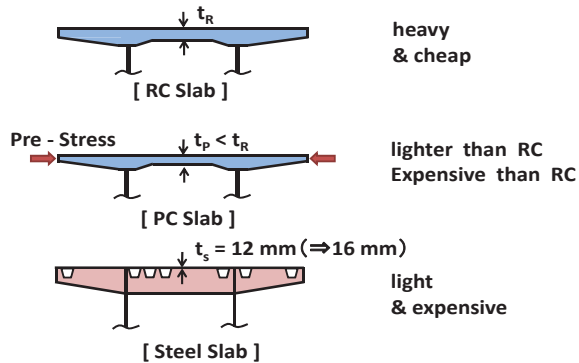


## Slabs

## Internal Stress



## Slabs for roadway

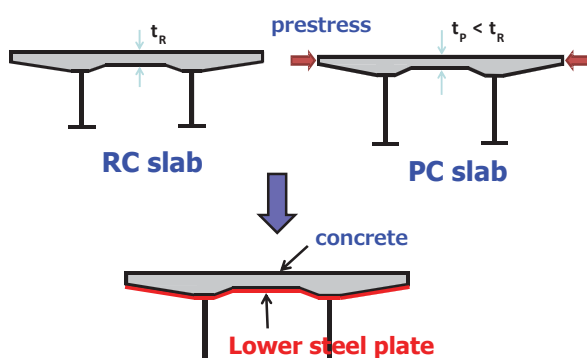


- Lower plate type -

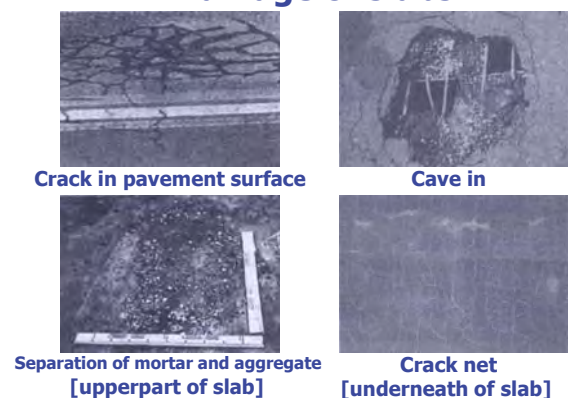


- Double plate type (sandwich type) -

## Steel-concrete composite slab

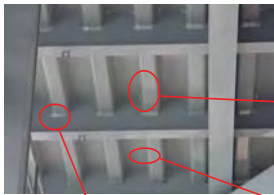


## Damage of slabs

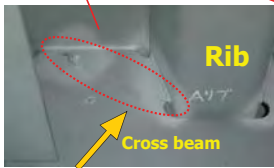




### Fatigue damage of steel deck plate



Steel deck and longitudinal ribs



Rib

Cross beam

Cross beams and longitudinal ribs



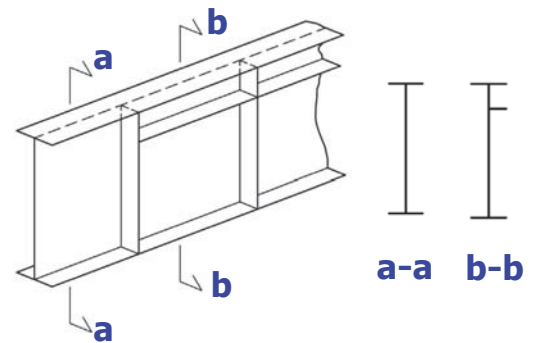
Welding part of longitudinal ribs

## I-girder bridges

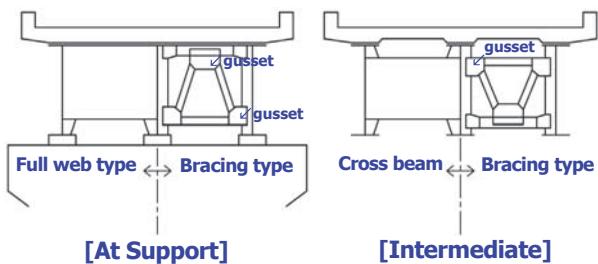
### Plate girder bridges

- I-girder and box girder bridges -

### I-girder Section



### Cross Section of I-girder



short span bridges



### Composite I and box girder bridges

with a few main girders and  
simple transverse stiffening system

Girder bridge  
for light rail  
(London)





## Composite 2-I-girder bridge in France



Courtesy by Prof. Raoul (SETRA, France)  
Haute-Colme (TGV north)



Seibair on two-girder bridges Japan 1104

11

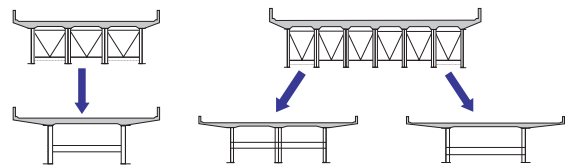
## composite 2-I-girder bridge for TGV

## Composite 2-I-girder bridge in France

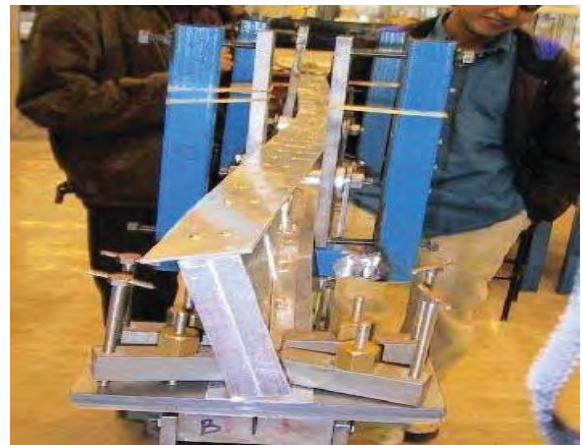
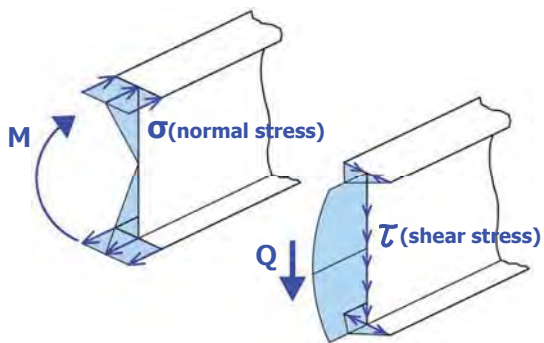


## Composite I-girder bridge (span:30~60m)

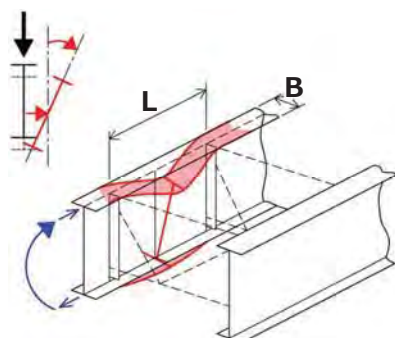
[in JAPAN]



## Internal Stress of I-girder

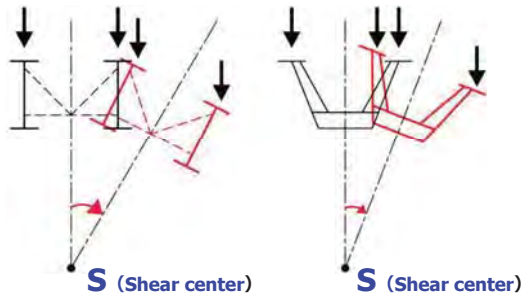


## Lateral torsional buckling

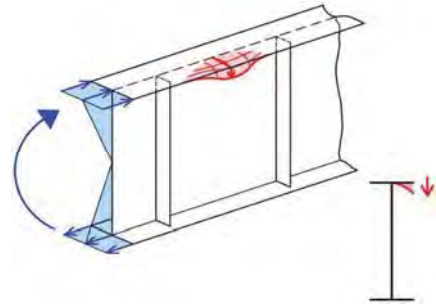




## Global lateral torsional buckling

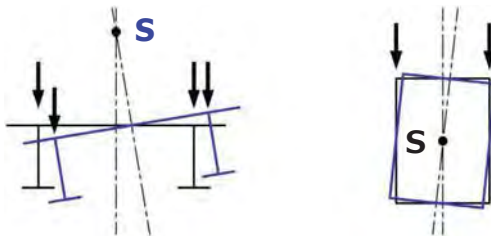


## Local Buckling at Flange-PL

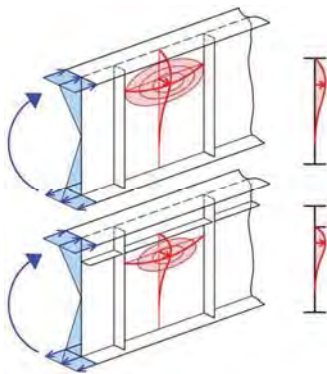


## Torsion Deformation

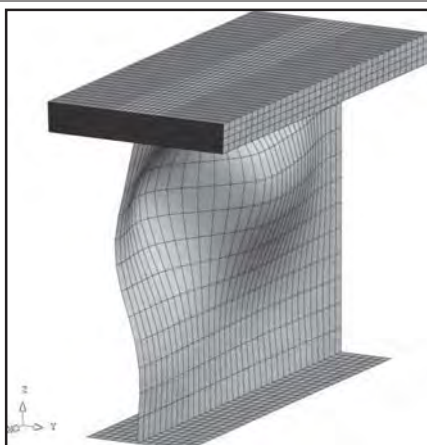
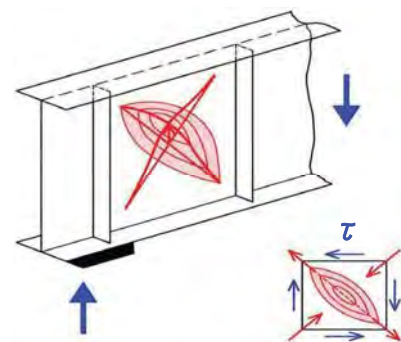
no possibility of lateral torsional buckling



## Buckling at Web-PL



## Shear Buckling at Web-PL





## Design(safety check)

determine the proportion [size and thickness]

$$f \leq f_{ult.}/\gamma$$

**f** : stress ( $\sigma$ ,  $\tau$ )

**f<sub>ult.</sub>** : strength

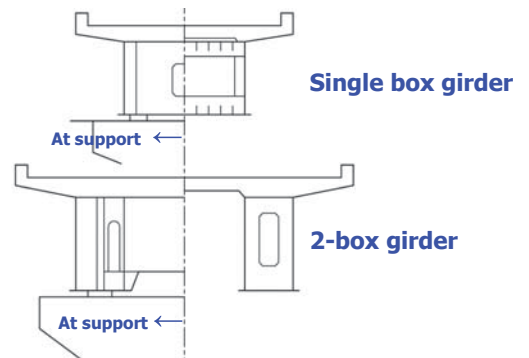
**$\gamma$**  : safety factor

$$S^* \leq R^*$$

**S\*** : factored action

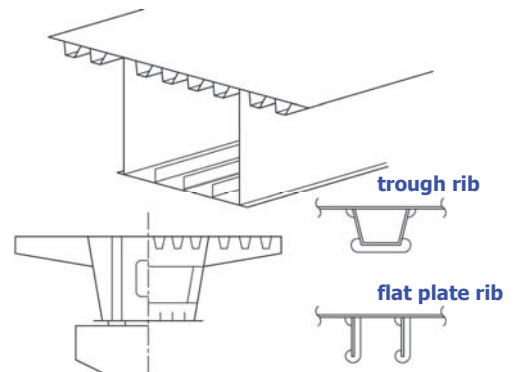
**R\*** : factored strength(resistance)

## Cross Section of Box-girder



## Box-girder bridges

## Steel Plate Deck Box-girder

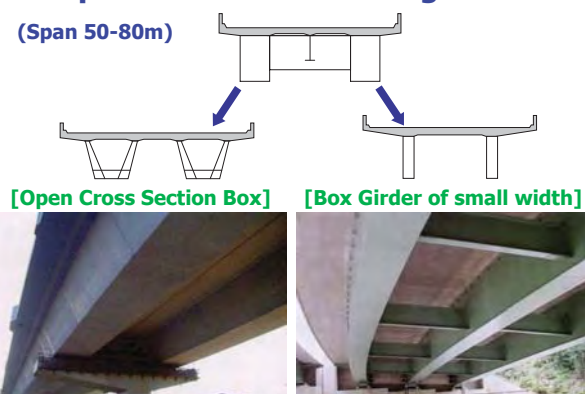


## Omonogawa Bridge



## Composite Box-Girder Bridge [in Japan]

(Span 50-80m)



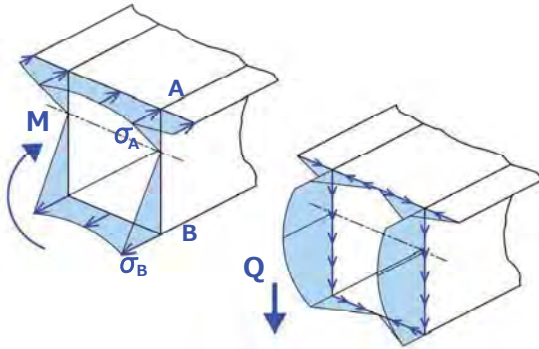
## Narrow Width Box Girder Bridge

(Omit the intermediate Cross Beams)



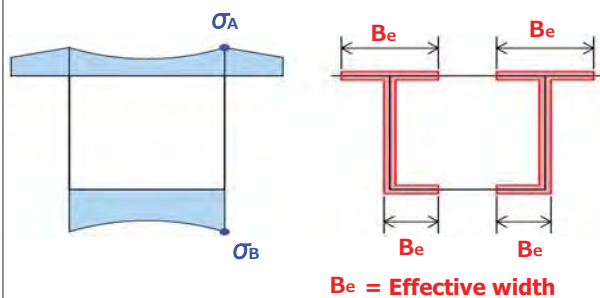


## Internal Stress of Box-girder



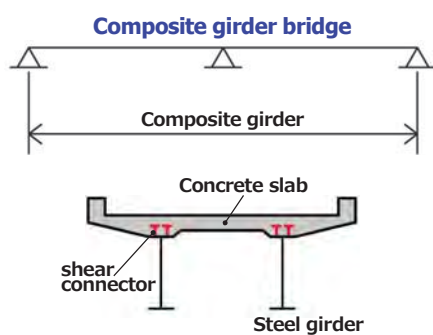
## Steel-concrete hybrid girder bridges

## Effective Width by Bending Stress

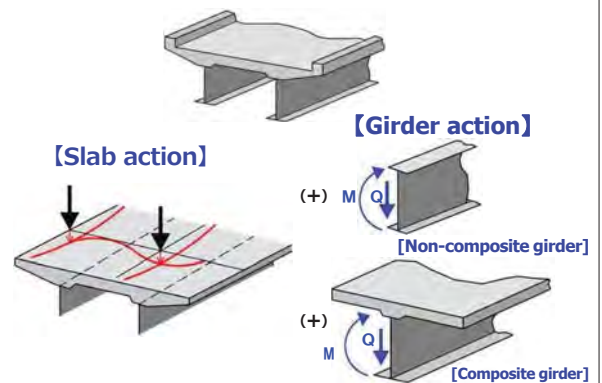


## composite girder bridges

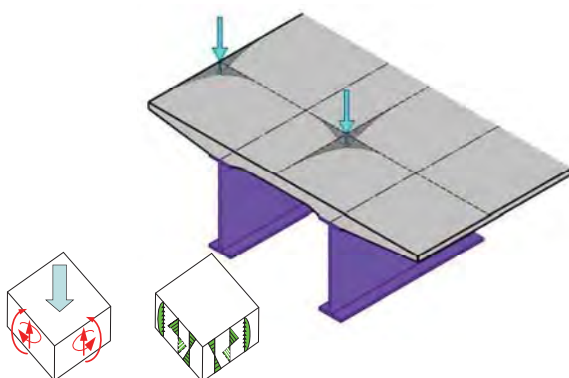
## Composite Structure



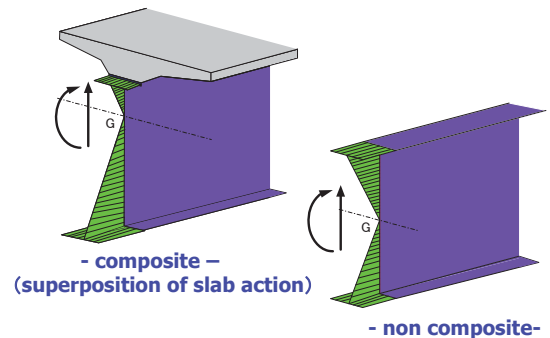
## Slab and Girder action



## Slab action

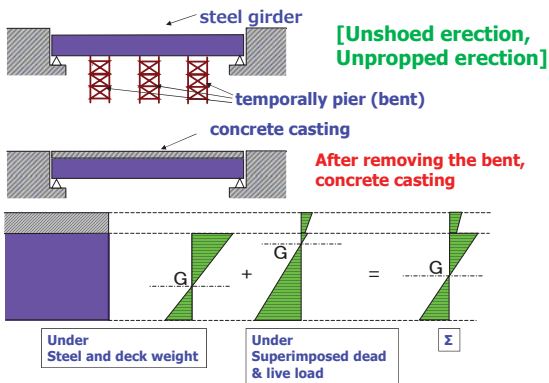


## Girder action

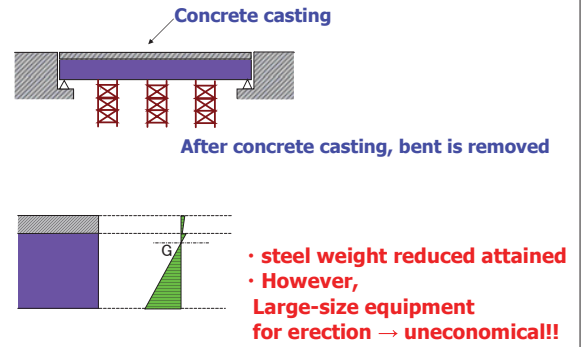




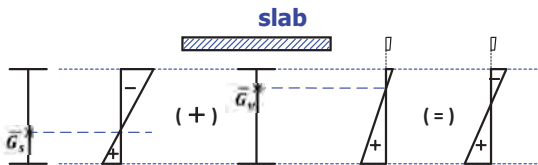
### Composite girder bridge (composite action under live loading)



