1-8-4 Counterpart's Coopertion and Training

(1) Local headquaters

The study team was given a wide range of services and conveniences in conducting the survey operations from the counterparts, including the management of the survey operations. The counterparts who participated in the consultations over the survey operations and the management of the operations with the study team at the headquaters were as follows:

Brig. MD. Mahbubul Karim	(Surveyor General)
Col. Shahedul Islam Mondal	(Surveyor General, Fourth Year Study)
Mr. A.K.M. Shamsul Alam	(Director Defence Survey)
Maj. Kh. Aftab Hossain	(Director Defence Survey, Second and Third Year Study)
Lt. Col. Kazi Shafayetul Haque	(Director Defence Survey, Fourth Year Study)
Mr. Noor Mohammad Mia	(Section Chief of Defence Survey)
Mr. Abu Naser Wahid	(Section Cheif of Defence Survey, Third and Fourth Year Study)

2) Training and Cooperation with Counterpart Personnel

During field study, counterpart staff not only participated in the study team headquarters and work groups, but also cooperated in operational preparations for each process, material organization, negotiations with local residents and field study priorities.

The study team tranferred technology by working together with counterpart staff.
Counterpart personnel who participated in the study are listed in the following table.

Work group	SOB C/P	Position
Control point surveying	MR. ABU NASER WAHID	Technical Assistant
Control point surveying	MR. MD. RAZIUDDIN	Sub Assistant
Control point surveying	MR. SHAHJAHAN ALI	Surveyor
Control point surveying	MR. M.A. KADER	Surveyor
Control point surveying	MR. SHAIKH SYED AHMED	Surveyor
Control point surveying	MR. AZMAT HOSSAIN	Surveyor
Control point surveying	MR. AFAZUDDIN AHMED	Sub Assistant
Control point surveying	MR. SHAFIQUL ISLAM (Sr)	Technician
Control point surveying	MR. SHAFIQUL ISLAM (Jr)	Surveyor
Control point surveying	MR. ATIQUL ISLAM	Surveyor
Control point surveying	MR. HUMAYUN KABIR	Surveyor
Control point surveying	MR. MD SHAHJAHAN	Surveyor
Control point surveying	MR. MD SHAH ALAM	Surveyor
Control point surveying	MR. RATTAN	Sub Assistant

Work group	SOP C/P	Position
Control point surveying	MR. SHAFIQ	Sub Assistant
Control point surveying	MR. AZHER	Technician
Control point surveying	MR. MUJIB	Surveyor
Control point surveying	MR. SHAHJAHAN	Surveyor
Control point surveying	MR. TAJUL	Surveyor
Control point surveying	MR. MOSTAFA	Surveyor
Control point surveying	MR. ATAUR	Surveyor
Control point surveying	MR. KADER	Surveyor
Control point surveying	MR. ENAMUL	Surveyor
Levelling	MR. NAYON CHANDRA SARKER	Technical Assistant
Levelling	MR. GANESH CHANDRA ROY	Technical Assistant
Levelling	MR. MD. RAZIUDDIN	Sub Assistant Super.
Levelling	MR. AMIN	Technical Assistant
Levelling	MR. KADER	Surveyor
Levelling	MR. ALI	Surveyor
Levelling	MR. AZMAT	Surveyor
Levelling	MR. HASAN	Surveyor
Levelling	MR. ASHRAF	Surveyor
Levelling	MR. SHAFIQUL	Surveyor
Levelling	MR. MOSHARRAF	Surveyor
Levelling	MR. ALAM	Surveyor
Levelling	MR. AHMED	Surveyor
Levelling	MR. HASSAN	Surveyor
Levelling	MR. RAFIQ	Surveyor
Levelling	MR. S. ALAM	Surveyor
Levelling	MR. RUHUL ANIN	Surveyor
Levelling	MR. BAHAR	Surveyor
Levelling	MR. ASHRFA	Surveyor
Tidal station	MR. NAYAN CHANDRA SARKER	Technical Assistant
Tidal station	MR. GANESH CHANDRA ROY	Technical Assistant
Tidal station	MR. SHAHJAHAN ALI	Surveyor

1-8-5 Training Courses Provided for Counterparts

Futhermore, the following counterpart personnel received special training in Japan. In order to comprehend the survey system practiced in Japan, these trainees visited the Ministry of Construction, Geographic Survey Institute, Japan Survey Association, Japan Map Center and survey instrument manufacurers, where they received explanations of study, facilities and other details.

Name	Training term	Training program
Mr. NOOR MUHAMMAD MIAN	1993. 3. 7 ~ 4. 16	Plan and management of geodetic surveying
Mr. ABU NASER WAHID	1993. 7. 5 ~ 8. 11	Technical explanations and facility study tours for geodetic network training
Mr. ABDUL QUADIL	1994. 6. 6 ~ 7. 9	- ditto -
Mr. MD. RAZIUDDIN	1995. 3. 27 ~ 4.23	- ditto -

1-8-6 Significance of the Results from the Study

In this study the precise control points were installed in almost even density (one at an interval of about 30 km) in the entire inland area, excluding the Chittagong Hill Tracts and the coastal area in the south, and the geodetic coordinates system was established based on the standard datum of geographical coordinates obtained by GPS. Also, the mean sea level of the Bay of Bengal was determined by the tidal observation at Chittagong, and the bench marks were installed along the main national roads of a total extension of about 2,400 km at the distribution ratio of one unit at an interval of about 5 km, and orthometric heights were computed by the 1st order levelling.

(1) Integration of survey results and renovation

The final survey results based on the datum station installed in Bangladesh form the geodetic skelton that leads to the determination of "geodetic coordinates" and "elevations" and contribute to the renewal and integration of the existing survey results in the country. Also, the results will greatly contribute to the development of land, flood protection measures and map production.

(2) Correction of horizontal datum of geographical coordinates

While the standard datum of geographical coordinated enables to determine the exact position on the world geodetic coordinates system (geocentric coordinates and geodetic longitude and latitude), a list of the survey data on the UTM (Universal Transverse Mercator) values has been prepared for all the control points so that the present map projection can be converted to Transverse Mercator Projection in the future, which Bangladesh desires.

(3) Utilization of existing survey data

In the light of the provisional use of the existing published survey data and existing maps available in Bangladesh, with due respect to the old reference ellipsoid data (Everest-1830), we have computed the transformation parameter necessary for converting the data to the

WGS-84 reference ellipsoid so that Bangladesh can convert to the new geodetic system any time when it so desires in the future.

(4) Utilization of local geoid map

A geoid map has been produced by computing the geoid undulation on the Bangladesh geodetic network after linking orthometric heights obtained from the newly determined bench marks network to the 51 bench marks.

This geoid map will provide important materials when Bangladesh intends to conduct GPS surveying on its own initiative.

2. TECHNICAL REPORT

This study is a field survey that was started in March 1992 (Phase I) following a request for technical assistance from the government of Bangladesh in connection with the "Study on the Geodetic Survey of Bangladesh".

The objectives of the study were to determine the mean sea level in the Bay of Bengal, to add terrestrial "positions" and "elevations" to a geodetic control point network that was established more or less uniformly throughout the country, and to attempt to rearrange and unify the results of state geodesy. The survey work was completed in March 1995 (Phase IV).

The work of tidal observation involved measuring analog and digital data in parallel. For this we introduced a computerized system for aggregation. Meanwhile, levelling survey involved the use of digital automatic levels equipped with data collectors. For control point surveying, the GPS (Global Positioning System) space technology was used. For these surveying systems we employed the most up-to-date and high-precision surveying technology currently available in Japan. Therefore, we can say that we have completed one of the world's highest level geodetic control networks in Bangladesh.

The "vertical datum" and the "horizontal datum" were set up in the capital city of Dhaka, located more or less in the centre of the geodesic control point network.

In February 1994 (Phase III) an unveiling ceremony was held to commemorate the completion of work on the geodetic datum and the vertical datum, symbols of the geodesic control point network. In their addresses representatives from the Bangladesh side expressed their joy on the completion of their country's own "horizontal and vertical datum" as well as their thanks for the cooperation from Japan.

This was the first attempt by the Japanese government to offer technical cooperation in the field of major geodesic surveying in order to build a state geodesic control point network such as the one in this study. It was achieved by aggregating the surveying technology available in Japan.

The results of surveying for the geodesic control point network will be a vital state asset for the government of Bangladesh and will contribute to national land development and flood control measures as well as surveying for cartography and so on.

Therefore, when this study is complete, we will make proposals to the government of Bangladesh for maintenance and management plans on control points over the long term, in order to avoid loss or damage to them. Our hope is that they will be put to positive use in a variety of ways, such as making links with the world's geodesic networks, surveys on movements of the earth's crust, surveys on ground subsistence, and academic and scientific research.

This study proceeded according to plan, on the basis of amicable cooperation between the representatives of both countries. In doing so we were able to intensify human interchanges throughout the joint survey work between the two countries. In addition, technical guidance was given to Bangladesh technical personnel, and we were able to take steps to improve surveying technology and to effectively transfer new surveying technology.

2-1 Geodesic Control Point Network and Facility Construction

We conducted a study on the positions for installation of surveying facilities and control points (triangulation and levelling) that would form the backbone of the national land geodesic control point network. We constructed surveying facilities and installed control points and bench marks, as listed below.