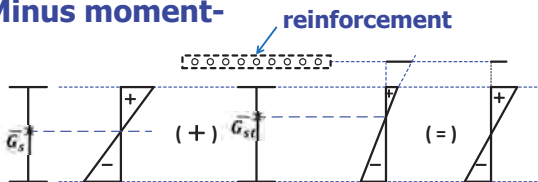
### Composite girder bridge (composite action under both dead and live load)



### -Plus moment-



### -Minus moment-



### Sear Connectors for composite action

#### Headed studs

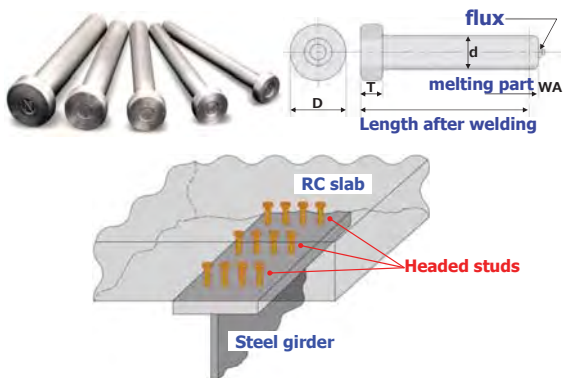
#### Perforated steel plates

### In case of non composite design

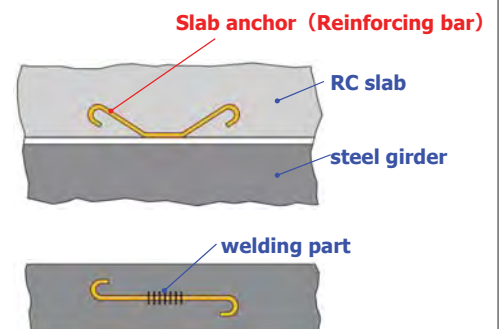
#### slab anchor [bent reinforcement]

↑ Preventing separation between slab and steel girder

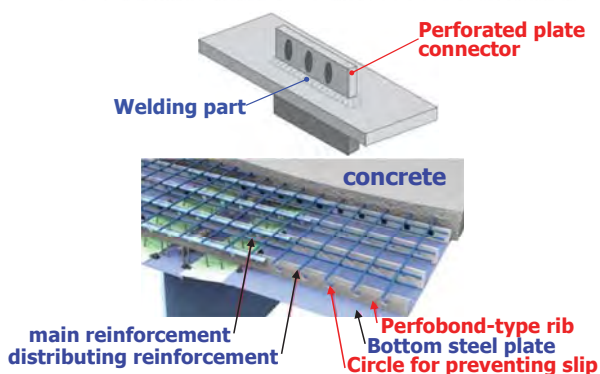
### Headed studs



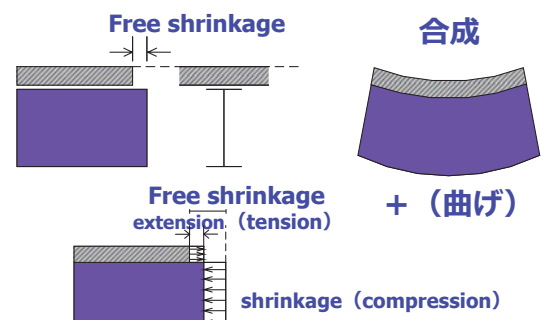
### Non-composite design(bent of RB)



### Perforated steel Plate Connector

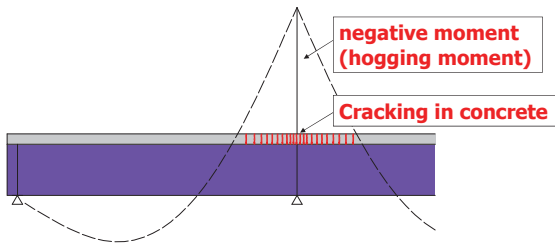


### Stress transfer due to creep and shrinkage





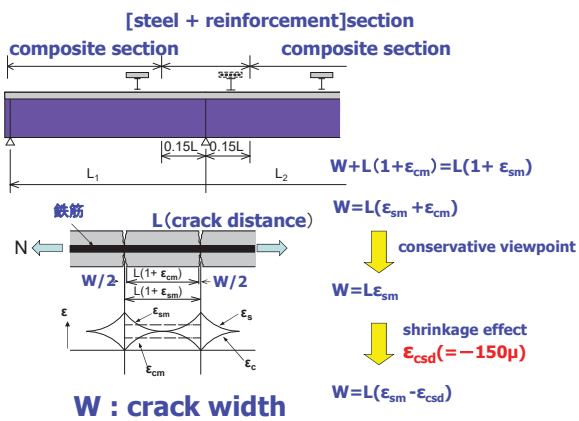
## Crack width control design



JHBS :  $\rho_s = 2\%$

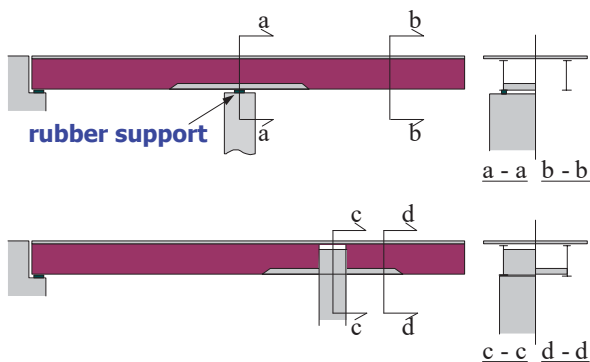
海外  
NEXCO } : control design

$$\rho_s = \frac{\text{reinforcement area}}{\text{slab area}}$$

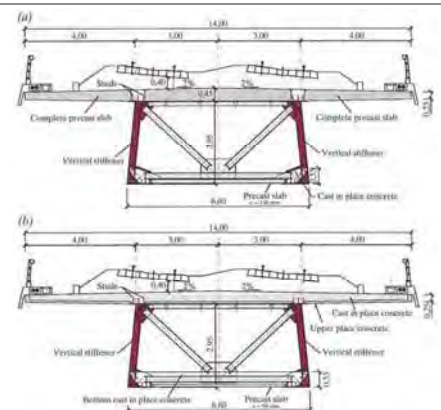


## Double composite bridges

### Double composite girder bridges



### Ex. (lower deck installed at intermediate support)



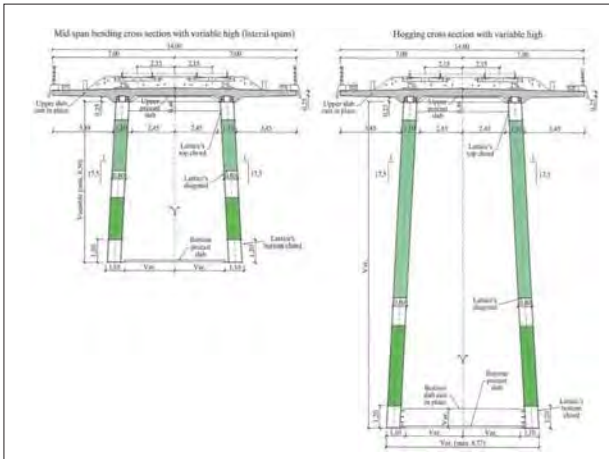
(a) span center (b) intermediate support





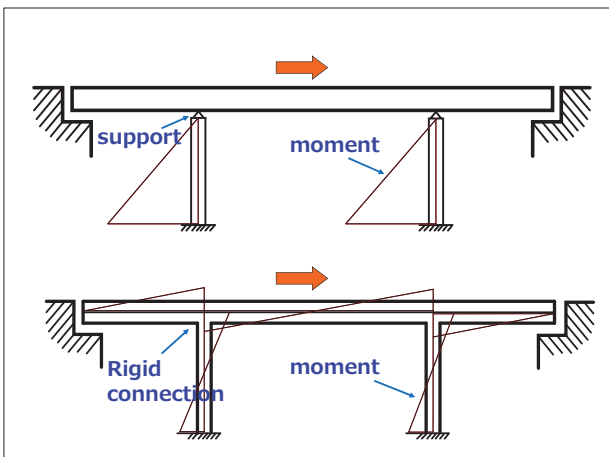
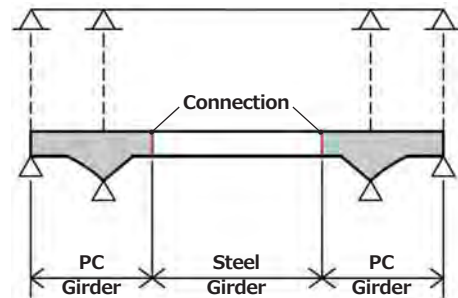
**Fig.2: Lateral view of viaduct over River Ulla**

## Rigid frame type bridges



## Mixed Structure

### Mixed girder bridge



## Shinkawa Br.

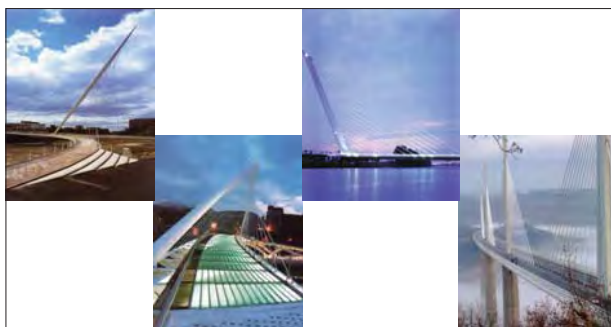




## PC Box-girder bridge with steel corrugated webs

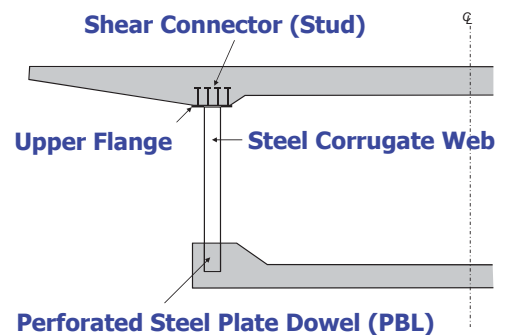


## PC Box-girder with Steel Truss Webs



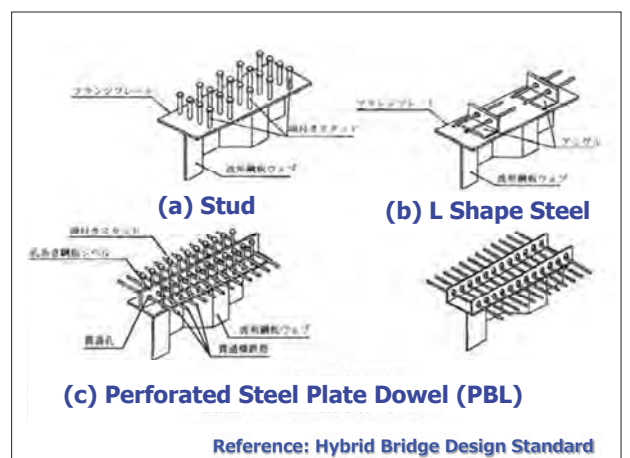
Thank you for your kind attention

## Joint Method of Steel & Concrete (Recent Example)



## Appendix (1)

### PC Box-girder with Steel Corrugated Web - Joint of Slab & Steel Web -





#### (d) PBL of the Steel Corrugate Web

**Reference: Hybrid Bridge Design Standard**

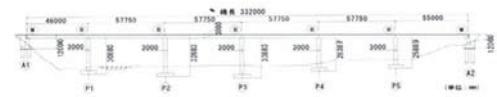


图 1.1.1 构造一般图

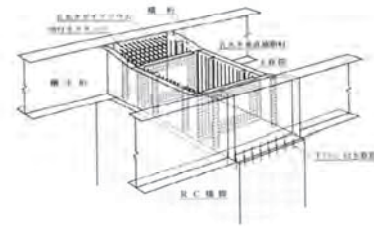


圖 1.1.2 剛結部概要圖

### (e) Rigid-connecting Structure

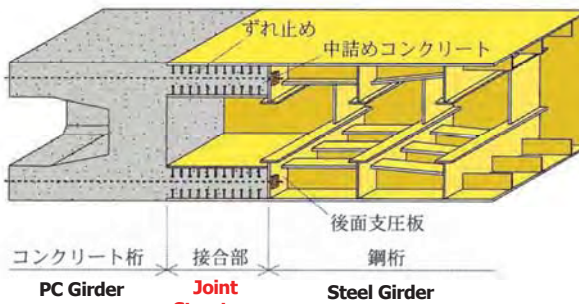
## Appendix (2)

## Rigid frame Structure -Joint of RC Pier & 2-girders Bridge-

## Appendix (3)

## Mixed Girder Joint Structure

### -Joint Structure of PC Box Girder & Steel Box Girder-



### (f) Joint Structure of Mixed Girder

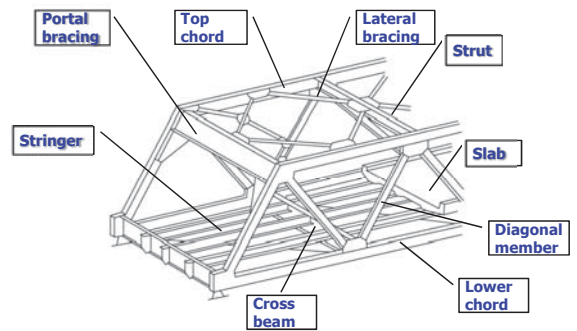
**Reference: Hybrid Bridge Design Standard**



[10-2-2]

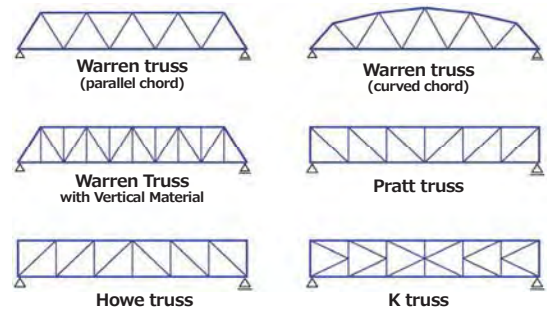
## Truss & Arch Bridges

### Name of Truss Members

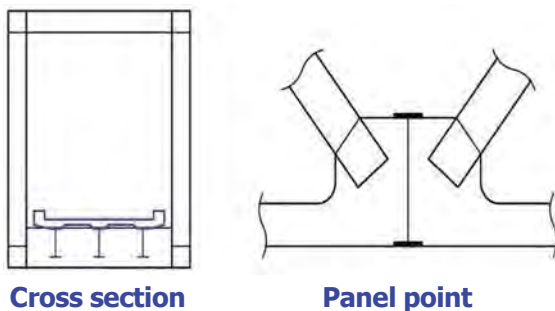


## Truss Bridges

### Types of typical Truss Bridge



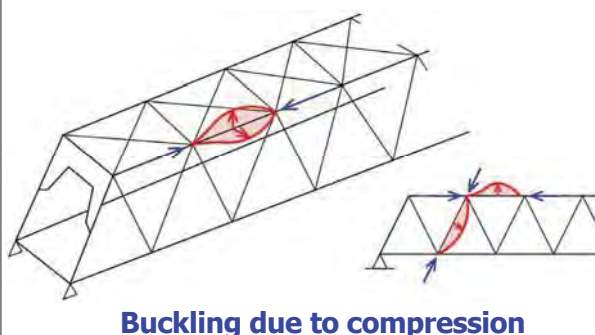
### Cross section and Panel point



### Forth Railway Br. (Edinburgh, UK)



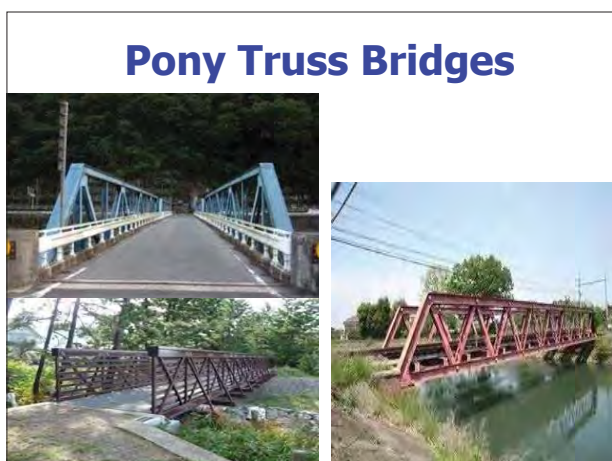
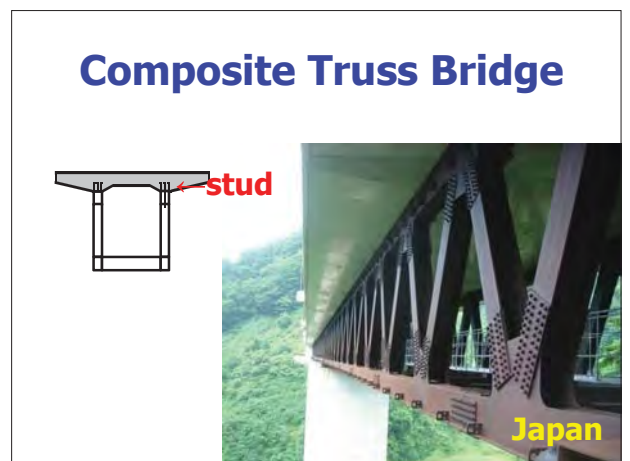
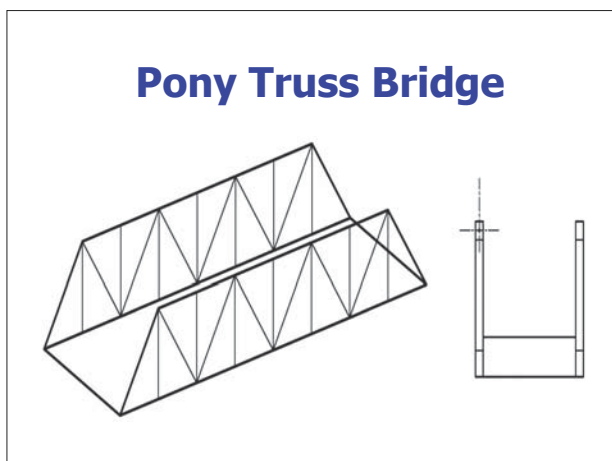
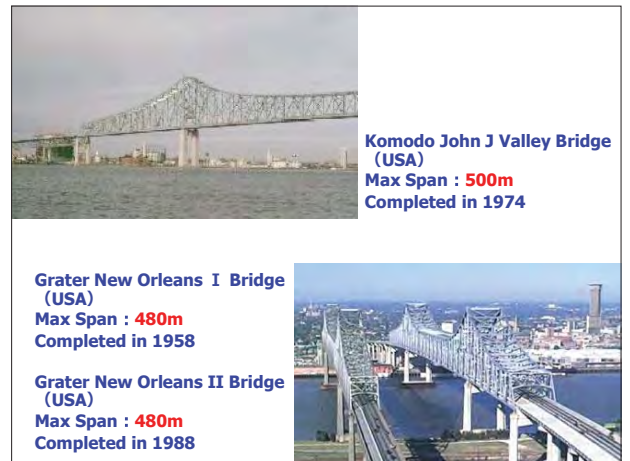
### Buckling of Truss Member