We contracted a local civilian construction company and conducted bidding, concluded contracts, and executed the work.

We submitted site photographs and work process charts in order to control the processes, quality, safety etc. of the construction work, and implemented work execution management as appropriate on site.

1) Construction of tidal observation station (including auxiliary tide gauge station) .... (Photo __)
2) Construction of vertical datum .... (Photo __)
3) Renovation of geodetic datum (including protective facilities) .... (Photo __)
4) Installation of bench marks (228 standard type, 233 smaller type) .... (Photo __)
5) Installation of control points (26 type A, 89 type B) .... (Photo __)
(for designs etc., refer to Appendix P/0)

2-1-1 Construction of Tidal Observation Station

1) Construction of standard tidal observation station

Before constructing the standard tidal observation station, we conducted sea bed boring surveys, water depth surveys, and topographical surveys, and, after considering factors such as topography, geology, and marine conditions, we chose a location approx. 60m south of the piers of the CUFL (Chittagong Urea Fertilizer Ltd.) at the estuary of the River Karnafuli in Chittagong.

The results of the boring survey showed that there was good ground that could give sufficient support at a point about 18-25m below the river bed surface. Here we constructed the main body of the standard tidal observation station based mainly on the survey location, and as an auxiliary facility we built a structure consisting of a pier that approached the tidal observation station from the landward side. Before constructing it, we studied the standards, strength, quality and other aspects of locally procured construction materials.

In this location there seems to be little accumulation of sediments and no obstacles to navigation by ships or the like, since it is sandwiched between two large piers of the fertilizer factory.

We determined the depth of the well for tidal observation and the size and location of water inlet holes, and undertook detailed design on the basis of data collected locally, sea bed geology boring surveys, and so on.

We made the lower extremity (bottom) of the observation well about 1m deeper than the minimum tide level, the upper extremity about 5m higher than the normal maximum tide level, and the diameter of the observation well 1.0m.

The size of the observation room in the tidal observation station was set at 4m x 4m and the ceiling height at 3m. We attached a steel window with a diameter of 50 cm to enable a level staff to be installed directly near the centre of the ceiling, and added a glass window on the south side in order to allow light in.

The structure of the tidal observation station was so made as to withstand the local topographical, geological, meteorological, and marine conditions, and also to enable continuous observation of tide levels over the long term.

A measuring staff (tide pole) was installed on the supporting pillar of the cat walk near the tidal observation station for visual observation, and a standard mark (metal marker) was attached at the top of the well.

We set up a raised platform above the observation well, installed a tidal observation gauge (Fuess type, Kyowa Shoko Co., LFT-5 type), and started tide level observations.

Meanwhile, a "tidal station bench mark (TBM)" of the same standard as the bench mark (smaller type) was installed to the landward side of the tidal observation station.

2) Construction of the auxiliary tide gauge station

After a marine condition survey conducted around the estuary of the River Karnafuli in Chittagong, the tidal observation station was located about 1.5km upstream from the estuary of the Karnafuli River. Therefore it was expected to be susceptible to the effects of increased water levels during the rainy season. Thus there was a concern that the rise and fall in sea level due to tidal movements in the open sea might show somewhat different variations in view of the nearby topography and marine conditions. In view of this we constructed an auxiliary tide gauge station on the coast of the Bay of Bengal near the river estuary, in order to verify actual tide level observations.

The auxiliary tide gauge station was built in a location about 800m into the Patenga Beach offing. In addition, we installed an auxiliary measurement staff (tide pole) at a point about 400m into the offing. (See Figure 6)

The interior structure of the protective piles of the auxiliary tide gauge station was of steel reinforced concrete, using three steel pipes with a diameter of 400mm, with a length of 18m below the sea bed and 10 m above it. Further, for reinforcement they had a beam structure whereby steel pipes with a diameter of 300mm were linked together at a point 8m above the sea bed. In addition, the structure was such that it could withstand cyclones and waves.

The structure of the steel pipe for attaching the tide gauge was similarly of self-standing steel-reinforced concrete, and it was linked to each of the protective piles by slightly slackened 25mm wire, thus giving it a structure whereby it could withstand excessive lateral movement.

A tide gauge platform was installed 0.81m above the measuring staff zero point, and the total length of the main body of the tide gauge was 0.36m. Thus the standard observation level of the pressure-sensing tide gauge (the zero level of the tide gauge) was set at a position 1.17m above the measuring staff zero position in normal installation conditions.