### Quebec Br. (span=549m, 1919, Canada)









## Steel-truss Web PC Box-girder Bridge



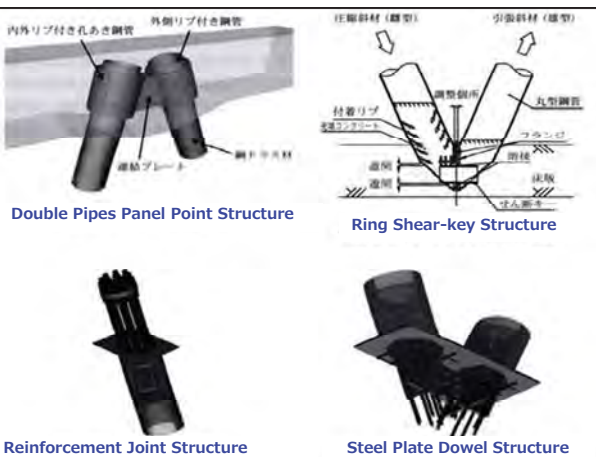
France



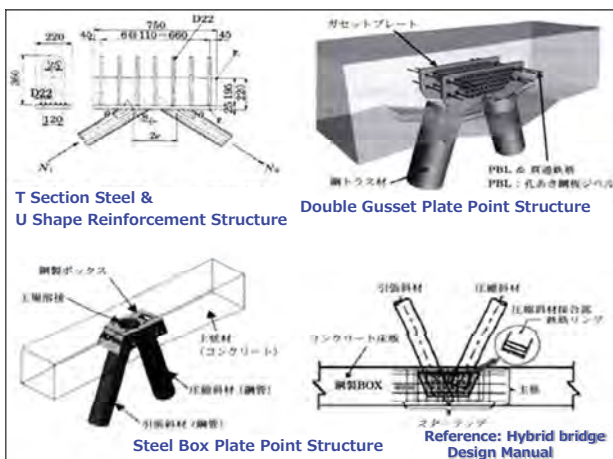
Pipe Truss  
Web



Ex. of  
Panel Point Structure



Arch Bridges



Timber bridges

Stone bridges

Iron bridges





**Kintai Br. (Japan)**



**Megane Br.**



**Isahaya Megane Br.**



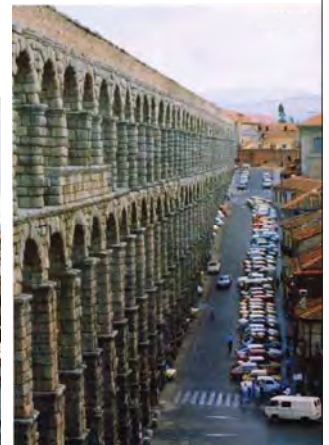
**Tsujun Br.**

[Megane] = eye-glass

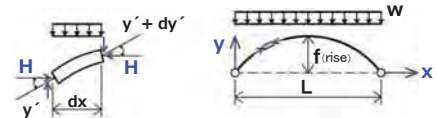


**kaminomori Br. (Japan)**

## Stone Bridges



## Formula of Arched Structure



$$Hy' = Wdx + H(y' + dy')$$

$$y'' \left( = \frac{d^2 y}{dx^2} \right) = - \frac{W}{H}$$

$$[ \text{at } X=0, y=0 ] , [ \text{at } X=L, y=0 ]$$

$$y = - \frac{W}{2H} X (X - L)$$

$$[ \text{at } X=L/2, y=f ]$$

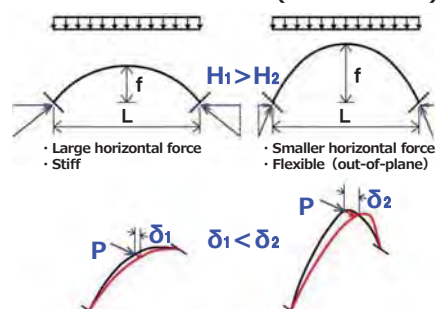
$$H = \frac{WL^2}{8f}$$

## Iron Br.



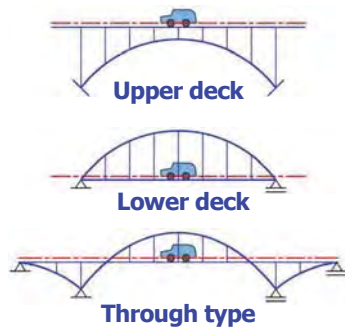
## Arch rise

At the design,  $L/f = (1/6 \sim 1/10)$





## Position of the traffic road

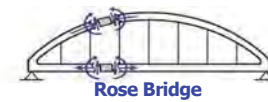


## Structural Type (Part 1)

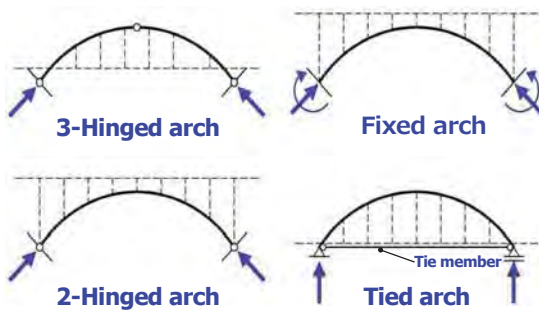
### [Unstiffened arch]



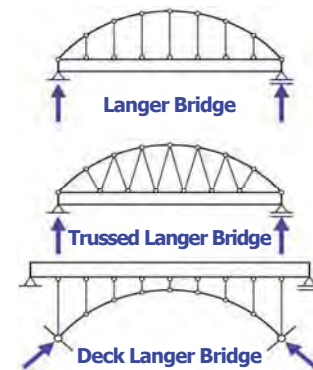
### [Stiffened arch]



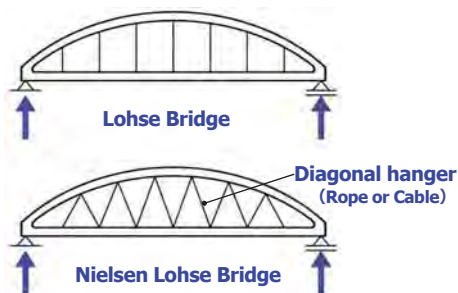
## Bearing Support Condition



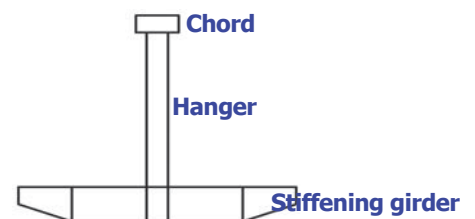
## Structural Type (Part 2)



## Structural Type (Part 3)

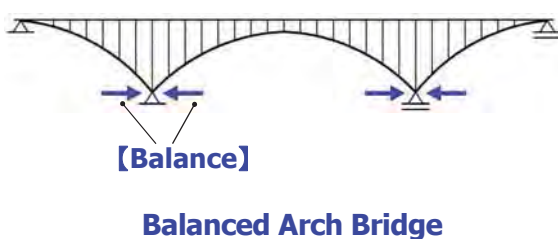


## Structural Type (Part 5)



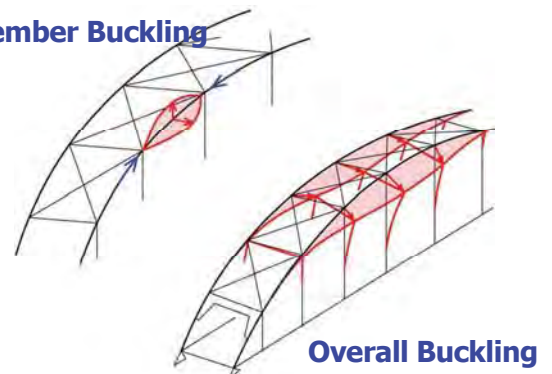
### Cross section of arch bridge with single rib

## Structural Type (Part 4)



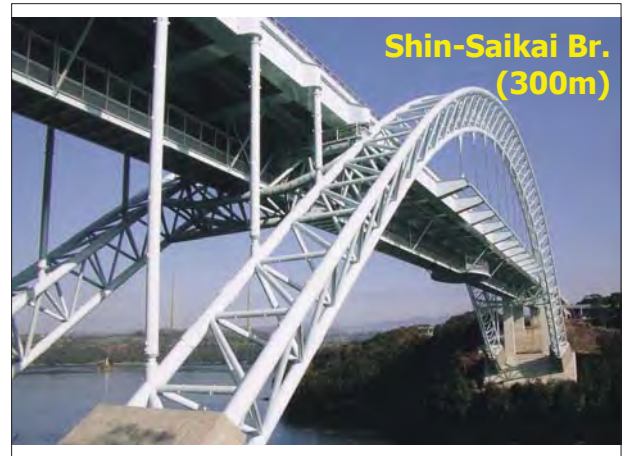
## Buckling of Truss Bridge

### Member Buckling





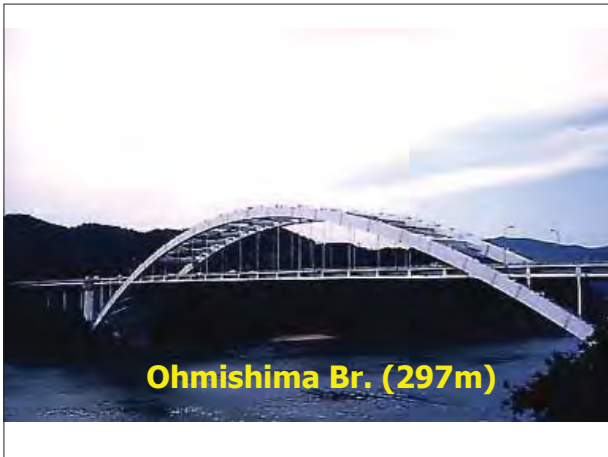
## Unstiffened arch



## Shin-Kizugawa Br.



## Ohmishima Br. (297m)



## New River Gorge Br.



## Sydney Harbor Bridge



## Bayonne Br.



## Chaotianmen Br.





## Lupu Br.



Crosses Huangpu River  
Total length 3,900 m (12,795 ft.)  
Width 28.7 m (94 ft.)  
Longest span **550 m** (1,804 ft.)  
Clearance below 46 m (151 ft.)  
Opened June 28, 2003



## Colorado River Bridge (USA)

- Hoover Dam Bypass Project
- Longest RC Arch Bridge in USA  
4<sup>th</sup> in the world
- Arch Span ; **323m**
- The Dam located in the state  
border of Arizona and Nevada.
- Completed in Oct. ,2010
- Constructor ; Obayashi Corp. &  
PSM Construction USA JV



## Designed by Robert Maillart



## Salginatobel Bridge (Switzerland) Completed in 1930

- RC Road Bridge
- Upper deck 3-Hinged Arch Bridge
- Bridge Length ; 132.3m
- Total Width ; 3.5m
- Arch Span ; **90.04m**
- Arch Rise ; 12.99m
- Pioneer of RC Arch Bridge

Completed in Oct. ,2010

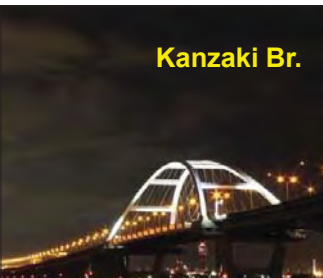


## Stiffened arch bridge



Nada Br.

## Kanzaki Br.



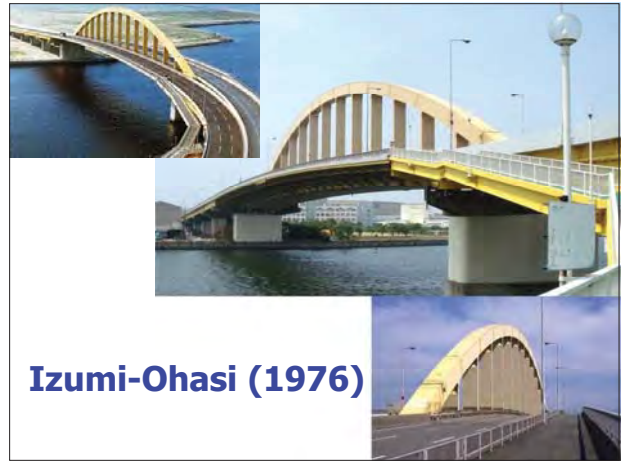
Rokko Island Br. (2002)



**Gosiki-sakura Br. (2002)**



**Double deck Nielsen Lohse Bridge**



**Izumi-Ohashi (1976)**

## **Mono-chord arch**

**Narita Br. (1979)**



**Puente Muffle Br. (1992, Spain)**



**Bach De Roda-Felipe II Bridge  
Barcelona, Spain, 1987**

**Inclined arch rib**



**Alameda Bridge &  
Underground Station  
Valencia, Spain, 1991-95**



**Inclined arch rib**

**Nuevo acceso al puerto  
de Ondarroa  
Ondarroa, Spain, 1995**



**Inclined hanger**



## Pedestrian Bridges

**Campo Volantin Footbridge  
Bilbao, Spain, 1997**



**Paris, France**



## Floating Bridges

**Yumemai Bridge  
(swing bridge)**

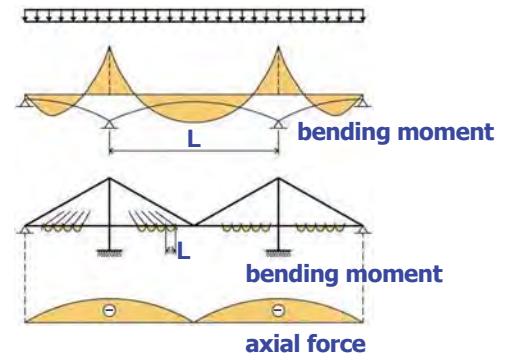




[10-2-3]

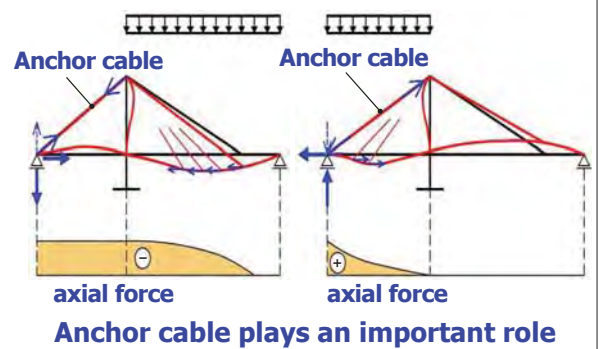
## Cable-stayed Bridges & Suspension Bridges

### Stress Resultants



## Cable-stayed bridges

### Stress Resultants and Deformation



### Continuous cable-stayed bridges



### World longest cable-stayed bridges (1,104m)



### Curved cable-stayed bridges





## Design Parameter

- Number of Cables
- Cable arrangements
- Number of cable plane
- Tower shape
- Continuous cable-stayed Br.
- Curved cable-stayed Br.
- the world record of cable-stayed Br.
- Material (Composite, Mixed)
- Extra-dosed Br.
- Pedestrian Br.



**[Number of cables]**

## Multi-cable system



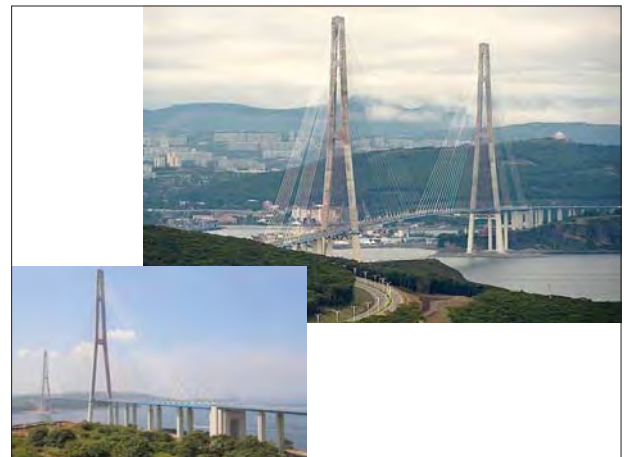
**stayed by a few\* cables**

**\*small number**

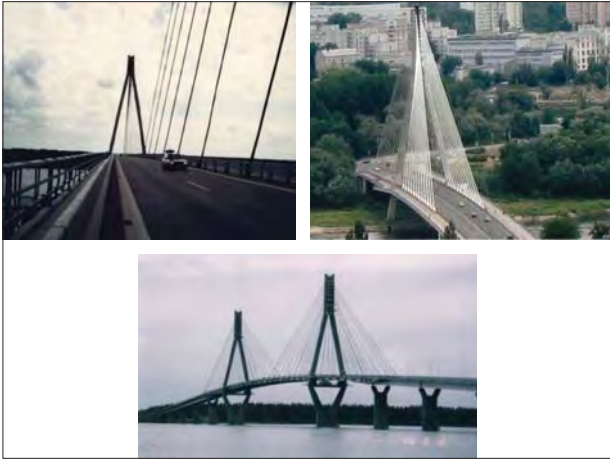


**[Cable arrangement]**

## Fan-type







**Harp-type**



**Radial-type**



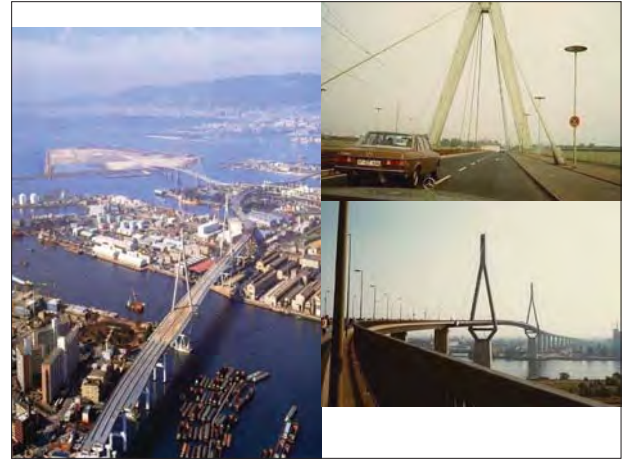
**[Number of plane]**

**Single vertical plane**



**two planes**

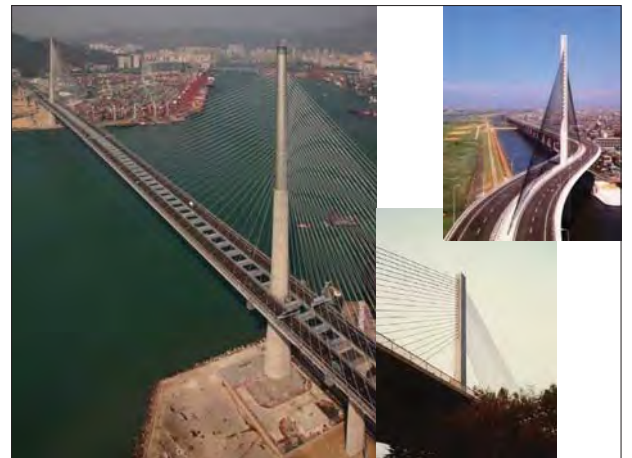




**[Tower shape]**

**"A" shaped Tower**

**"H" shaped tower**



**Single tower type**

**Two towers type**





Other type of tower shape



spatial type tower



Inverted Y-shaped tower



Neckertalbrücke-1 Br. (Germany)

Tower underneath the girder





**[Continuous cable-stayed bridges]**



**[Curved Cable-stayed bridges]**



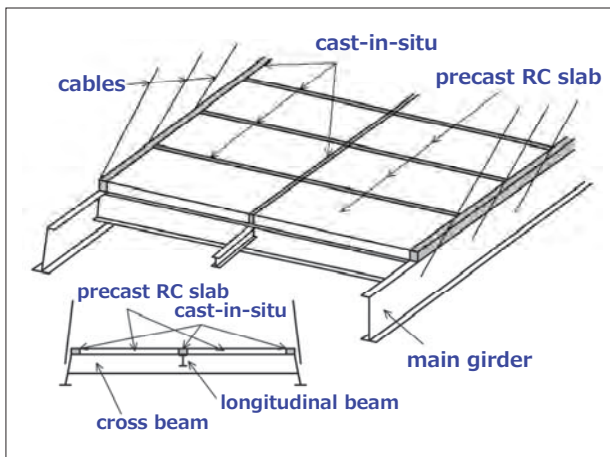




**[Longest bridges  
in the world and Japan]**



**[Composite Cable-  
stayed bridges]**



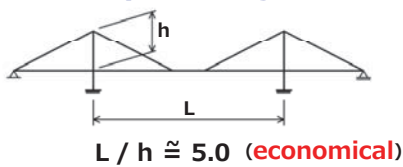


## [Mixed-type cable-stayed bridges]

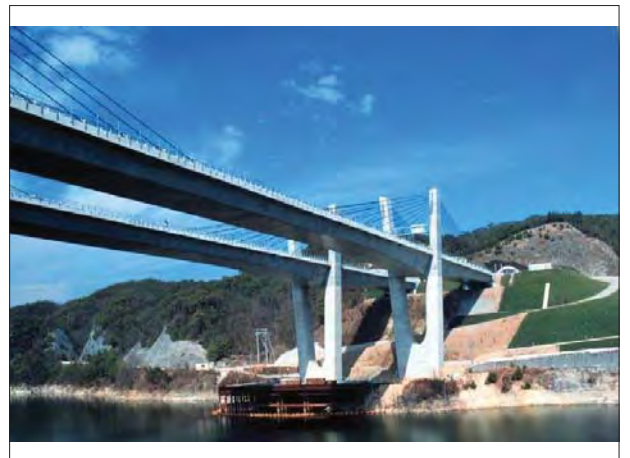
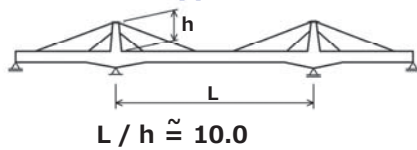


## [Extradosed PC Bridges]

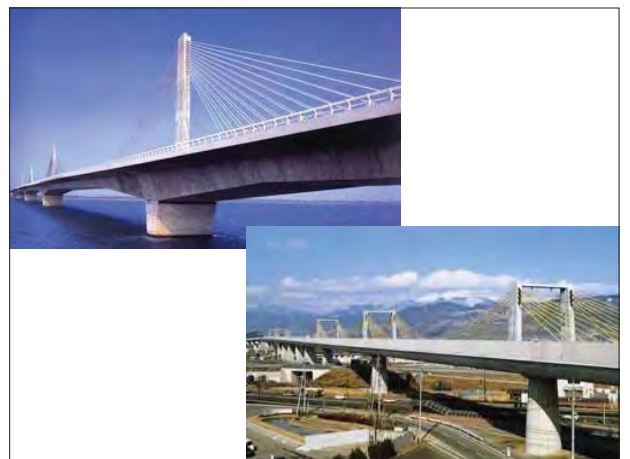
### Cable-stayed bridges



### Extradosed type



Odawara Blue way Bridge

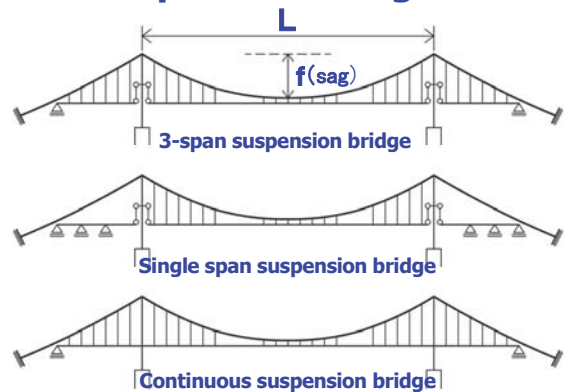




[Pedestrian cable-stayed bridge]

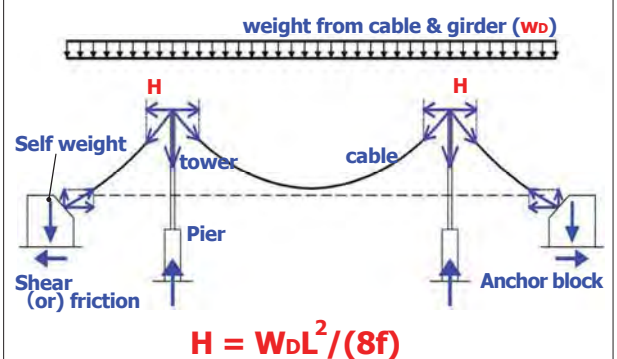


## Suspension Bridges



## Suspension Bridges

### Dead Load and Force Transfer





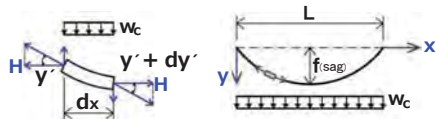
## Unique suspension bridge



Diagonal hanger cable system



## Cable configuration of Suspension Bridge



$$Hy' = w_c dx + H (y' + dy')$$

$$y'' \left( = \frac{d^2 y}{dx^2} \right) = - \frac{w_c}{H}$$

$$[ \text{at } X=0, y=0 ], [ \text{at } X=L, y=0 ]$$

$$y = - \frac{w_c}{2H} X (X - L)$$

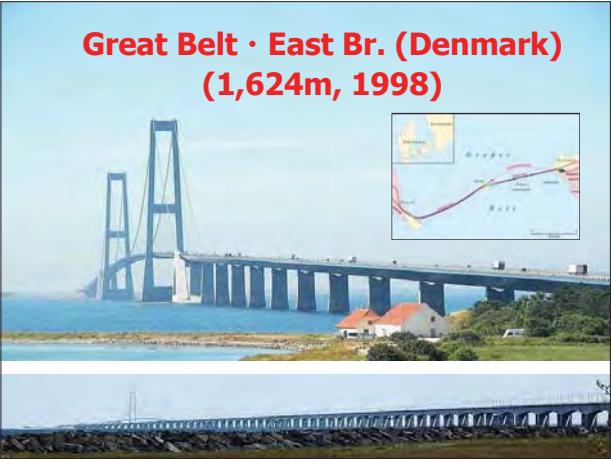
$$[ \text{at } X=L/2, y=f ]$$

$$H = \frac{w_c L^2}{8f}$$

## Big suspension bridges









[10-3-1]

# Structural Mechanics

## Depending on the body,

Point mass mechanics

Rigid body mechanics

Fluid mechanics\*

Soil mechanics\*

Rock mechanics\*

Structural mechanics\*

\*,\* : continuum mechanics

## Mechanics

is science

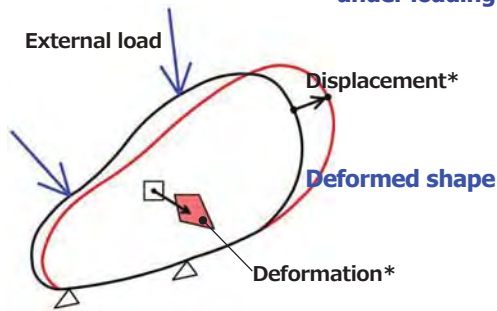
studying analytically and experimentally  
the phenomena of the body  
under (subjected to) applied loading



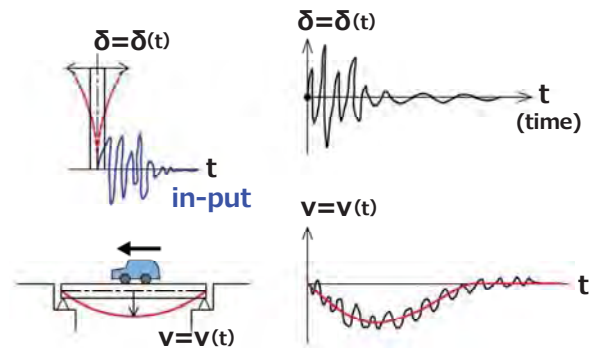


## Structural Mechanics

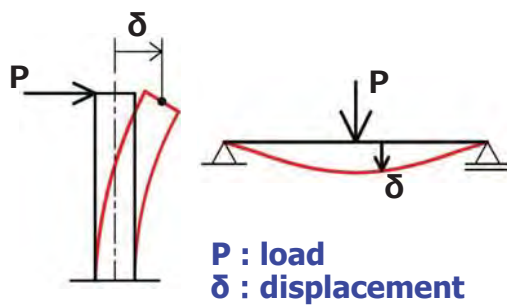
studying the response\* of the body (structures) under loading



## Dynamic Response



## Static response



Structures are modeled by following mechanical member or element

[1]axial member

(a) tension member (N)

(b) compression member (column) (N)

[2]beam member(M, Q)

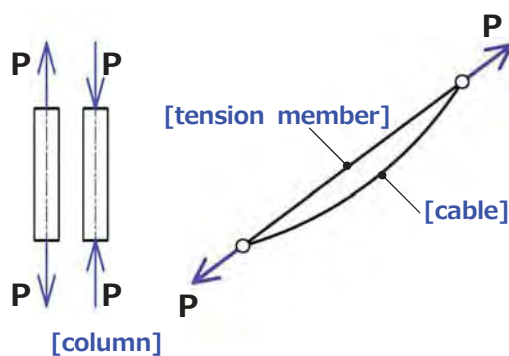
[3]beam-column member (N[compression], M, Q)

N : axial force

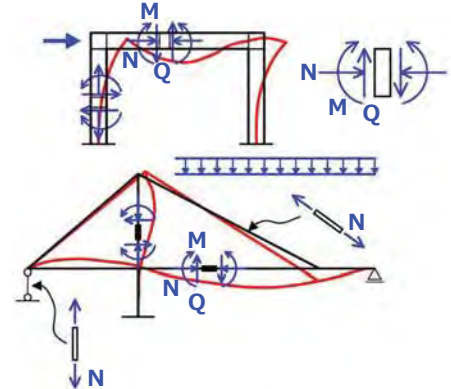
M : bending moment

Q : shear force

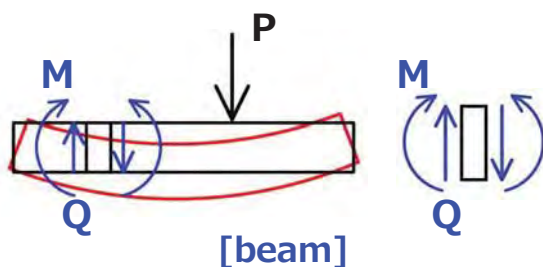
## compression & tension



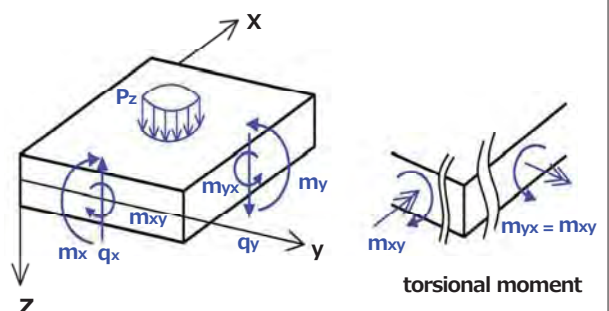
## [beam-column] {N, M, Q}



## Bending(M) and shear(Q)



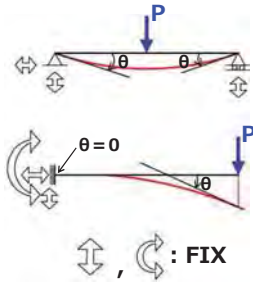
## [plate]



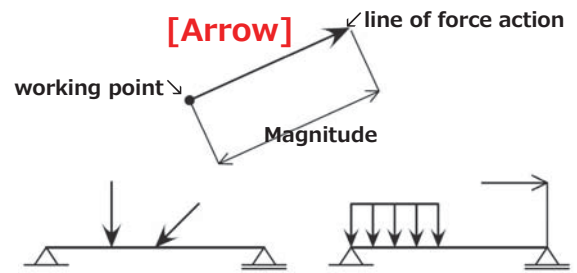


## Boundary conditions

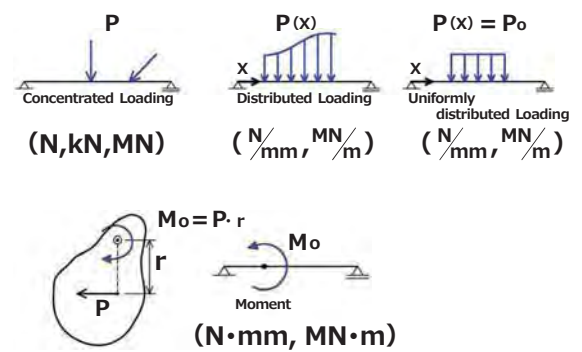
Type	translational Movement	Rotation
Move	O	O
Hinge	X	O
Fix	X	X