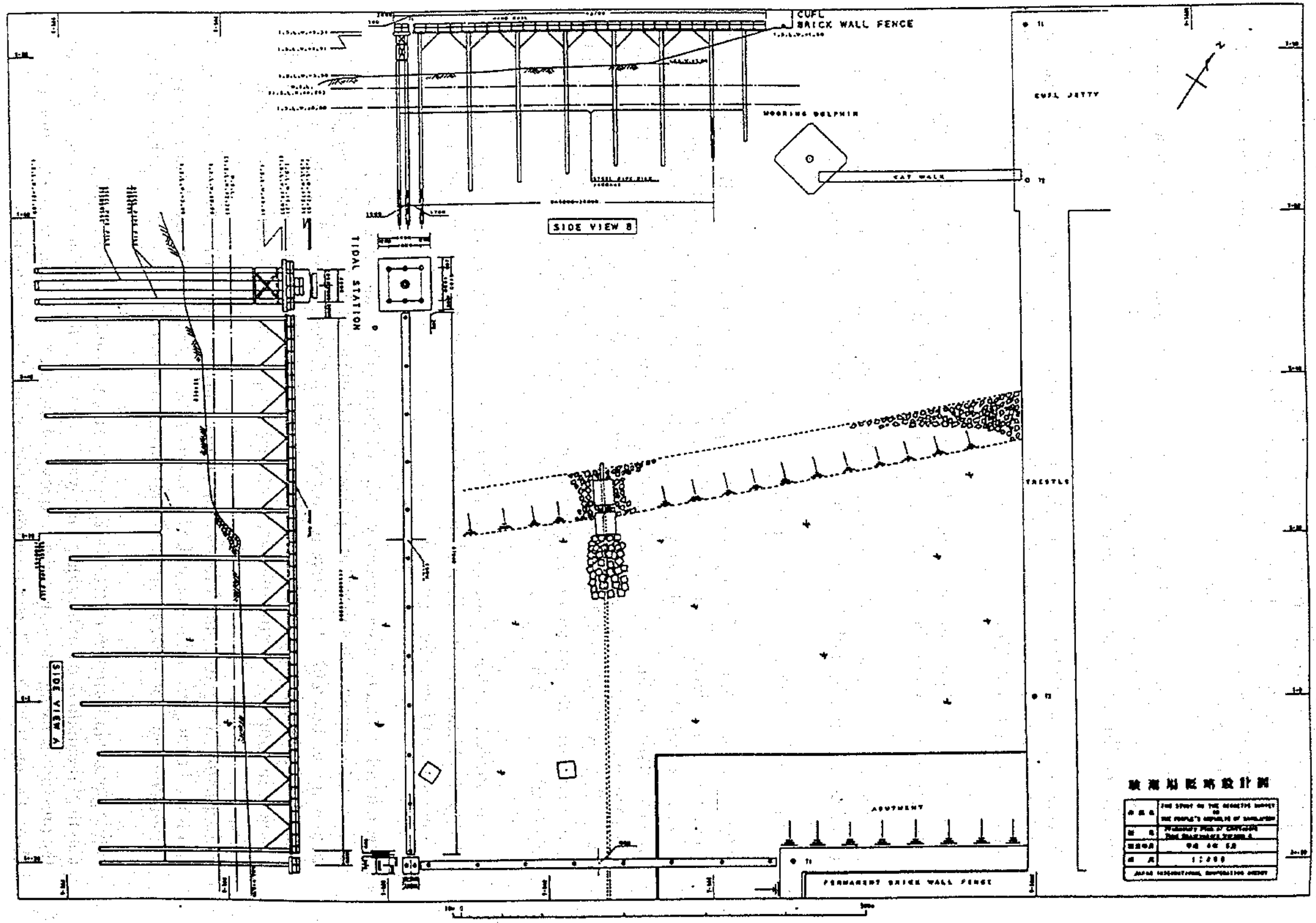


Figure 5 Layout of the standard tide observation station

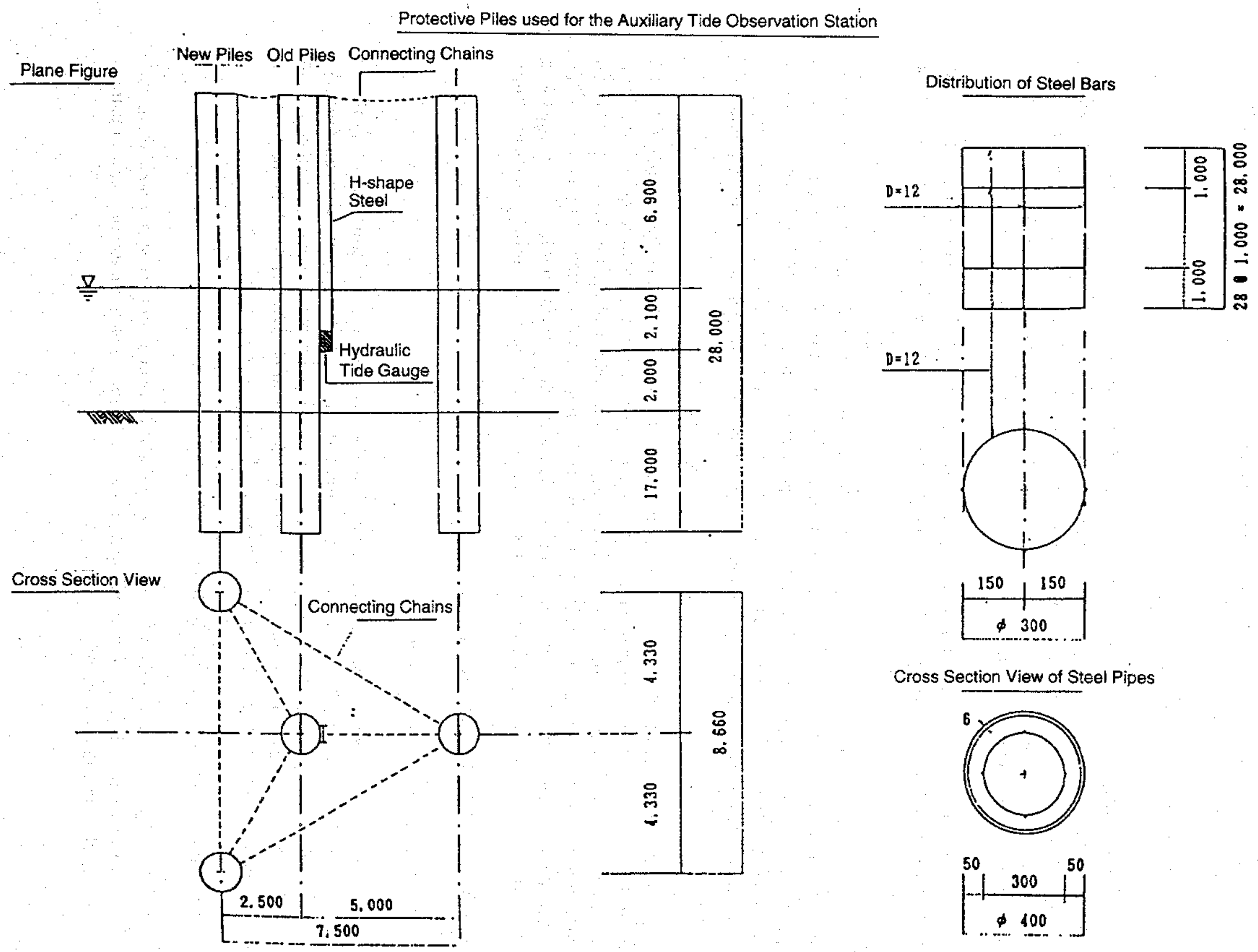


Figure 6 Layout of the auxiliary tide observation station

We studied tide level data from the auxiliary tide gauge station in comparison with the mean tide level of the standard tidal observation station, and used these as vital reference data for determining the mean sea level.

2-1-2 Construction of Vertical Datum

The vertical datum was constructed next to the geodetic datum in a corner of the Gulshan-2 Tank Park. As for the soil strata at this point, according to data from the Geological Survey of Bangladesh, the area comprises a natural levee with a hard red clay zone about 10 metres below the surface and a highly compacted gravel layer below that. Thus it is on firm ground with sufficient resistance for supporting the vertical datum and buildings.

To construct the vertical datum we drove a 600mm diameter cast in site concrete pile to a depth of 16m from the surface to the supporting foundation and built the vertical datum on top of this. (See Figure 7)

We inserted a stainless steel metal marker into the centre of the datum, and to prevent destruction by natural disasters and erosion by storms we built a housing facility of hard, good quality brick to protect the datum. We built this datum housing facility to such a height that a 3 metre 1st order levelling staff could be installed in it, and made it such that levelling observation could be implemented directly. In addition, the point for installation of the level inside the site was laid with concrete.

In a position between the geodetic and vertical data we constructed two reference points (A, B) using the same criteria as the bench marks (type A). These were just in case of unforeseen situations, as well as to monitor time-lapse changes in the height of the vertical datum. We drove 600mm diameter concrete piles to a depth of 16m down to the supporting foundation, as in the case of the vertical datum, and built the points on top of this. By repeatedly carrying out levelling using these reference points it will be possible to monitor subsidence and other phenomena in the vertical datum.

2-1-3 Geodetic Datum Renovation

1) Renovation of the geodetic datum

As for the geodetic datum that forms the point of departure for the control point network, we repaired the 1st order triangulation point (formerly the provisional national datum Gulshan Point) that was installed in the Gulshan-2 Tank Park in Dhaka City and renovated it with concrete to make it suitable as a national geodetic datum. We engraved the name of the geodetic datum in marble, and set this on the front.

2) Construction of protective facilities

In order to preserve and protect the horizontal and vertical datum, we surrounded them with a 1 metre high brick wall and put up a protective facility (fence) on top of this wall using a steel frame and barbed wire.