## Force



## Loading



## Composition of force (by chart)



## Composition of force (by analysis)

$$R^2 = (P_2 + P_1 \cos \beta)^2 + (P_1 \sin \beta)^2$$

$$= P_1^2 + P_2^2 + 2P_1 P_2 \cos \beta$$

$$R = \sqrt{P_1^2 + P_2^2 + 2P_1 P_2 \cos \beta}$$

$$\tan \alpha = \frac{P_1 \sin \beta}{P_2 + P_1 \cos \beta}$$

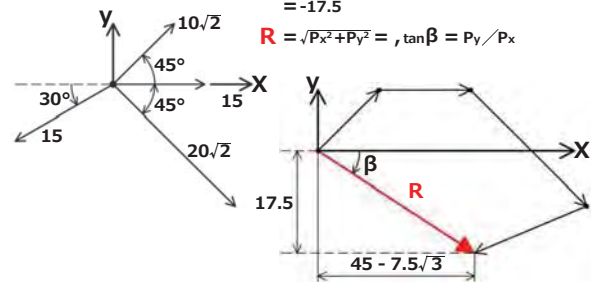
$$P_x = 10/\sqrt{2} \left( \frac{1}{\sqrt{2}} \right) + 15 + 20/\sqrt{2} \left( \frac{1}{\sqrt{2}} \right) - 15 \cos 30^\circ$$

$$= 45 - 7.5\sqrt{3}$$

$$P_y = 10/\sqrt{2} \left( \frac{1}{\sqrt{2}} \right) - 20/\sqrt{2} \left( \frac{1}{\sqrt{2}} \right) - 15 \sin 30^\circ$$

$$= -17.5$$

$$R = \sqrt{P_x^2 + P_y^2}, \tan \beta = P_y / P_x$$



## Resolution of force

$$P_1 \cos \alpha + P_2 \cos \beta = R$$

$$P_1 \sin \alpha = P_2 \sin \beta$$

$$P_1 = \frac{R \sin \beta}{\sin \theta}, P_2 = \frac{R \sin \alpha}{\sin \theta}$$

$$\theta = \frac{\pi}{2} \rightarrow \sin \theta = 1$$

$$P_1 = R \sin \beta = R \cos \alpha$$

$$P_2 = R \sin \alpha = R \cos \beta$$

$$P_x = \sum P_i \cos \alpha_i$$

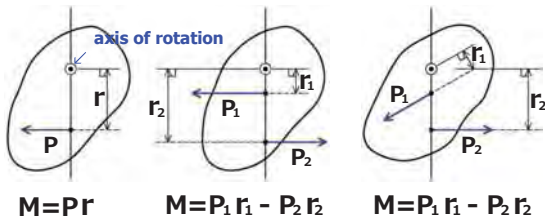
$$P_y = \sum P_i \sin \alpha_i$$

$$R = \sqrt{R_x^2 + R_y^2}$$

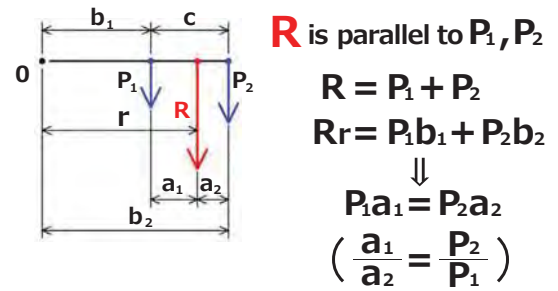
$$\tan \beta = P_y / P_x$$



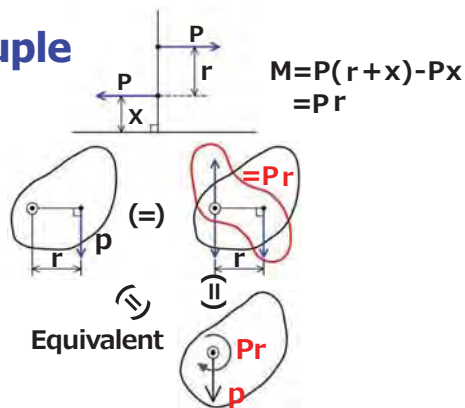
## Moment



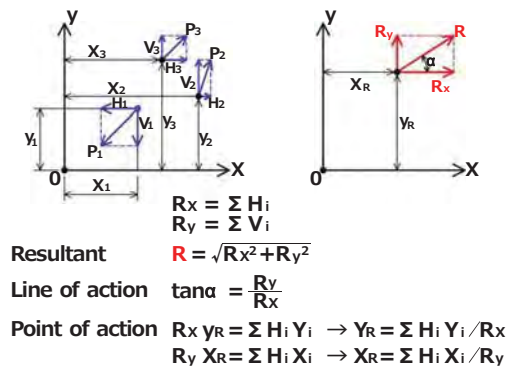
## Composition of parallel force



## Couple



## Composition of forces whose working point are not the same



## Equilibrium of Force

[2D]



3-degree of freedom  
2 (translational) movements  
and rotation

[3D]



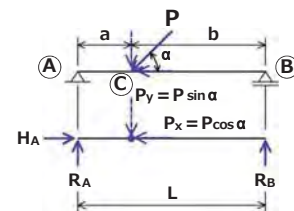
6-degree of freedom  
3 (translational) movements  
and 3 rotations

In equilibrium State (2D)

$$\sum H = 0 \text{ (Horizontal direction)}$$

$$\sum V = 0 \text{ (Vertical direction)}$$

$$\sum M = 0 \text{ (rotation [Moment])}$$



$$H_A = P \cos \alpha$$

$$R_A + R_B = P \sin \alpha$$

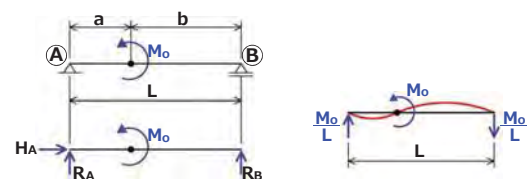
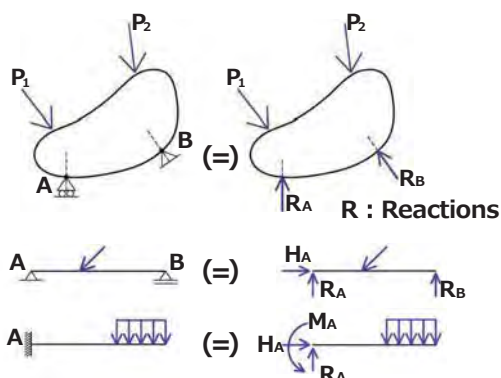
Moment at A  $R_B L = P_y a \rightarrow R_B = \frac{a}{L} P_y$

Moment at B  $R_A L = P_y b \rightarrow R_A = \frac{b}{L} P_y$

$(R_A + R_B = \frac{a+b}{L} P_y = P_y)$

Moment at C  $R_A a = P_b b$

## Reactions



$$H_A = 0$$

$$R_B L + M_o = 0 \rightarrow R_B = -\frac{M_o}{L}$$

$$R_A = -R_B \rightarrow R_A = \frac{M_o}{L}$$

$([R_A, R_B] \text{ does not depend on } [a, b])$



## Stable or unstable

$$n_E = r - (j_e + 3)$$

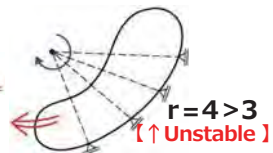
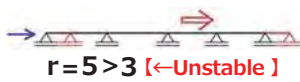
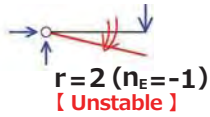
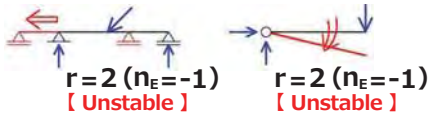
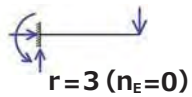
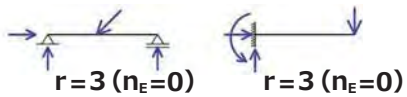
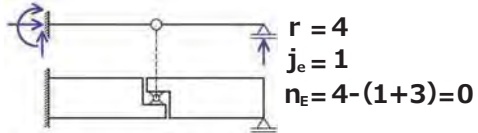
$r$  : Number of reaction

$j_e$  : Number of hinge

$$n_E < 0 \quad \text{Unstable}$$

$$n_E = 0 \quad \text{Stable} \\ \text{(Externally) determinate}$$

$$n_E > 0 \quad \text{Stable} \\ \text{(Externally) indeterminate}$$

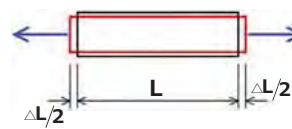




[10-3-2]

# Stress & Strain

## Strain

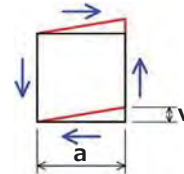


Normal strain

$$\epsilon = \frac{\Delta L}{L}$$

$\epsilon > 0$  Tension

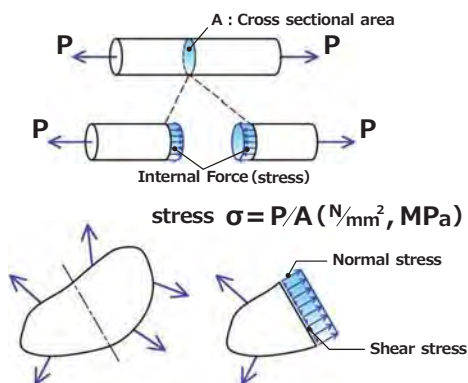
$\epsilon < 0$  Compression



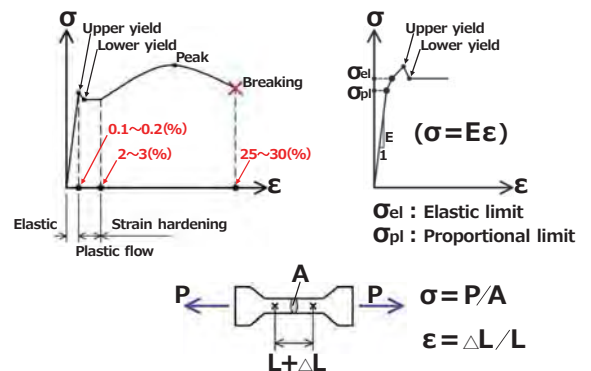
Shear strain

$$\gamma = \frac{v}{a}$$

## Stress



## Stress and strain relationship (Steel)



## Hook's law

[proportional(linear) relationship]

$$\sigma = E\epsilon$$

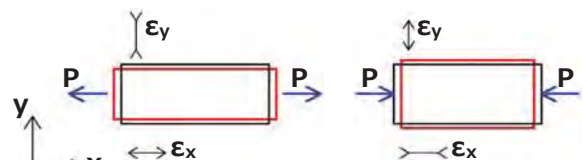
$$\tau = G\gamma$$

$E$  : Young's modulus of elasticity  
= 200,000(N/mm<sup>2</sup>) [steel]

$G$  : shear modulus of elasticity  
= 77,000(N/mm<sup>2</sup>) [steel]

$$G = E / \{2(1 + \nu)\}$$

## Poisson's Ratio

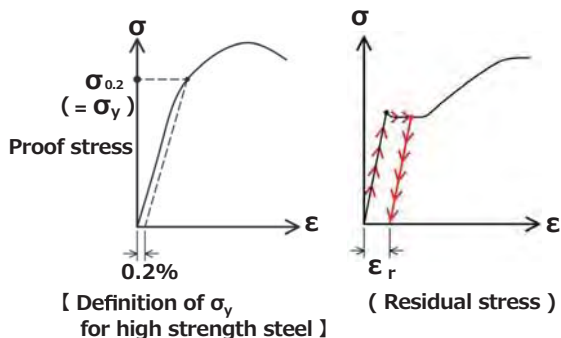


$$\nu = -\epsilon_y / \epsilon_x$$

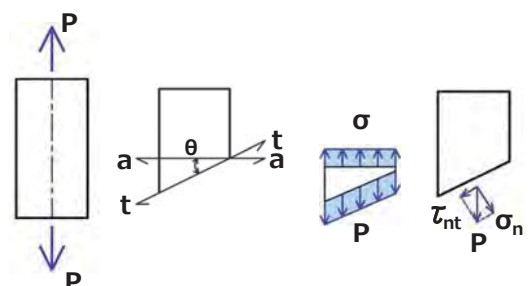
$\nu$  : Poisson's ratio

$m (= 1/\nu)$  : Poisson's number

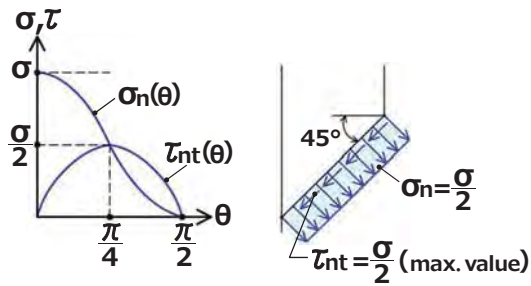
$\nu = 0.3$  (steel)



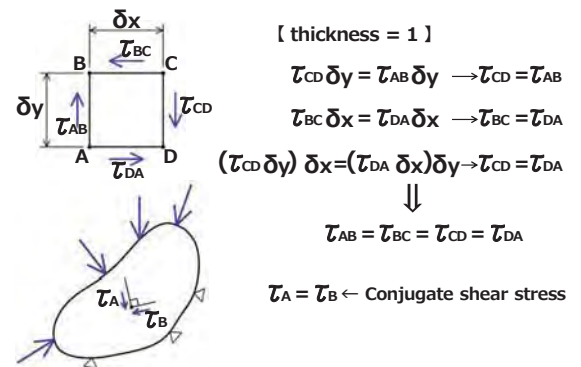
## Pure tension



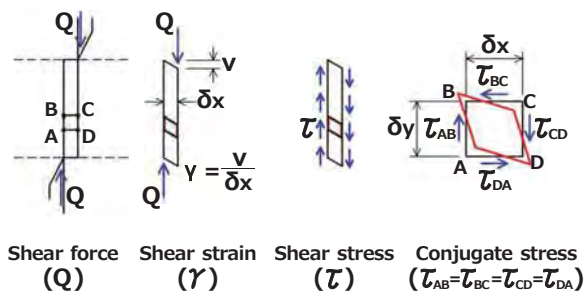




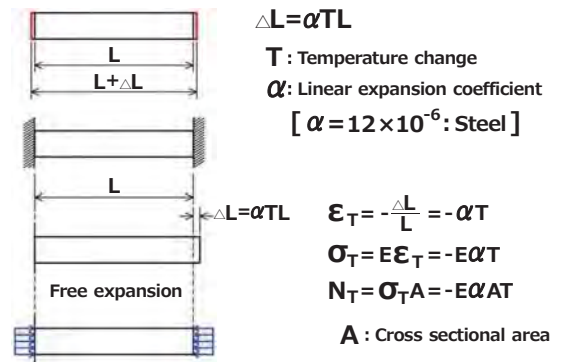
## Conjugate shear stress



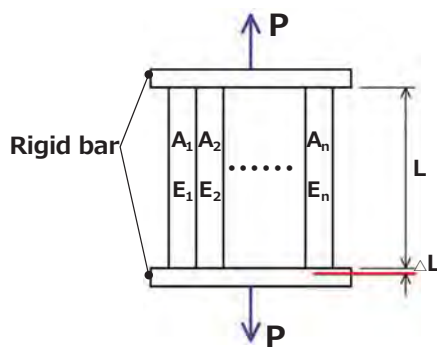
## Shear



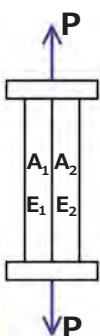
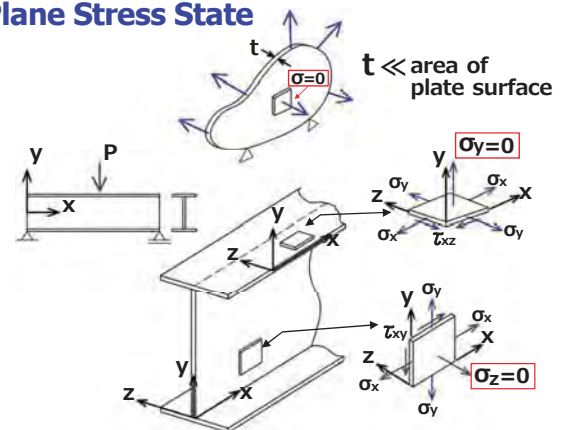
## Stress due to temperature change



## Composite member



## Plane Stress State



Since strain( $\epsilon$ ) in both bars is same,

$\epsilon = (P_1/E_1 A_1) = (P_2/E_2 A_2)$

From equilibrium condition,

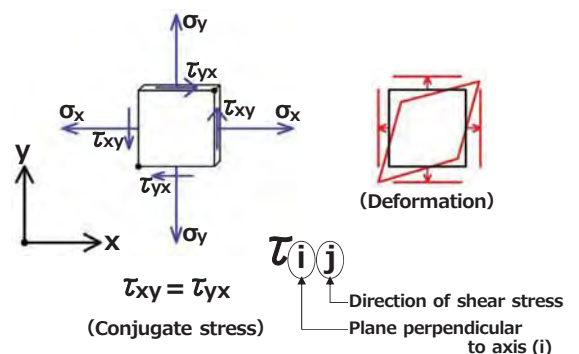
$$P_1 + P_2 = P$$

$$P_1 = (E_1 A_1 / \alpha) \cdot P$$

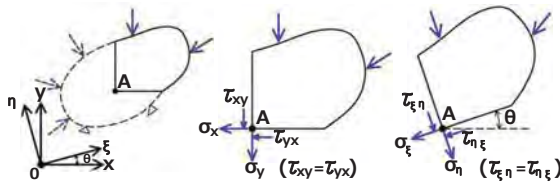
$$P_2 = (E_2 A_2 / \alpha) \cdot P$$

$$\alpha = E_1 A_1 + E_2 A_2$$

## Definition of sign (plus[+], minus[-])







$$\sigma_{\xi} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{\eta} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

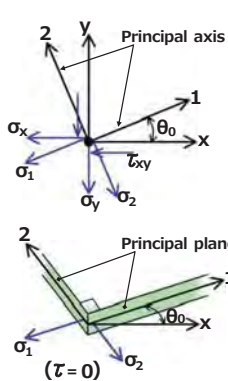
$$\tau_{\xi\eta} = \tau_{\eta\xi} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

max., min. Principal stress

$$\frac{d^2\sigma_{\xi}}{d\theta^2} = \frac{2\{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2\}}{>0} \cdot \frac{\cos\theta}{\sigma_y - \sigma_x}$$

From the sign (+ or -) of  $\left(\frac{\cos\theta}{\sigma_y - \sigma_x}\right)$ ,

max. (or) min. is defined.



$$\sigma_{\xi} = \sigma_{\xi}(\theta), \sigma_{\eta} = \sigma_{\eta}(\theta)$$

$$\Downarrow \left\langle \frac{d\sigma_{\xi}}{d\theta} = \frac{d\sigma_{\eta}}{d\theta} = 0 \right\rangle$$

Peak stress ( $\sigma_1, \sigma_2$ : Principal stress)

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tau_{\xi\eta} = \tau_{\eta\xi} = 0$$

$$\theta_0 = \frac{1}{2} \tan^{-1} \left( \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right)$$

$$(\sigma_1 + \sigma_2 = \sigma_x + \sigma_y = \sigma_{\xi} + \sigma_{\eta})$$

↑ Stress invariant

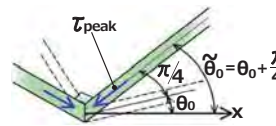
$$\tau_{\xi\eta} = \tau_{\eta\xi}(\theta) = \tau_{\eta\xi}$$

Conjugate stress

$$\Downarrow \left\langle \frac{d\tau_{\xi\eta}}{d\theta} = 0 \right\rangle$$

$$\tau_{\max., \min.} = \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

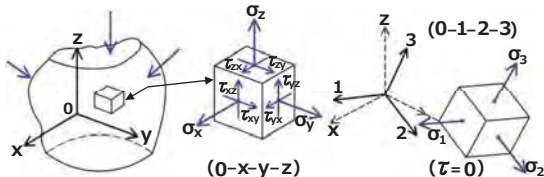
$$= \frac{\sigma_1 - \sigma_2}{2}$$



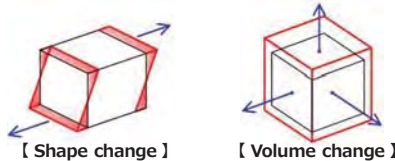
$$\tan(2\tilde{\theta}) = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\Downarrow \left\langle \tan(2\tilde{\theta}) \cdot \tan(2\theta_0) = -1 \right\rangle$$

$$\tilde{\theta}_0 = \theta_0 + \frac{\pi}{4}$$



Stress(deformation) can be divided into two Deformation(strain)



Characteristic equation

$$\sigma^3 - J_1\sigma^2 + J_2\sigma - J_3 = 0$$

where,  $J_1, J_2, J_3$  are stress invariant, which does not depend on co-ordinate

$$J_1 = \sigma_x + \sigma_y + \sigma_z = \sigma_1 + \sigma_2 + \sigma_3 = 0$$

$$J_2 = \frac{1}{6} \{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\}$$

$$= \frac{1}{6} \{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\} = \frac{1}{2} (\sigma_1^2 + \sigma_2^2 + \sigma_3^2)$$

$$J_3 = \sigma_1 \sigma_2 \sigma_3$$

Strain energy ( $U_0$ ) divided into two portion.

$$U_0 = U_{os} \text{ (Shear strain energy)} + U_{ov} \text{ (Volume strain energy)}$$

↑  $U_{os}$  has a close relation to yielding

$$U_{so} = \frac{1+\nu}{E} J_2 \Leftarrow J_2 \text{ has a close relation to yielding}$$

Since yielding depends on [ Shape change ], We deal with deviatoric stress ( $\sigma'_x, \sigma'_y, \sigma'_z$ )

$$\sigma'_x = \sigma_x - \sigma_m, \sigma'_y = \sigma_y - \sigma_m, \sigma'_z = \sigma_z - \sigma_m$$

$$\sigma_m = \frac{\sigma_x + \sigma_y + \sigma_z}{3} : \text{mean stress}$$

Principal deviatoric stress  $\{\sigma'_1, \sigma'_2, \sigma'_3\}$  and direction of principal axis  $\{n_1, n_2, n_3\}$  can be determined from

$$\begin{bmatrix} \sigma'_x - \sigma' & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma'_y - \sigma' & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma'_z - \sigma' \end{bmatrix} \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

To have  $\{n_1, n_2, n_3\}$  no-zero solution.

$$\det \begin{bmatrix} \sigma'_x - \sigma' & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma'_y - \sigma' & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma'_z - \sigma' \end{bmatrix} = 0$$

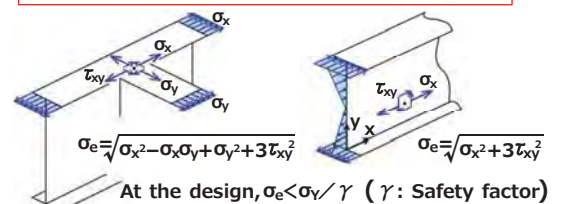
has to be satisfied.

$$J_2 = \frac{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2}{6} + \tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2 = \frac{\sigma_y^2}{3}$$

$$\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x\sigma_y - \sigma_y\sigma_z - \sigma_z\sigma_x + 3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2) = \sigma_y^2$$

$$\frac{\sigma_e}{\sigma_y} = \sqrt{\frac{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x\sigma_y - \sigma_y\sigma_z - \sigma_z\sigma_x + 3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}{\sigma_y^2}}$$

↑  $\sigma_e$ : Equivalent stress



At the design,  $\sigma_e < \sigma_y / \gamma$  ( $\gamma$ : Safety factor)



Von Mises Criterion (max. shear strain energy theory)

Yield criterion is expressed

$$F(J_2) = J_2 - k^2 = 0$$

Under uni-axial state

$$\sigma_1 = \sigma_Y \text{ (yield stress), } \sigma_2 = \sigma_3 = 0$$

$$J_2 = \frac{\sigma_Y^2}{3} - k^2 = 0 \Rightarrow k = \frac{\sigma_Y}{\sqrt{3}}$$

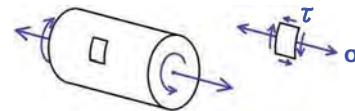
Under pure shear

$$\tau_{xy} = \tau_Y \text{ (yield shear stress)}$$

$$\sigma_x = \sigma_y = \sigma_z = \tau_{yz} = \tau_{zx} = 0$$

$$J_2 = \tau_Y^2 - k^2 = 0 \Rightarrow \tau_Y = k = \frac{\sigma_Y}{\sqrt{3}}$$

Ex.



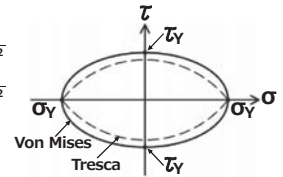
Von Mises  $\sigma_e^2 = \sigma^2 + 3\tau^2 = \sigma_Y^2$

Tresca  $\sigma_1 = \frac{\sigma}{2} + \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$

$$\sigma_2 = \frac{\sigma}{2} - \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$\sigma_1 - \sigma_2 = \sqrt{\sigma^2 + 4\tau^2} = \sigma_Y$$

$$\sigma^2 + 4\tau^2 = \sigma_Y^2$$



Experimental values show good agreement with those from Von Mises criterion.

Tresca criterion (max. shear stress theory)

in case of ( $\sigma_1$ (max.),  $\sigma_2$ ,  $\sigma_3$ (min.))

$$\sigma_1 - \sigma_3 = \sigma_Y = 2\tau_Y (=2k) \Rightarrow \tau_Y = \frac{\sigma_Y}{2}$$

$$\left( \because \frac{\sigma_1 - \sigma_3}{2} = \tau_{\max} = \tau_Y \right)$$

When one of the following is satisfied, yielding state. (develops)

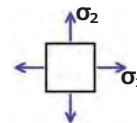
$$f_{1,2} = [\sigma_1 - \sigma_2] - 2k = 0$$

$$f_{2,3} = [\sigma_2 - \sigma_3] - 2k = 0$$

$$f_{3,1} = [\sigma_3 - \sigma_1] - 2k = 0$$

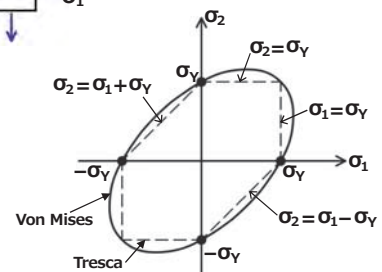
$$k = \tau_Y$$

Ex.



Von Mises

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$





## Cross-sectional Properties

$$A = \int_A dA : \text{Cross-sectional area}$$

$$S_x = \int_A y dA, S_y = \int_A x dA : \text{Geometrical moment of area}$$

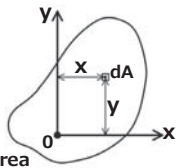
$$I_x = \int_A y^2 dA, I_y = \int_A x^2 dA : \text{Geometrical moment of inertia (Moment of inertia of area)}$$

$$I_{xy} = \int_A xy dA : \text{Product of inertia of area}$$

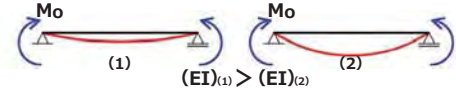
$$I_p = \int_A r^2 dA : \text{Polar moment of inertia of area}$$

$$r = \sqrt{I/A} : \text{Radius of gyration}$$

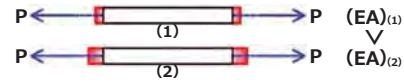
$$w = I/y : \text{Section modulus}$$



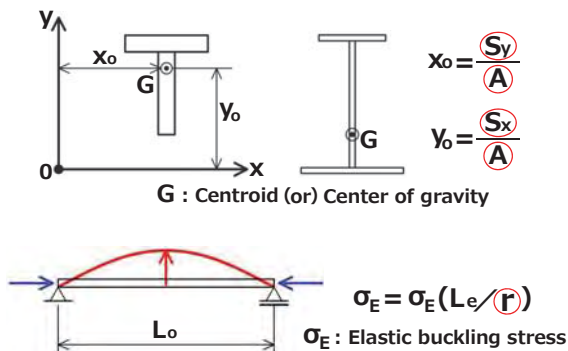
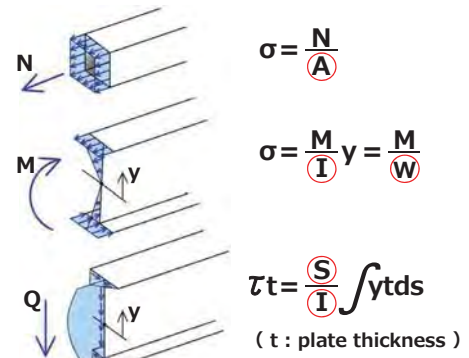
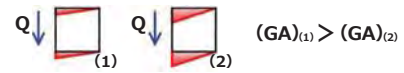
**EI : Flexural rigidity**



**EA : Axial rigidity**



**GA : Shear rigidity**



$$I_x = \int_A y^2 dA$$

$$I_y = \int_A x^2 dA$$

$$I_{xy} = \int_A xy dA$$

$$I_{x'} = \int_A y^2 dA = \int_A (y - y_0)^2 dA = I_x - 2y_0 S_x + y_0^2 A$$

$$I_{y'} = I_y - 2x_0 S_y + x_0^2 A$$

$$I_{x'y'} = I_{xy} - x_0 S_x - y_0 S_y + x_0 y_0 A$$

When point (origin) 0 is centroid,  $S_x = S_y = 0$

$$I_{x'} = I_x + y_0^2 A > I_x$$

$$I_{y'} = I_y + x_0^2 A > I_y$$

$$S_x = \int_A y dA$$

$$S_y = \int_A x dA$$

$$S_{x'} = \int_A y dA = \int_A (y - y_0) dA = S_x - y_0 A$$

$$S_{y'} = \int_A x dA = \int_A (x - x_0) dA = S_y - x_0 A$$

$$S_{x'} = S_{y'} = 0 \text{ (Center of gravity, Centroid)}$$

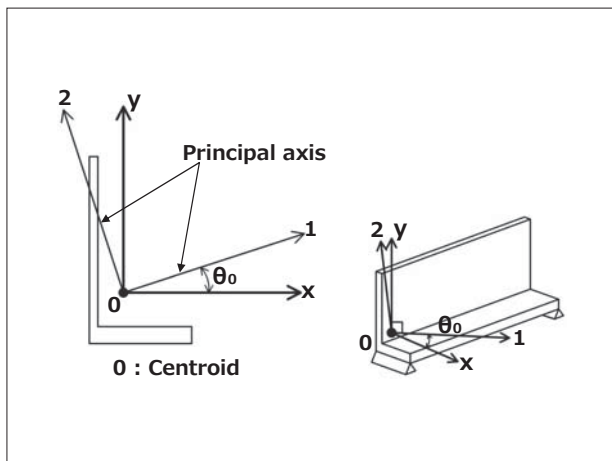
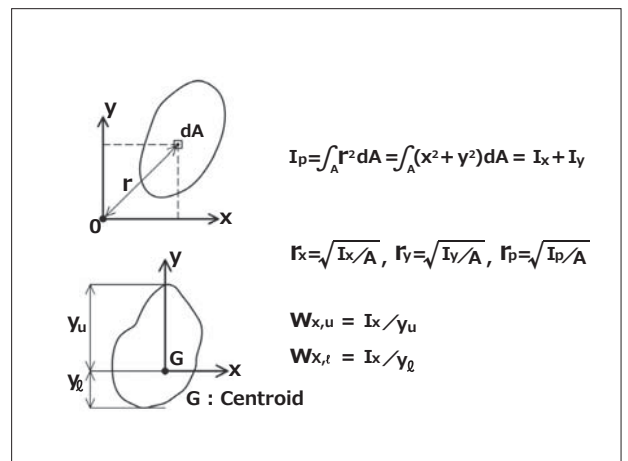
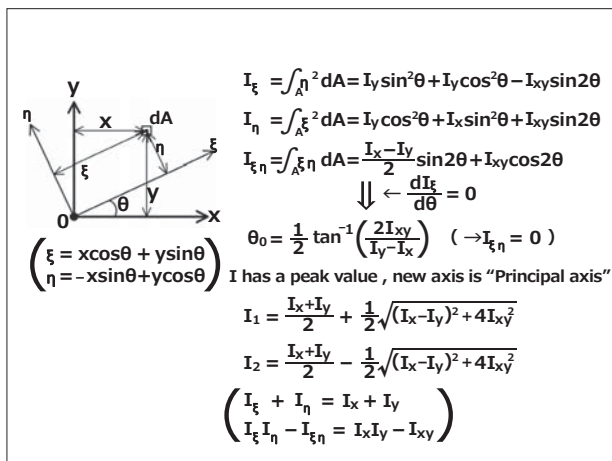
$$x_0 = \frac{S_x}{A}, y_0 = \frac{S_y}{A}$$

$$I_{x'} = \frac{BH^3}{3} > I_x = \frac{BH^3}{12}$$

$$I_x = \int_{-H/2}^{H/2} y^2 dA = B \left[ \frac{y^3}{3} \right]_{-H/2}^{H/2} = \frac{BH^3}{12}$$

$dA = B dy$



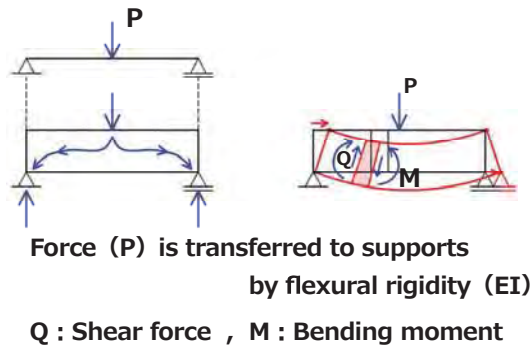




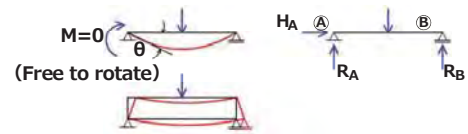
[10-4-1,2]

# Bending and Shear of Girder (Beam)

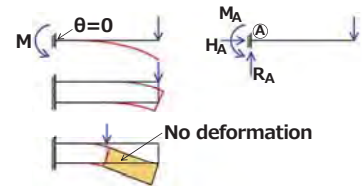
## Force transfer and Stress resultants



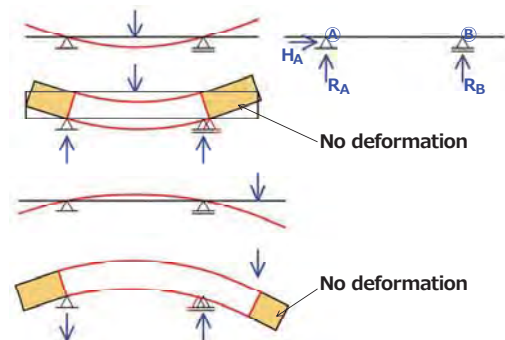
## [Simply supported beams (Simple beam)]



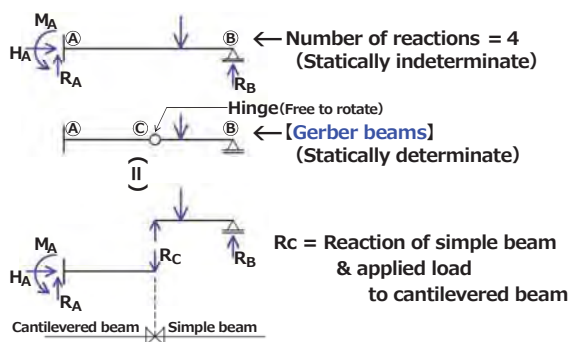
## [Cantilevered beams]



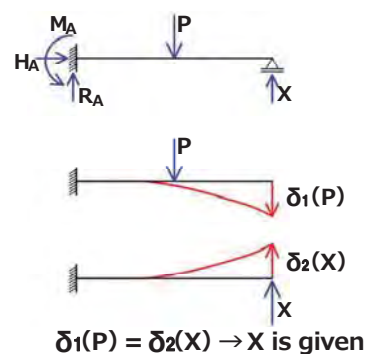
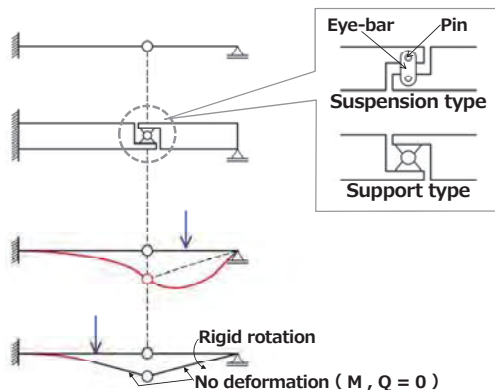
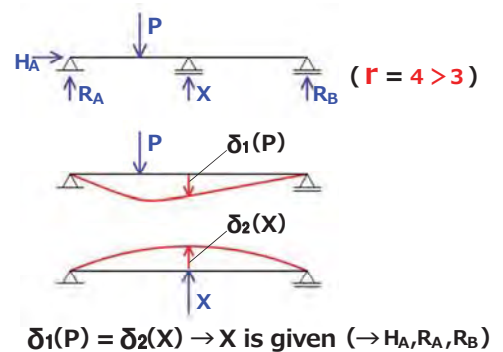
## [Simply supported beams with wings]