Layout of Vertical Datum Point

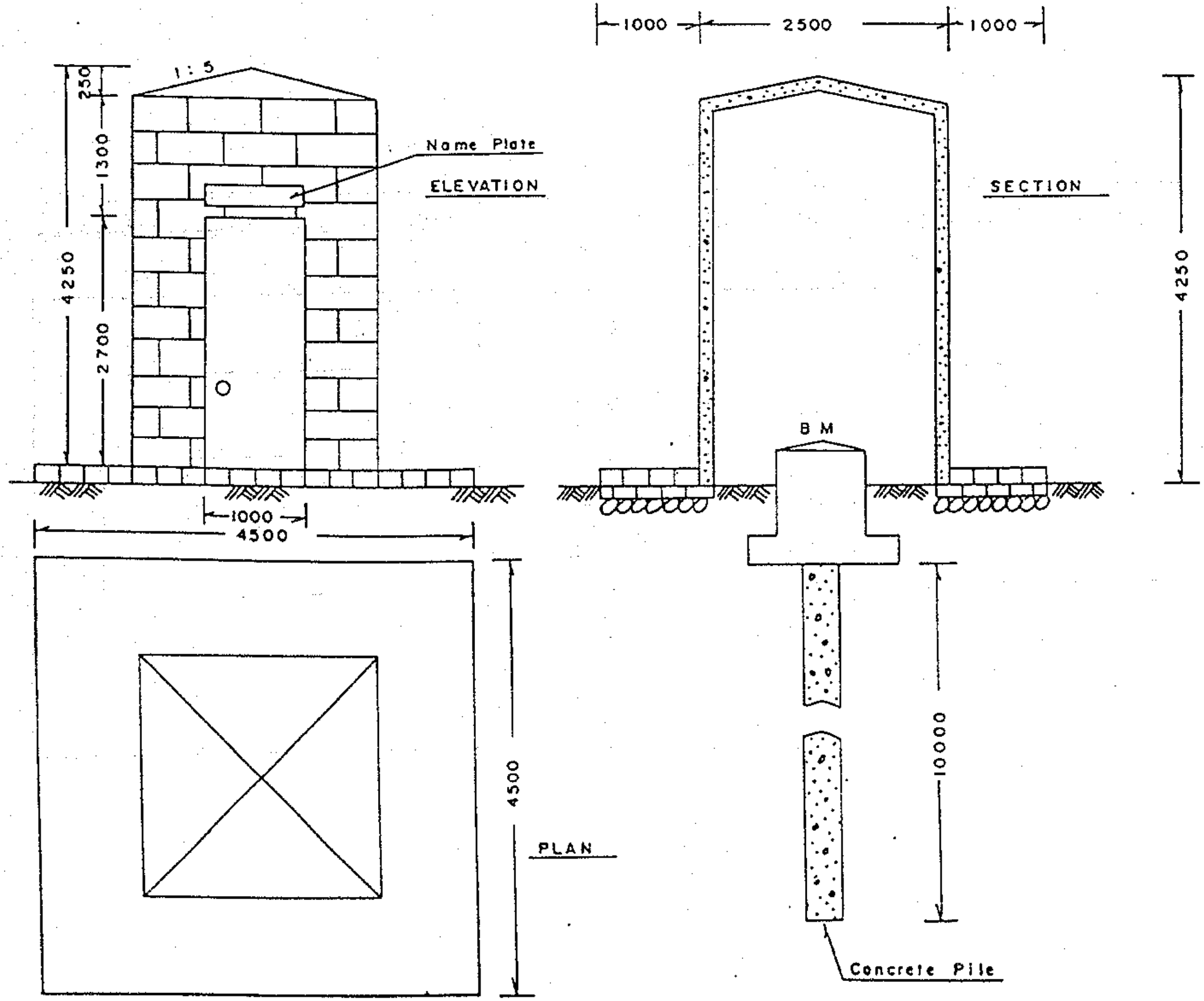


Figure 7 Layout of the datum station of levelling

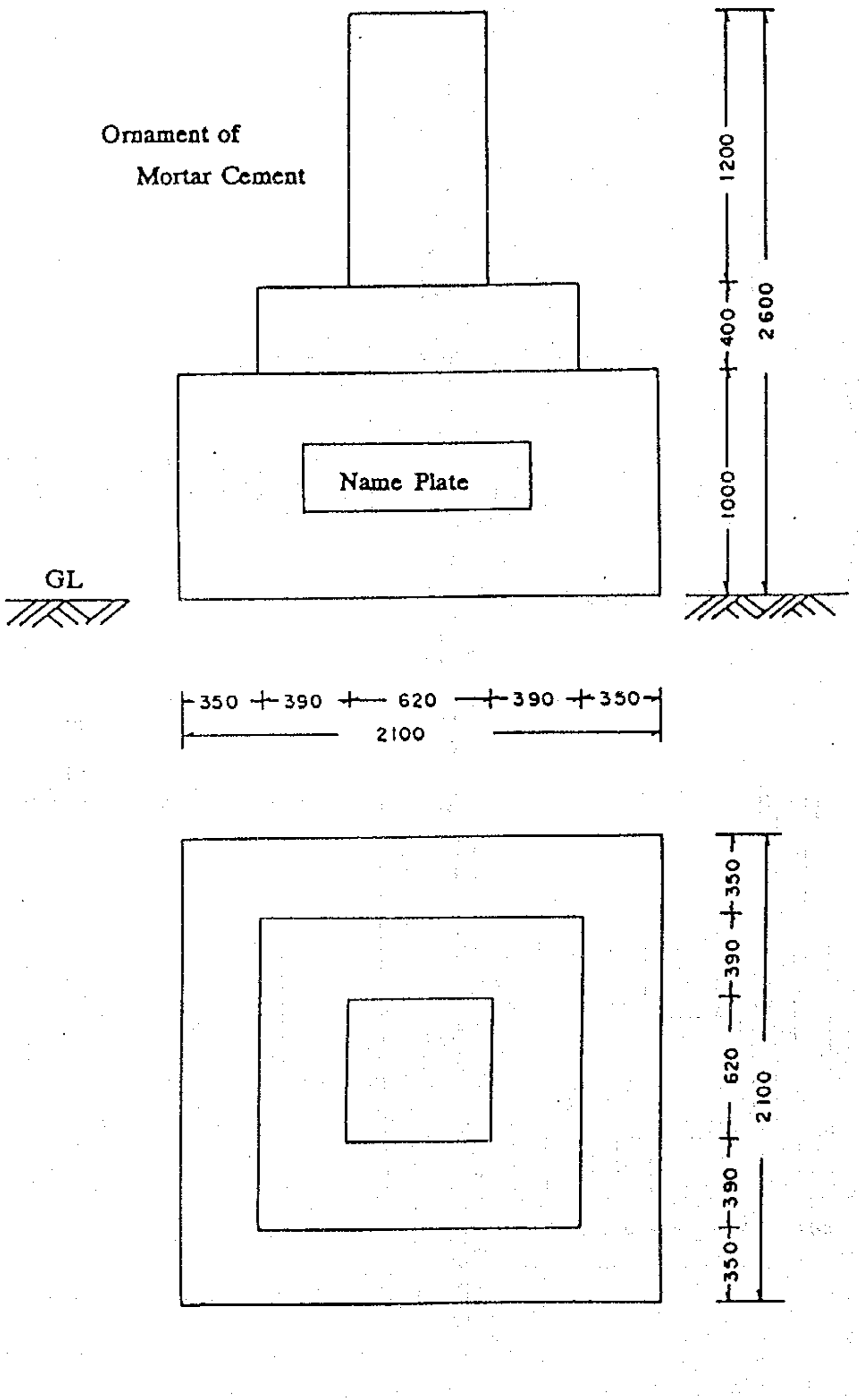


Figure 8 Layout of the standard datum PF geographical coordinates

2-1-4 Monumentation of Bench Marks

Bench marks monumented in the study area were mainly installed on the verges of main roads and on public land at intervals of around 5km along the levelling route, taking into account issues such as their use, conservation, topography, the state of transportation, preservation, and so on.

The total length of the levelling route was 2,388 km, while the number of bench marks installed was 228 of the standard type and 233 of the smaller type, totalling 461 bench marks in all.

For the specifications of bench marks (standard type), after deliberation with the SOB side we installed them at intervals of approximately 10 km, using a type that retained the basic specifications of bench marks already installed in Bangladesh.

Meanwhile, for the specifications of smaller type bench marks, we used a type that excluded the bottom part of the standard type bench marks, and installed them in alternation with the latter. (See Figure 9)

2-1-5 Monumentation of Control Points

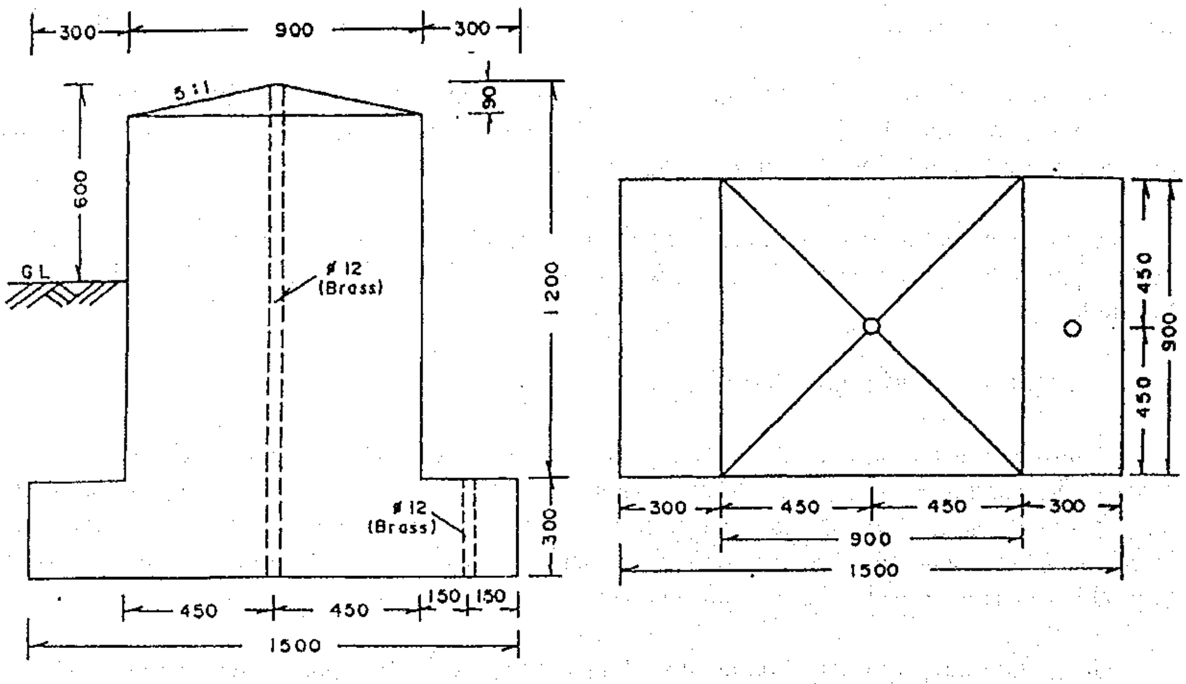
Control points monumented in the study area were mainly installed on public land (city offices, schools, etc.) at intervals of around 30km, taking account of factors such as their use, conservation, topography, the state of transportation, preservation, and so on. Also, we selected sites that were not affected by artificial radio wave interference, natural objects, or vegetation etc., and that were favorable in terms of utilization and preservation.

The total number of control points installed was 115, with 26 of type A and 89 of type B. For the specifications of control points (type A), after deliberation with the SOB side we installed them at intervals of approximately 50 km, using a type that retained the basic specifications of control points already installed in Bangladesh, and taking account of their existing distribution. Meanwhile, for the specifications of type B control points we used a smaller version of type A control points that excluded the base foundation concrete slab of the latter, and installed them so as to complement the type A control points to enable to distribute control stations in 30 km internals. (See Figure 10)