## [Gerber beams]



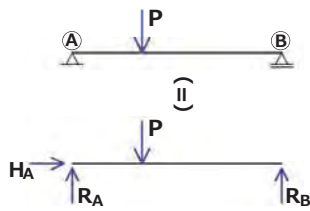
## Statically indeterminate beam





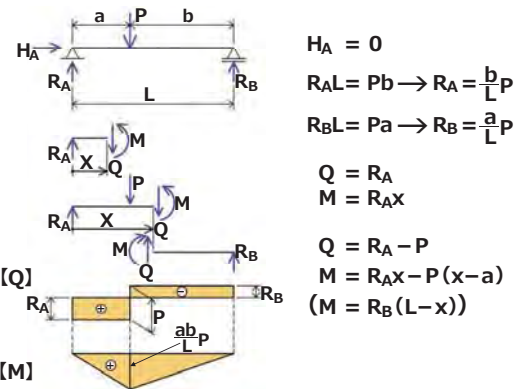
## Procedure

### [First step (Reactions)]

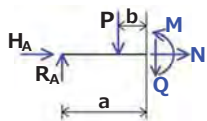


3 unknowns ( $H_A, R_A, R_B$ )

3 conditions ( $\Sigma H = 0, \Sigma V = 0, \Sigma M = 0$ )



### [Next step (Stress resultants N, M, Q)]

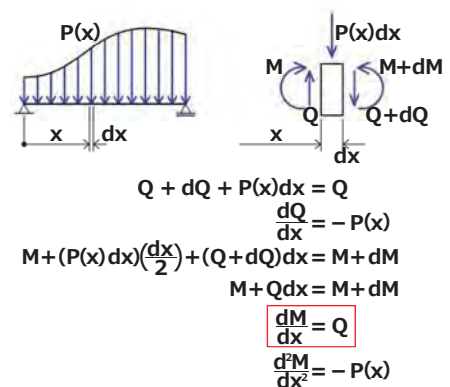
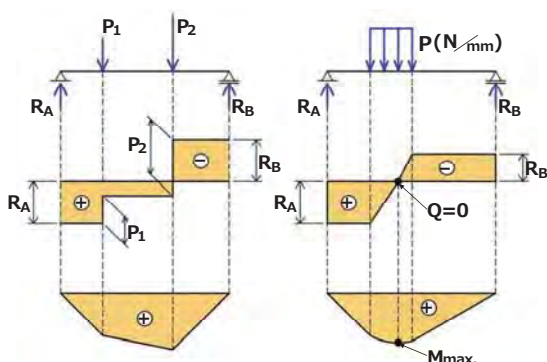
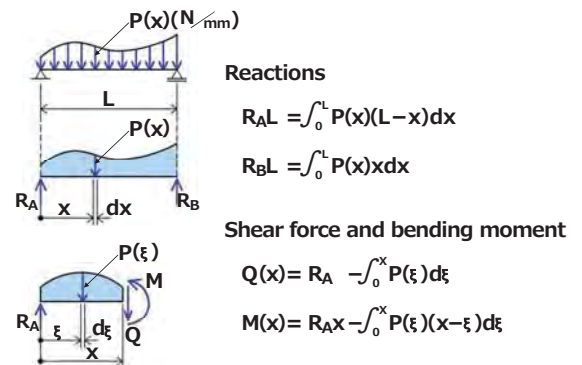
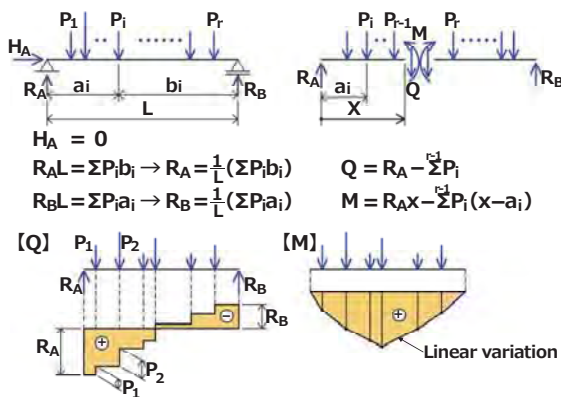
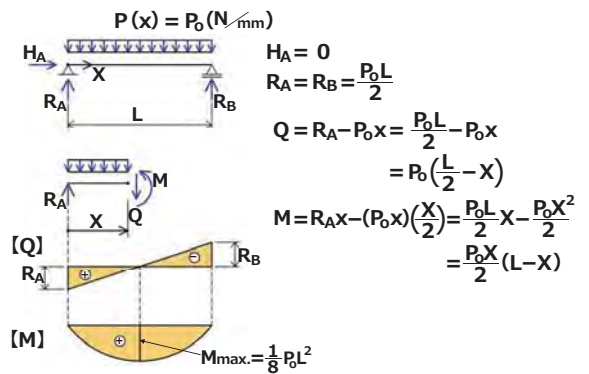


In equilibrium state  
Stress resultant  
(Internal force)  
 $N, M, Q$  are obtained

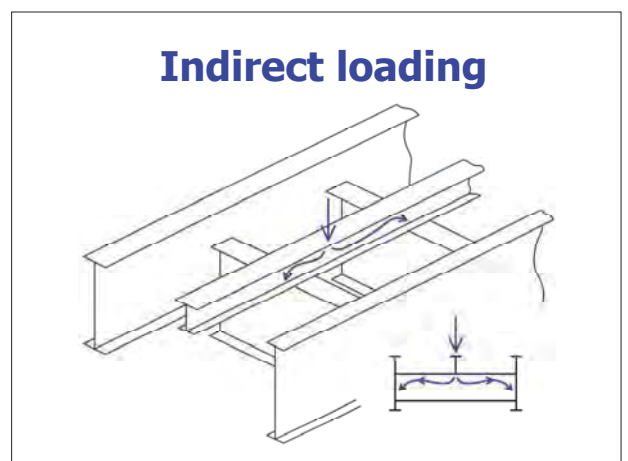
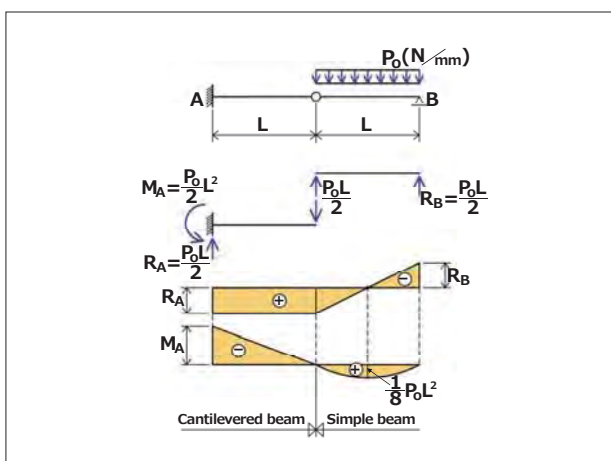
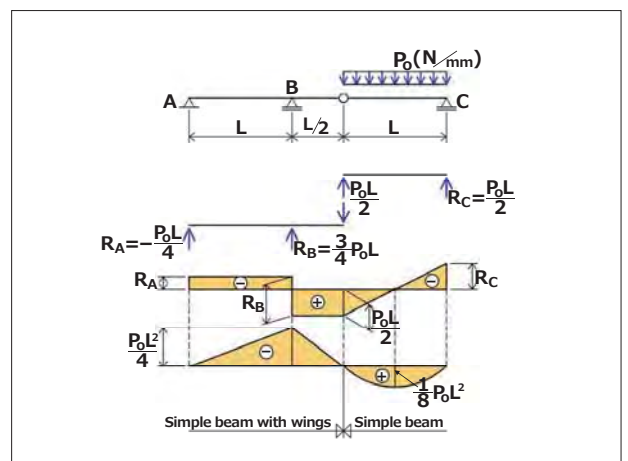
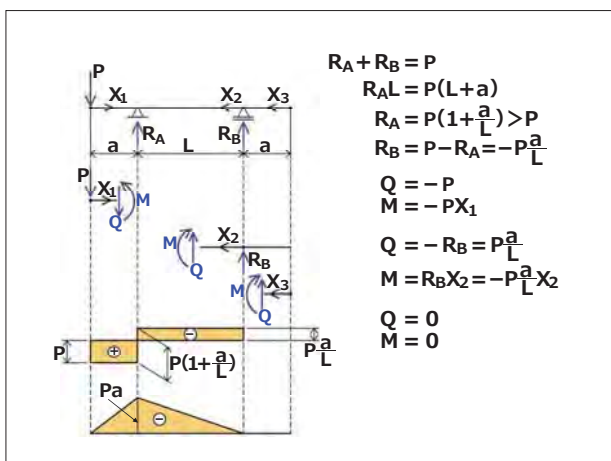
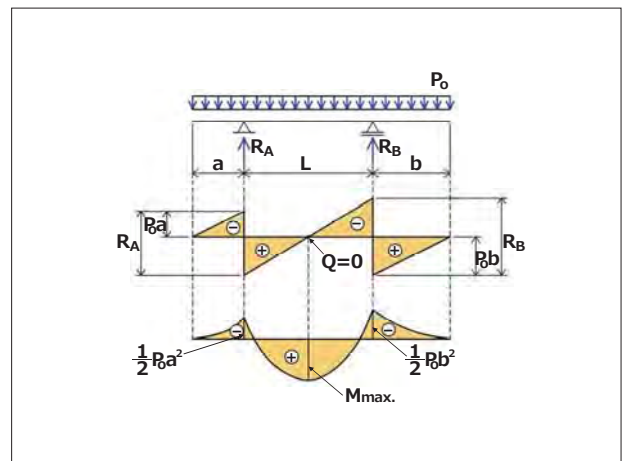
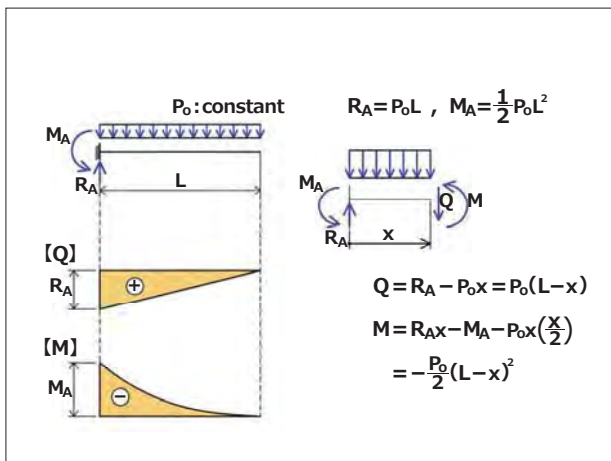
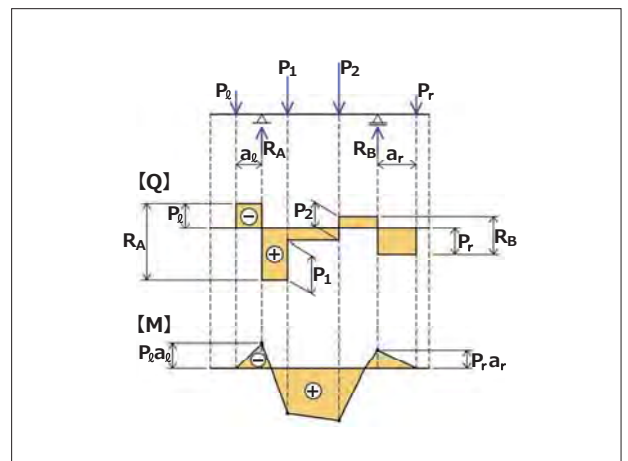
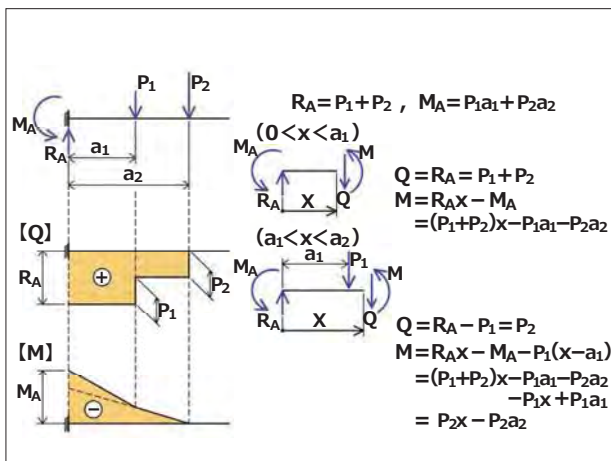
$$\begin{aligned}
 N + H_A &= 0 \rightarrow N = -H_A \\
 Q + P &= R_A \rightarrow Q = R_A - P \\
 M + Pb - R_A a &= 0 \rightarrow M = R_A a - Pb
 \end{aligned}$$

Plus (+) sign of deformation  
bending moment Shear

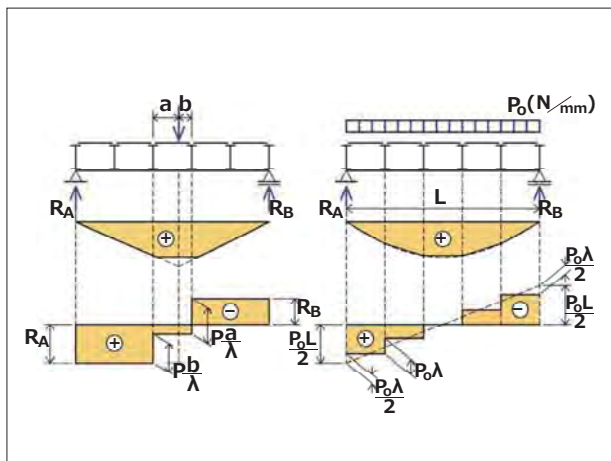
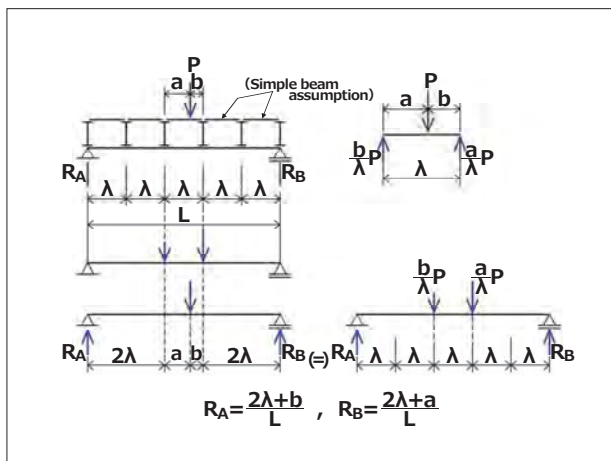
When (-) sign is obtained,  
direction is reverse.











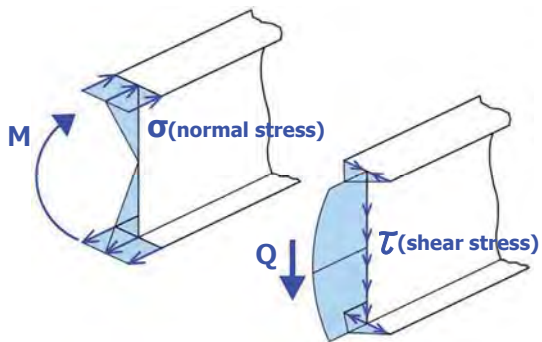


[10-4-3]

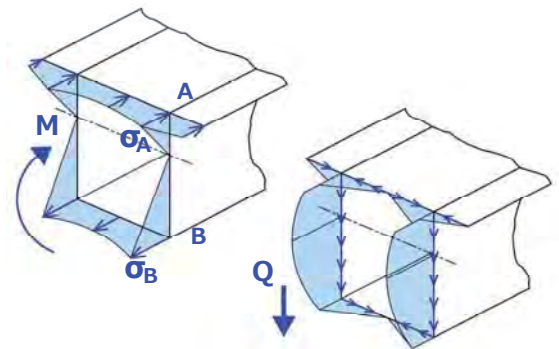
## Girder Stress



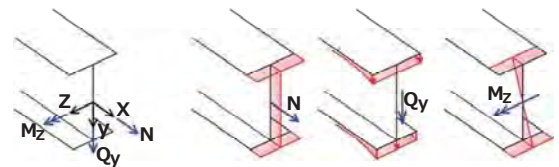
### I-girder



### Box-girder



### Stress and Stress Resultants Relation



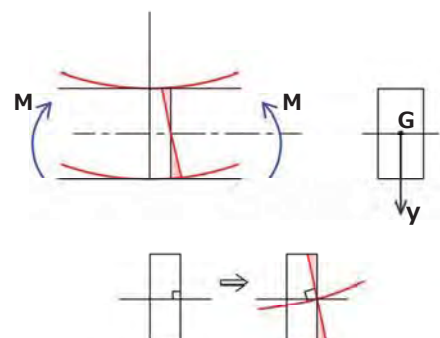
$$N = \int_A \sigma dA \quad \text{Axial force}$$

$$Q_y = \int_A \tau_{xy} dA \quad \text{Shear(ing) force}$$

$$M_z = \int_A \sigma y dA \quad \text{Bending moment}$$

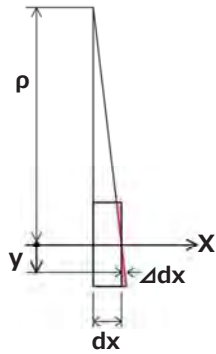


### Bernoulli-Euler Hypothesis





## Radius of Curvature



$$\rho : dx = y : \Delta x$$

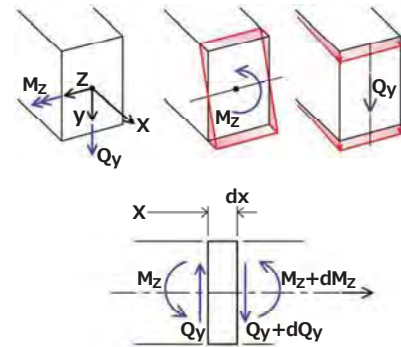
$$\epsilon_x = \frac{\Delta dx}{dx} = \frac{y}{\rho}$$

$$\sigma_x = E \epsilon_x = E \frac{y}{\rho}$$

$\rho$ : radius of curvature

$K (= 1/\rho)$ : curvature

## Shear(ing) Stress



## Normal Stress

$$M_z = \int_A \sigma_x y dA = E \frac{1}{\rho} \int_A y^2 dA = \frac{EI_z}{\rho} \quad \left( \frac{1}{\rho} = \frac{M_z}{EI_z} \right)$$

$$\sigma_x = E \frac{y}{\rho}$$

$$M_z = EI_z \frac{1}{\rho}$$

$$\sigma_x = \frac{M_z y}{I_z}$$

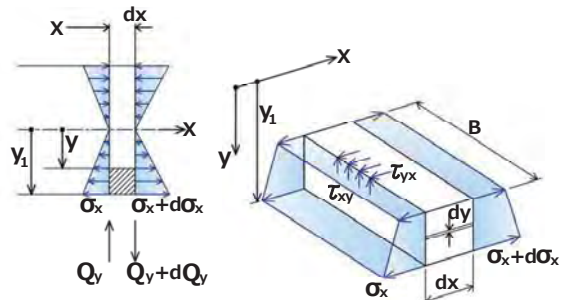
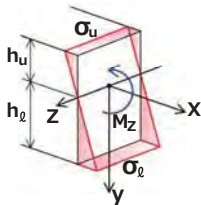
$M$ : Bending moment

$I_z$ : Geometrical moment of inertia

$y$ : Distance from neutral axis

$$\sigma_u = \frac{M_z}{I_z} h_u = \frac{M_z}{W_u} \quad (W_u = I_z/h_u)$$

$$\sigma_l = \frac{M_z}{I_z} h_l = \frac{M_z}{W_l} \quad (W_l = I_z/h_l)$$



$$\int_y^{y_1} (\sigma_x + d\sigma_x) B dy = \int_y^{y_1} \sigma_x B dy + \tau_{yx} B dx \quad (\tau_{yx} = \tau_{xy})$$

$$\tau_{yx} B dx = \int_y^{y_1} d\sigma_x B dy$$

$$\sigma_x = \frac{M_z}{I_z} y \rightarrow d\sigma_x = \frac{dM_z}{dx} \cdot \frac{y}{I_z} dx$$

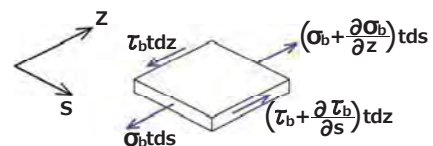
$$\tau_{yx} B dx = \int_y^{y_1} \left( \frac{dM_z}{dx} \cdot \frac{y}{I_z} dx \right) B dy$$

$$\tau_{yx} B = \int_y^{y_1} Q_y \cdot \frac{y}{I_z} B dy \quad \left( \frac{dM_z}{dx} = Q_y \right)$$

$$\tau_{yx} (= \tau_{xy}) = \left( \frac{Q_y}{I_z B} \right) \int_y^{y_1} y B dy$$

$$= \frac{Q_y}{I_z B} S_z(y) \quad \left( \int_y^{y_1} y B dy = S_z(y) \right)$$

## Shear flow theory



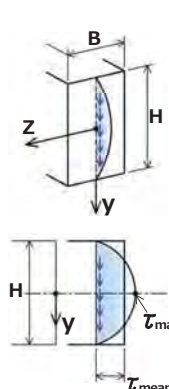
$$\frac{\partial(\sigma_b t)}{\partial z} + \frac{\partial(\tau_b t)}{\partial s} = 0$$

$$\tau_b \cdot t = - \frac{\partial(\sigma_b t)}{\partial z} ds$$

$$\Downarrow \quad (\sigma_b = \frac{M}{I} y)$$

$$\tau_b \cdot t = \frac{Q}{I} \int y t ds$$

## Ex. Rectangular section



$$S_z(y) = B \int_y^{H/2} y dy = B \left[ \frac{y^2}{2} \right]_y^{H/2}$$

$$= \frac{B}{2} \left( \frac{H^2}{4} - y^2 \right)$$

$$\tau_{xy} = \frac{Q_y}{I_z B} S_z(y) = \frac{Q_y B}{I_z B} \left( \frac{H^2}{4} - y^2 \right)$$

$$= \frac{Q_y}{2 I_z} \left( \frac{H^2}{4} - y^2 \right)$$

max.  $\tau_{xy}$  is produced at  $y=0$

$$\tau_{xy, \max.} = \tau_{xy} |_{y=0} \quad \left( I_z = \frac{BH^3}{12} \right)$$

$$= \frac{1}{2} \frac{12}{BH^3} \frac{H^2}{4} Q_y$$

$$= \frac{3}{2} \frac{Q_y}{BH} \quad (A = BH)$$

$$= \frac{3}{2} \frac{Q_y}{A}$$

$$= 1.5 \tau_{\text{mean}} \quad \left( \tau_{\text{mean}} = \frac{Q_y}{A} \right)$$

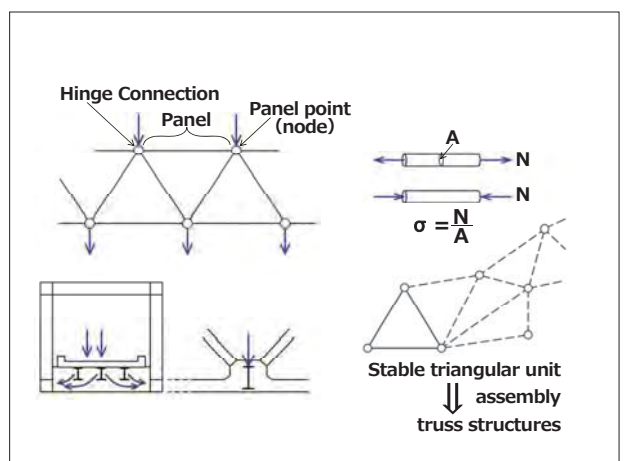
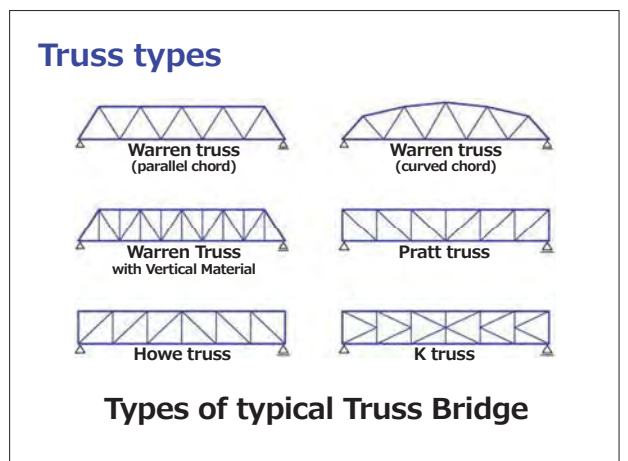
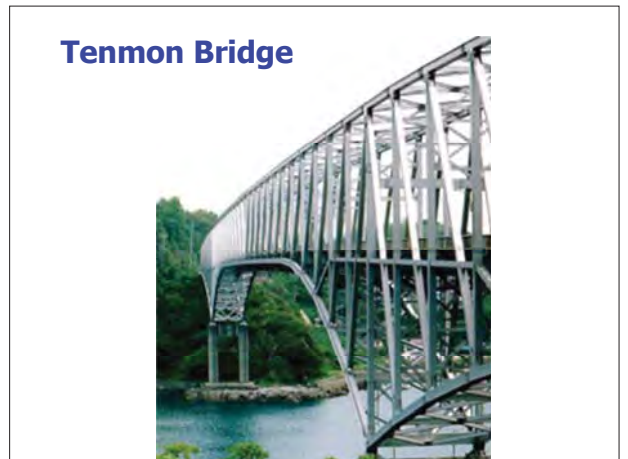
## Shear stress by shear flow theory

$\tau_1 = \frac{S}{I} \frac{bh}{4}$	$\tau_1 = \frac{S}{I} \frac{bh}{2}$	$\tau_1 = \frac{S}{I} \frac{bh}{4}$
$\tau_2 = \frac{S}{I} \frac{bh}{2} \frac{I_z}{I_w}$	$\tau_2 = \frac{S}{I} \frac{bh}{2} \frac{I_z}{I_w}$	$\tau_2 = \frac{S}{I} \frac{bh}{4} \frac{I_z}{I_w}$
$\tau_{\text{max}} = \frac{S}{I} \left( \frac{h^2}{8} + \frac{bh}{2} \frac{I_z}{I_w} \right)$	$\tau_{\text{max}} = \frac{S}{I} \left( \frac{h^2}{8} + \frac{bh}{2} \frac{I_z}{I_w} \right)$	$\tau_{\text{max}} = \frac{S}{I} \left( \frac{h^2}{8} + \frac{bh}{4} \frac{I_z}{I_w} \right)$

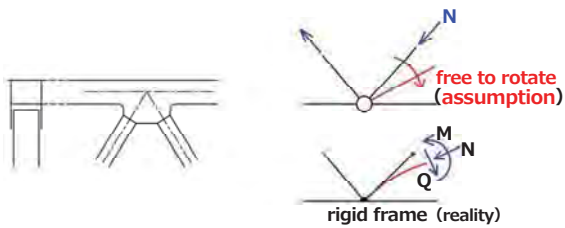


[10-5-1,2]

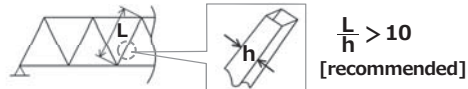
## Axial Force of Truss Structures



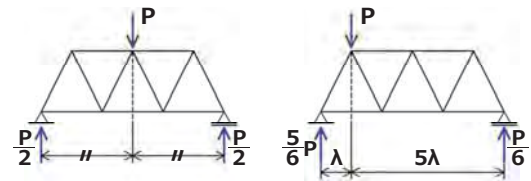




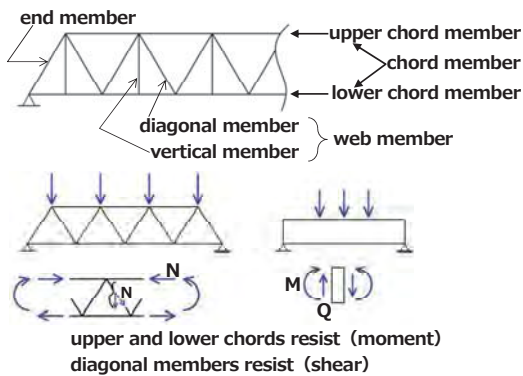
In order to (M) be smaller value.



## Reactions



The same procedure as applied to beam reactions.

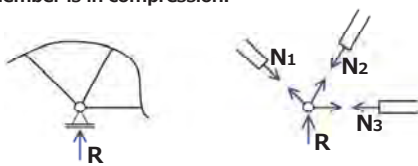


## Method to obtain axial force in truss structures

- Joint method
- Section method
- Ritter (moment) method

### [joint method]

First, tension force (+) in member is assumed. If obtained (calculated) sign is reverse (minus), member is in compression.

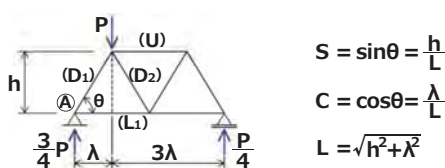


At node, equilibrium conditions ( $\Sigma H=0$ ,  $\Sigma V=0$ ) are applied.

$$\begin{aligned} \Sigma V=0 \quad D_2 S &= -P - D_1 S = -P + \frac{3}{4}P = -\frac{1}{4}P \\ &\rightarrow D_2 = -\frac{1}{4S}P \\ \Sigma H=0 \quad U &= (D_1 - D_2)C \rightarrow U = -\frac{C}{2S}P \end{aligned}$$

$$\begin{aligned} \Sigma V=0 \quad D_3 S &= -D_2 S \rightarrow D_3 = \frac{1}{4S}P \\ \Sigma H=0 \quad L_2 &= L_1 + (D_2 - D_3)C \rightarrow L_2 = \frac{C}{4S}P \end{aligned}$$

$$\begin{aligned} \Sigma V=0 \quad D_4 S &= -\frac{P}{4} \rightarrow D_4 = -\frac{1}{4S}P \\ \Sigma H=0 \quad L_2 &= -D_4 C \rightarrow L_2 = \frac{C}{4S}P \end{aligned}$$



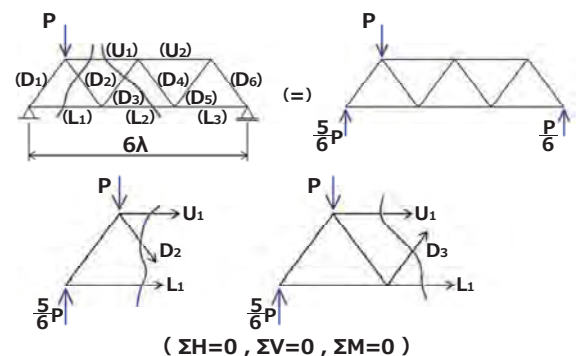
$$\begin{aligned} S &= \sin \theta = \frac{h}{L} \\ C &= \cos \theta = \frac{\lambda}{L} \\ L &= \sqrt{h^2 + \lambda^2} \end{aligned}$$

At first, we start node point with two unknowns

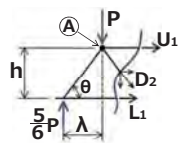
at A,

$$\begin{aligned} \Sigma V=0 \quad D_1 S + \frac{3}{4}P &= 0 \rightarrow D_1 = -\frac{3}{4S}P \\ \Sigma H=0 \quad D_1 C + L_1 &= 0 \rightarrow L_1 = -D_1 C = \frac{3C}{4S}P \end{aligned}$$

### [Section method]







$$S = \sin\theta = \frac{h}{L}$$

$$C = \cos\theta = \frac{L}{L}$$

$$L = \sqrt{h^2 + \lambda^2}$$

$$\Sigma H = 0 \quad U_1 + L_1 + D_2 C = 0$$

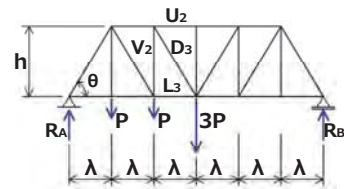
$$\Sigma V = 0 \quad D_2 S = \frac{5}{6}P - P = -\frac{1}{6}P \rightarrow D_2 = -\frac{1}{6S} \cdot P$$

$$\Sigma M = 0 \quad (\text{at point A})$$

$$L_1 h = \frac{5}{6}P \lambda \rightarrow L_1 = \frac{5\lambda}{6h} \cdot P (= \frac{5C}{6S} \cdot P)$$

$$U_1 = -L_1 - D_2 C = -\frac{5C}{6S} \cdot P + \frac{C}{6S} \cdot P$$

$$\rightarrow U_1 = -\frac{2C}{3S} \cdot P (= -\frac{2\lambda}{3h} \cdot P)$$



$$S = \sin\theta = \frac{h}{L}$$

$$C = \cos\theta = \frac{\lambda}{L}$$

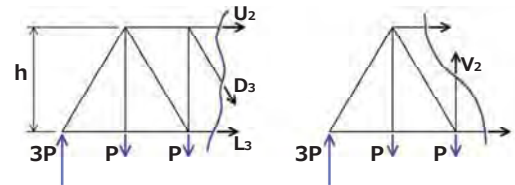
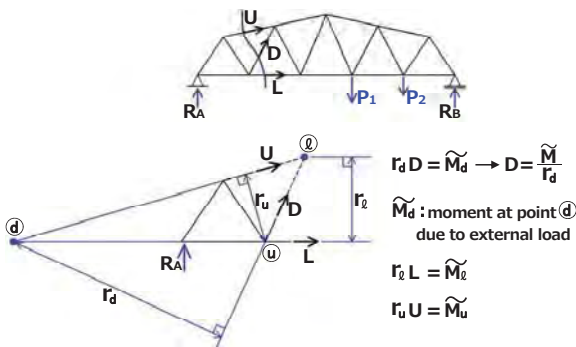
$$L = \sqrt{h^2 + \lambda^2}$$

$$R_A + R_B = 5P$$

$$R_B (6\lambda) = P\lambda + P(2\lambda) + 3P(3\lambda) = 12P\lambda \rightarrow R_B = 2P$$

$$R_A = 5P - R_B = 3P$$

### [Ritter (moment) method]



$$D_3 S + 2P = 3P \rightarrow D_3 = \frac{1}{S}P$$

$$V_2 + 3P = 2P$$

$$\rightarrow V_2 = -P$$

$$L_3 h + P\lambda = 3P(2\lambda)$$

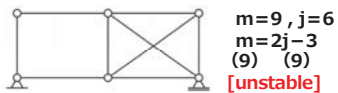
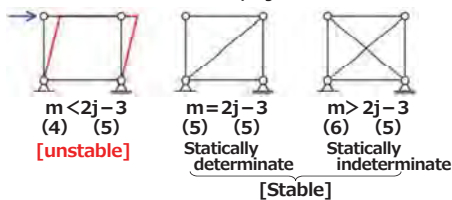
$$\rightarrow L_3 = 5P \frac{\lambda}{h} = 5 \frac{C}{S} P$$

$$U_2 = -D_3 C - L_3 = -\frac{C}{S} P - 5 \frac{C}{S} P$$

$$\rightarrow U_2 = -6 \frac{C}{S} P$$

### (Internal) stable, unstable

m : number of members, j : number of nodes



### (Total) stable, unstable, determinate, indeterminate

(Internal)  $m \geq 2j - 3$  stable

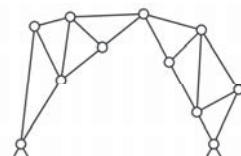
(External)  $r \geq 3$  stable

(total system)

$m + r = 2j$  stable, statically determinate

$m + r > 2j$  stable, statically indeterminate

$m + r < 2j$  unstable



$$m=18$$

$$j=11$$

$$r=4$$

$$m + r = 22 = 2j$$