1st Order Bench Mark

Scale 1:20

Standard Type



Smaller Type

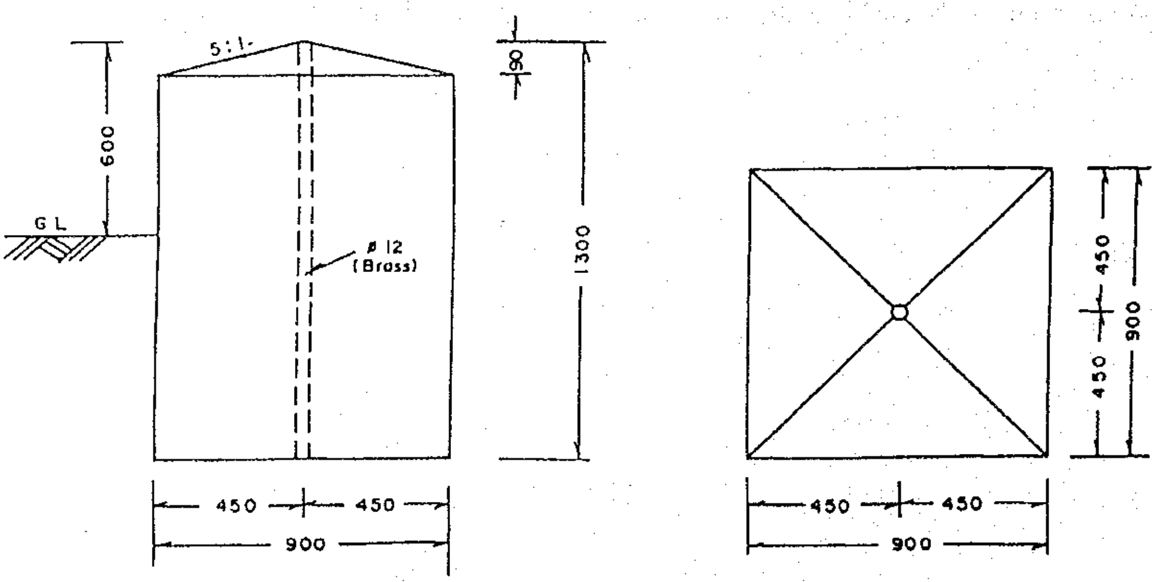
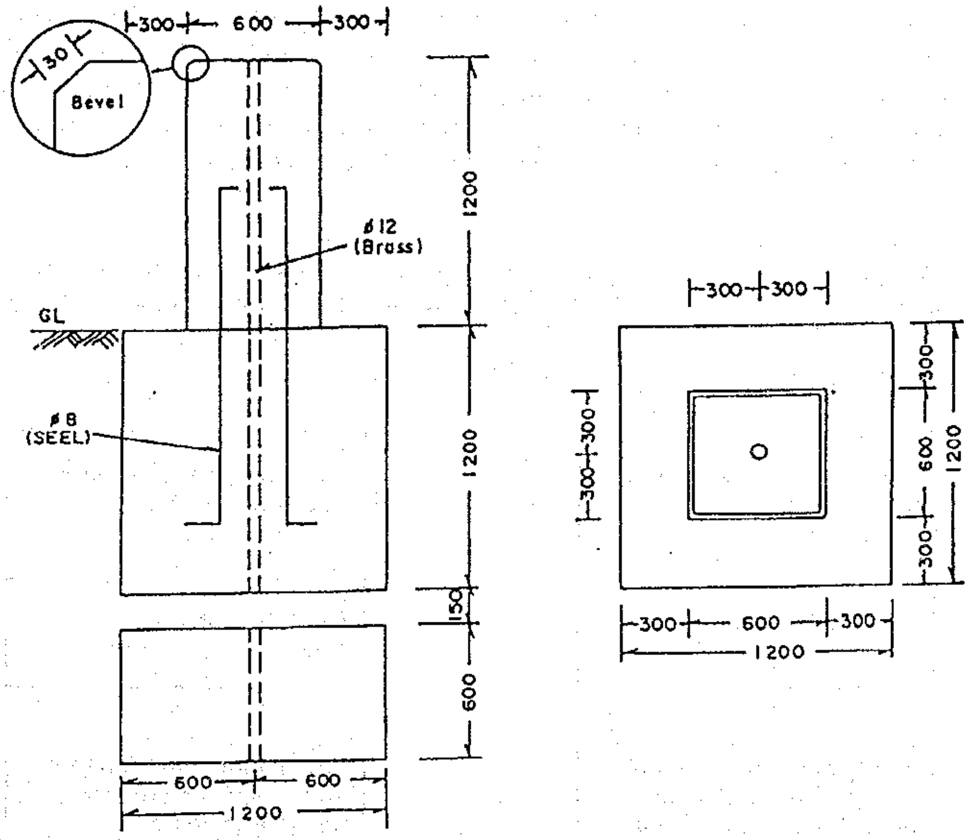


Figure 9 Layout of bench mark

A–type Scale 1:30



B–type

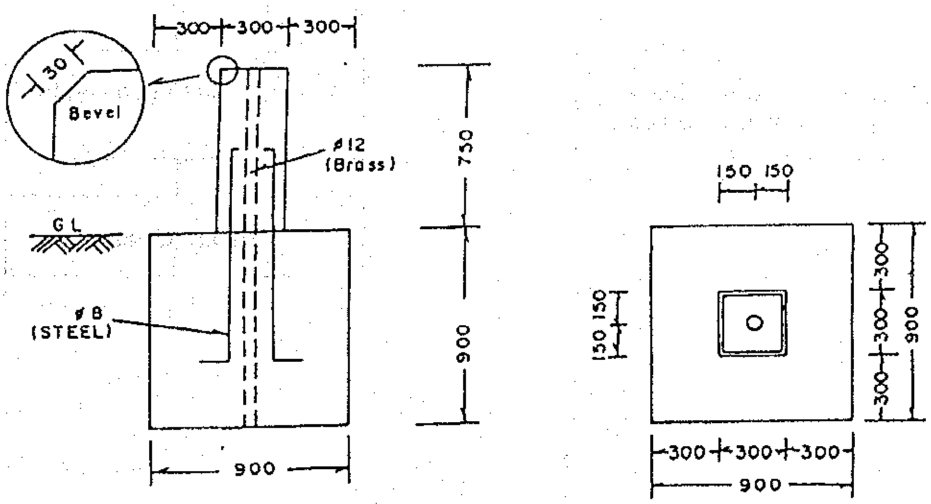


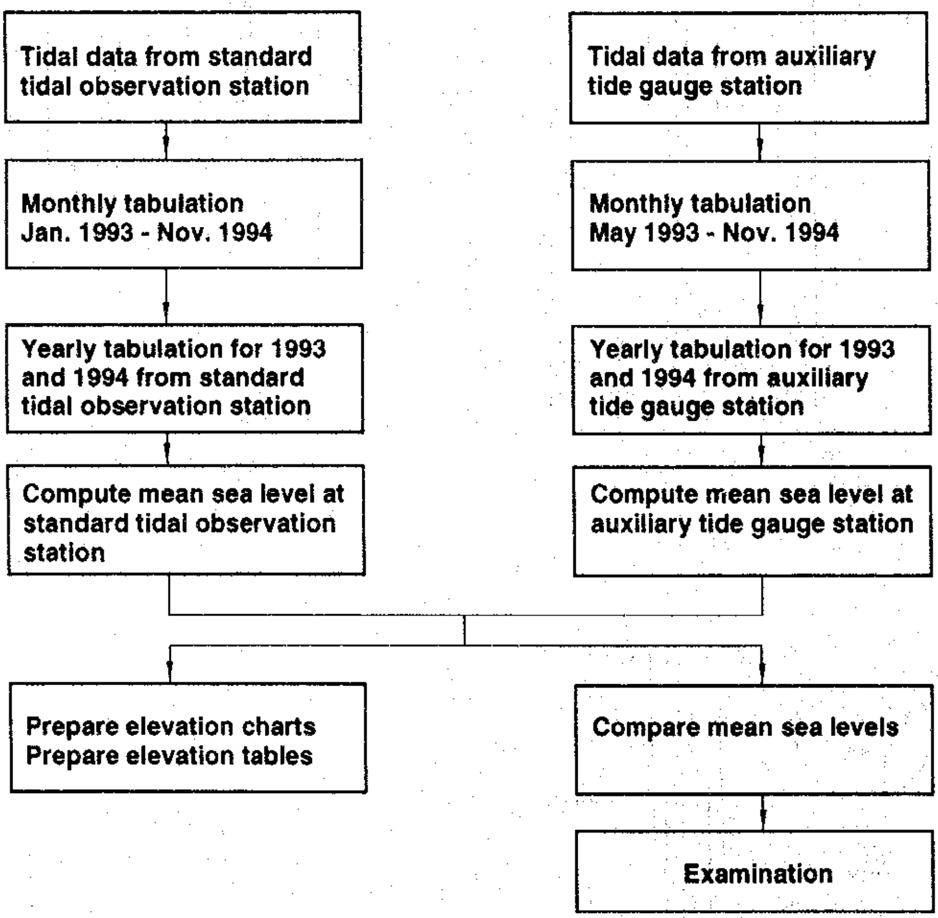
Figure 10 Layout of bench mark

2-2 Tide Level Observation and Analysis

In order to determine the mean sea level to provide the standard level for land elevations, we carried out tide level observations in the Bay of Bengal. Variations in tide level in the Bay of Bengal reach a maximum of about 7 metres, thus observing and analyzing the tide level was extremely difficult work. Once the construction of the standard tidal observation station was complete and the tide gauge had been installed, although a certain amount of observation was lost due to faults caused by mechanical breakdowns, power disruptions, software error, natural disasters etc., by repairing and adjusting these faults we were eventually able to obtain data for a total of 22 months, or close to the 2 years of the plan.

Meanwhile, the auxiliary tide gauge station ultimately yielded observations for only 15 months, due to breakdowns caused by natural disasters and faults involving power circuits and software error.

1) Procedure for computing mean sea level



2-2-1 Reference Level Check and Employed Equipment

1) Standard observation station

We observed the relative heights of the tide gauge, fixed point (standard mark) within the tidal observation station, the tide pole, and the water level near the tidal observation station via direct levelling. The observation accuracy was $\pm 4\text{mm}\sqrt{S}$ (S:km).

For the tide gauge in the standard tidal observation station, we installed a tide gauge (LFT-5 Fuess type) adjusted to conform to the maximum local tide level variance.

This tide gauge is completely powered by batteries, thus it suits the local conditions in a country where power cuts are common. In addition, this device has two functions, namely one whereby it records data on print-out paper in analog format, and one whereby it records the data in a memory which can then be computer processed via floppy disk. Thus if one of the functions were to break down the other would provide a back-up.

The primary local analysis of tide levels was carried out using analytical software incorporated into a tide level analyzer (NEC PC-9801FA).

2) Auxiliary tide gauge station

Surveying of the height of the auxiliary tide gauge station was carried out via direct levelling, to an accuracy of $\pm 10\text{mm}\sqrt{S}$(S:km). For the auxiliary tide gauge we installed a pressure-sensing tide gauge (Union UWLR-2) about 1 metre below the minimum tide level.

This tide gauge detects water level through a pressure sensor and converts this into a tide level reading. The data are recorded on a built-in memory card, computer processed, and analyzed after adjustment with the data of atomospheric pressure.

2-2-2 Tide Level Observation

The chief engineer for tide observation and the SOB were responsible for the maintenance and management of the locally-constructed tide observation stations. When the chief engineer was absent, the JICA Study Team dispatched a staff member to care of the facility instead. The analysis of the tide levels at the site was carried out by the SOB counterpart and the data were sent to Japan for the final analysis by the chief engineer.

Tidal observation in future will be carried out by Survey of Bangladesh (SOB) personnel trained in the tide level observation through technology transfer.

The instruction and training for the SOB counterparts was carried out on the basis of a specially prepared English works manual. In the actual training, we gave necessary instruction on issues such as methods of observation, checks of observation data, data arrangement, and equipment maintenance.

After the end of this study, the SOB will continue to carry out tide level observation over the long term.

2-2-3 Arrangement of Tide Level Data

We carried out analysis on the basis of locally observed tide level data and related records, and worked out the mean sea level (elevation datum).

The data were analyzed by compiling data in monthly units from the processed software of tide level observations (CPR-T UWLR-2), which were then arranged into monthly tabulation. As well as this, we analyzed the data from the auxiliary tide gauge station and calculated the elevation of the tidal station bench mark, as valuable reference material for computing the mean sea level.

Moreover, in conducting this tidal observation work we referred to the work regulations for tidal observation work (Geographical Survey Institute of the Ministry of Construction).

1) Method of computing mean sea level

$$\text{Mean Sea Level} = \frac{\text{Sum of all tide level data during the observation period } (\Sigma H)}{\text{Total number of data during the observation period } (\Sigma n)}$$

2) Mean sea level at the standard tidal observation station (Fuess-type tide gauge)

Observation period	Sum of tide levels	Total data nos.	Mean sea level
Jan. 18, 1993 - Nov. 30, 1994	47,757.97 m	13,698 nos.	3.486 m

3) Mean sea level at the auxiliary tide gauge station (pressure-sensing tide gauge)

Observation period	Sum of tide levels	Total data nos.	Mean sea level
May 28, 1993 - Nov. 30, 1994	31,096.08 m	8,873 nos.	3.505 m

4) Comparison of mean sea levels

From an analysis of mean sea levels we took the tide level in the standard tidal observation station as the standard and compared the tide level at the auxiliary tide gauge station for the same period. As a result, we ascertained that the mean sea level at the auxiliary tide gauge station was 17mm lower. This difference is negligible and does not allow identification of effects such as those of marine or climatic phenomena, increased water levels through rainfall, and so on. However, since tide level observations were only made over a relatively short period we are not able to make judgments over the long term. Thus from now on the SOB ought to conduct long-term observations and carry out corrections and other research.

2-2-4 Tidal Observation Facilities

In order to verify subsidence caused by the dead weight of the structure of the standard tidal observation station & auxiliary tide gauge station, at regular intervals (twice a year) we measured the elevation difference between the tidal observation station and the land-based tidal station bench mark and thus monitored subsidence. But to date there has been no undue change in level nor any phenomenon of structural subsidence.

2-2-5 Transfer of Technology

The technology related to the tide observation was transferred to the SOB engineers based on the observation manual for providing technical guidance and information on the tide observation operations. More specifically, the major items transferred to the SOB counterparts were as follows:

1) How to operate the tide gauge and supplementary equipment

2) How to collect and arrange data

3) How to maintain and inspect the tide gauge and tidal observation station

For more detailed guidance, we gave instruction on the basis of O.J.T.

2-2-6 Determination of Mean Sea Level

We analyzed data observed over approximately two years, computed the mean sea level at the standard tidal observation station on the basis of the arranged tide level observations, and determined the elevation of the tidal station bench mark.

Determination of Mean Sea Level

1) Location (north latitude 22° 14', east longitude 91° 49')
   Tidal observation station: estuary of the River Karnafuli, Chittagong
   Auxiliary tide gauge station: Potenga Beach, Chittagong

2) Tide gauge
   Tidal observation station: LFT-5 (Fuess-type), Kyowa Shoko Co.
   Auxiliary tide gauge station: UWLR-2 (pressure-sensing) Union Engineering Co.

3) Analyzer and tide level analyzer: PC-9801FA NEC
   Analytical software: CPR-T (analysis & processing), UWLR-2 (read only)
   Tabular output: monthly tabulation

4) Data (types, period)
   Tidal observation station: 6-second intervals (digital), analog continuous recording, average for a total of 22 months, 28.1.1993 - 30.11.1994
   Auxiliary tide gauge station: 30-minute intervals (digital), average over 15 months, 28.5.1993 - 30.11.1994

5) Mean sea level (above the observation datum level)

$$\text{Mean Sea Level (MSL)} = \frac{\text{Total of tide levels } (\Sigma H)}{\text{Total data items } (\Sigma n)} = 3.486 \text{ m}$$

6) Elevation of tidal station bench mark

   Elevation of tidal station bench mark (TBM) H = 7.5766m

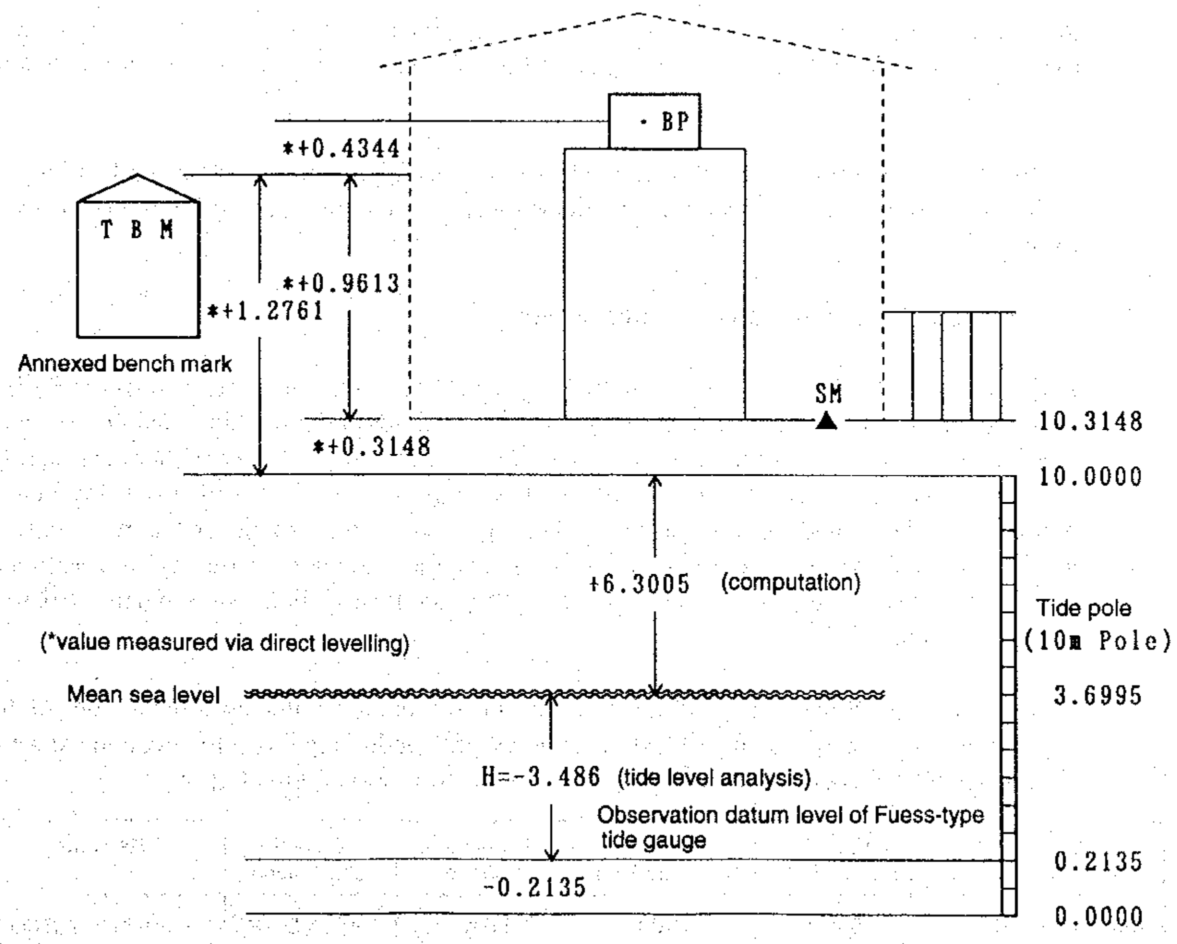
7) Tide level variance between the standard tidal observation station and the auxiliary tide gauge station
   After comparing the tide level variance between the standard tidal observation station installed at the estuary of the River Karnafuli and the auxiliary tide gauge installed in the Bay of Bengal, we observed that the tide level of the standard tidal observation station installed at the river estuary was about 2cm higher. Since this difference is only slight we computed the mean sea level on the basis of data from the standard tidal observation station at the river estuary.

   Although we were not able to identify the effects on tidal levels from marine or climatic phenomena or increased water levels due to rainfall in the vicinity of the tidal observation station during the observation period, we must continue to verify observations over the long term, as well as making corrections and conducting other research.

For more details about the analysis, please see the "Reference Data-1 on determinination of the mean sea level".

Elevation Chart for Standard Tidal observation Station

Name of Location	Altitude difference	Elevation	Remarks
Observation datum level	m	-3.4860	(Elevation difference)
	-3.4860	m	Tide level analysis
MSL (mean sea level)		0.0000	
	+6.3005		Computation
Tide pole (10m pole)		6.3005	
	+0.3148		Direct levelling
SM (standard mark)		6.6153	
	+1.3957		Direct levelling
BP (base point)		8.0110	
SM (standard mark)		6.6153	
	+0.9613		Direct levelling
TBM (tidal station bench mark)		7.5766	



2-3 Levelling

The leveling was conducted in accordance with the guide books under the title of "The Rules of Leveling Work Procedures" and "The Manual for Operations" of the same publication (Geographical Survey Institute, Ministry of Construction). Altitudinal differences between staffs were determined by using a well-adjusted automatic spirit level (WILD NA 3003) and staffs (Bar Code). The process was repeated to ensure the accuracy of alttitudinal differences. The elevation of the datum station of levelling located in Dahka by the mean sea level and elevation was computed by fixing the bench mark annexed to the standard tide observation station after calculating the simultaneous net adjustment for all the bench marks.

1) Direct levelling

Observation by levelling was started from the third year (Phase III), having left the bench marks unutilized for one year after installing them, in view of compaction subsidence of the benchmarks. In order to keep the effects of refraction to the very minimum we made the standard observation distance a maximum of 40 metres. In addition we did not collimate anything beyond 20 cm at the bottom of the staff or beyond 3 metres at the top.

We carried out test adjustment and function checks of the level about once a week, and made the accuracy of observation $\pm 4\text{mm}\sqrt{S}$(S:km) for both closure divergence and loop closure error.

As the datum level that resulted from the levelling, we calculated the mean sea level at the Chittagong tidal observation station, and made this the zero point (datum) for elevation. (See Figure 11)

2) Observation of unreconnoitred levelling routes

In the initial plan we were not going to make observations in this study in part of the Sylhet region in the northeast, because it consisted of vast wetlands and it was not feasible to secure levelling routes there. But fortunately in the middle of Phase III of the study there was hardly any meteorological obstruction and rainfall was slight, thus the work of observing bench marks as a whole made steady progress. Therefore, in a study made of "unreconnoitred levelling routes" in the Sylhet region in the northeast, the water level in the region was found to be extremely low, and we were able to pitch tents to secure levelling routes and thus to carry out observations.

Because we were able to carry out levelling for this region, it became possible to control the surveying accuracy by closing all levelling routes and conducting loop closure, and we were able to secure the accuracy by carrying out closure correction computation.

Level observation	From benchmark no.	To benchmark no.	Observation distance	Date of observation	Remarks
Unreconnoitred route	598	6100	32.5 Km	1994. 1.20~24	1st order level (Sylhet)

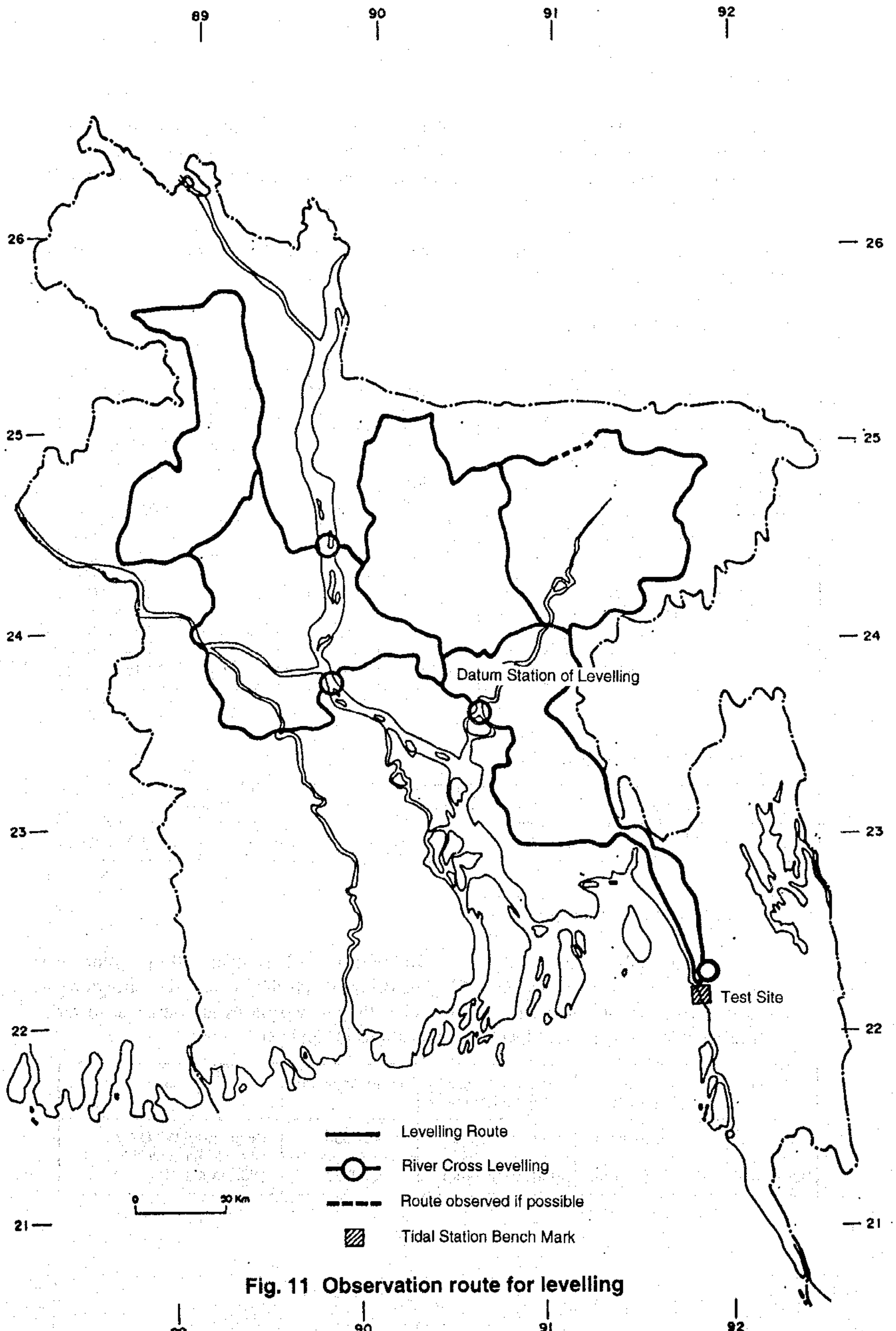


Fig. 11 Observation route for levelling

3) River cross levelling

For river cross bench marks, as far as possible we selected sites where the breadth of the river was narrow and the ground was good. We set the elevation difference between the observation points on either bank to a maximum of 1 metre, and used a line of sight by securing a minimum of 5 metres from the water surface.

River cross levels in seven locations were observed by the tilting screw method, using four levels (WILD N3).

(1) River cross level locations

River cross Level No.	From Bench Mark No.	To Bench Mark No.	Observation Distance	Observation Frequency	Standard Deviation	Remarks
(1)	TBM-1	501	0.8 Km	20 SET	±1.8 mm	CHITTAGONG
(2)	6042-1	6043-1	1.1 Km	30 SET	±1.1 mm	DAUDKANDI
(3)	6185-3	683-3	3.0 Km	100 SET	±1.8 mm	ARICHA
(4)	6144	6145-2	0.4 Km	20 SET	±0.3 mm	TANGAIL
(5)	6145-1	6146-1	1.7 Km	40 SET	±1.2 mm	SIRAJGANJ
(6)	6147-1	645-1	1.8 Km	40 SET	±1.2 mm	SIRAJGANJ
(7)	6028-1	531-1	0.8 Km	20 SET	±1.2 mm	BHAIRAD

(2) Method of river cross levelling

Observation Distance	Measuring Method	Remarks
Up to about 5 km	Tilting screw method (using four levels)	Simultaneous observation (WILD N3)

(3) Accuracy of river cross surveying

During the study period the Karnafuli Bridge that had been damaged by cyclone was restored, and the work on the No. 2 Meghna Bridge that had been under construction was completed. We could now carry out direct levelling in two places and were able to verify the accuracy of river cross levelling of locations (1) and (2).

Sector	River cross relative height (distance)	Direct level relative height (distance)	Discrepancy (m)	Remarks
TBM-501	-2.6096 (0.8 Km)	-2.6041 (45.3 Km)	+0.0055	River cross: WILD N3 Direct levelling: WILD (N3, NA3003)
6042-6043	-0.1020 (1.1 Km)	-0.0960 (2.4 Km)	+0.0060	

In a comparison between the two observations, the limit of loop closure ($\pm 4\text{mm}\sqrt{S}$(S:km)) was fully satisfied, and thus it is conjectured that the observations from river cross levelling achieve the same accuracy as those from direct levelling.

(4) Formula for calculation under the tilting screw method

$$l_0 = l_1 + (l_2 - l_1)\frac{m_0 - m_1}{m_2 - m_1}$$

$$\Delta H = l - l_0$$

where

ΔH : height difference

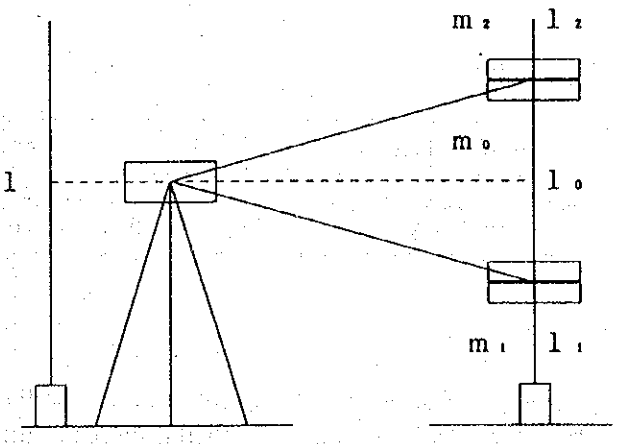
l_1, l_2 : staff scale positioned on bottom/top targets

m_1, m_2 : measurement at bottom/top targets (tilting screw scale)

m_0 : measurement at bubble concurrence (tilting screw scale)

l : backsight staff reading station

l_0 : foresight staff (opposite shore) reading station



Observation was carried out by taking the number of observation days (as calculated by the primary formula) and the number of observation sets as standard.

T = 2•S
n = 4•T

where T: number of observation days, S: observation distance in km, n: number of observation sets

Each set of observations according to the tilting screw method was carried out in two equal halves divided at around 13:00 hours.

4) Reciprocal levelling

For reciprocal bench marks, as far as possible we selected sites where the breadth of the river was narrow and the ground was good. We carried out observations after setting the height difference between the observation points on either bank to a maximum of 1 metre.

Reciprocal levels in seven locations were observed by the reciprocal levelling method, using one level (WILD N3, NA 3000 or NA 3003).

(1) Reciprocal level locations

Reciprocal level no.	From bench mark no.	To bench mark no.	Observation distance	Observation frequency	Deviation	Remarks
(1)	598-4-1	598-4	101 m	4 SET	-0.1 mm	MOHNGANJ NA
(2)	627	FM5126	260 m	4 SET	-1.9 mm	JAMALPUR N3
(3)	6095	597	330 m	4 SET	+0.5 mm	SUNAMGANJ N3
(4)	6088	590	107 m	4 SET	-1.2 mm	SYLHET NA
(5)	586	6085	148 m	4 SET	-1.0 mm	SAIDPUR NA
(6)	552	6050	146 m	4 SET	-0.5 mm	CHANDPUR N3
(7)	6100	601	115 m	4 SET	-0.1 mm	METRAKONA N3

(2) Method of reciprocal levelling

Observation 4 sets

Observation Distance	Measuring Method	Remarks
Up to about 450 m	Reciprocal levelling method (using one level)	5m method (WILD N3, NA 3003)

(3) Accuracy of reciprocal surveying

Since the deviation in average observations for each set of 4 at both observation points (both banks) was extremely small, we judged both the method and the frequency of observation to be appropriate, while the effects of refraction were also largely avoided.

(4) Formula for calculation using the reciprocal levelling method

$$\Delta H = \frac{1}{n} \sum_{1}^{n} a - \frac{1}{n} \sum_{1}^{n} b$$

where

ΔH : difference in relative height
a: backsight staff reading station
b: foresight staff (opposite shore) reading station
n: Numbers of readings

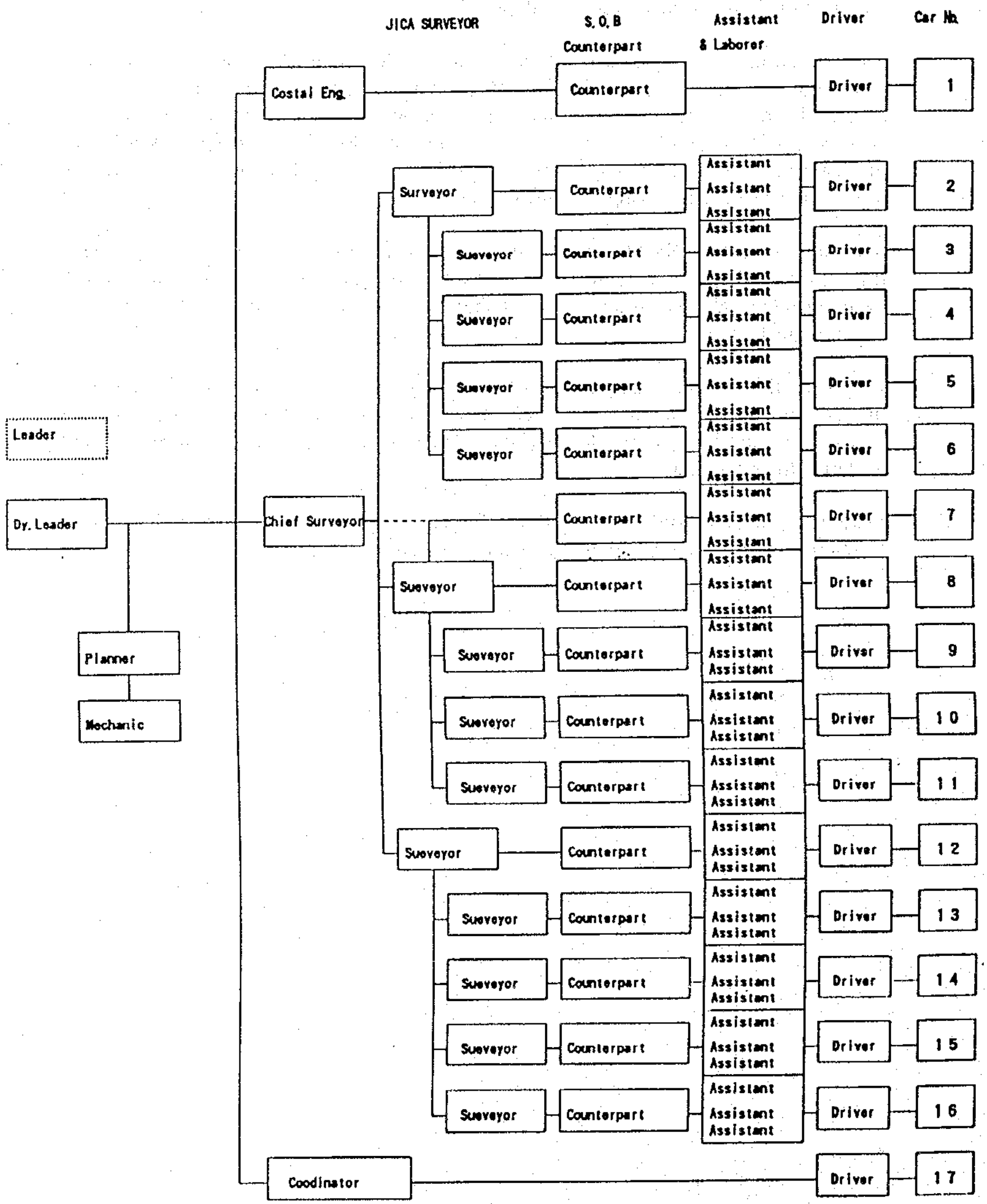
5) Observation Team Formation for levelling

The formation of observation teams in conduction levelling was made by taking into consideration accuracy, safety and technology transfer. In view of the fact that the most advanced and up-to-date surveying machinery was introduced, a Japanese registered surveyor was assigned to each team to ensure accuracy and locally employed assistants (one mechanic and two staff men) were also assigned. Also, locally hired hands were used for traffic control at the surveying site when necessary.

In the third year study eight teams and 14 teams in the fourth year study were formed. Each team was assigned with the SOB counterparts who were engaged in the negotiations with local inhabitants and the arrangement of lodging. A schematic diagram of Duty System is given in Fig.12.

In view of the heavy and bulky surveying instruments used for over-river levelling and the time consumed for preparations, a special team was formed by combining ordinary two teams into one and placed one team on the opposite side of the river so as to enable the levelling work to start at the same time.

In the latter of the fourth year study the SOB counterparts were asked to participate in the levelling work with the intention of technology transfer.



Figuer 12 A schematic diagram of local workforce assignment

2-3-1 Levelling Work Volume and Parameters for Observation

The levelling work volume and parameters for observation were as shown in the following table.

Category	1st Order Levelling
Total distance	2,386 km
Number of bench marks	461
Brake down:	
Standard type	228
Smaller type	233
Vertical datum	1 (Dhaka)
Reference points	2 (A, B)
Supplementary point	1 (tidal observation station)
River cross level sites	7 (9.6 km)
Reciprocal level site	7 (1.2 km)

Category	1st Order Levelling
Level	WILD NA3000, N3
Staff	WILD bar code
Collimating distance	Standard 40m
Reading unit	0.1 mm
Minimum reading	20 cm
Maximum reading	300 cm
Torelance	$\pm 4\text{mm}\sqrt{S}$ (S:distance)
Temperature reading	Units of 1 second
Level calculator	Data Collector
Number of levelling loops	5

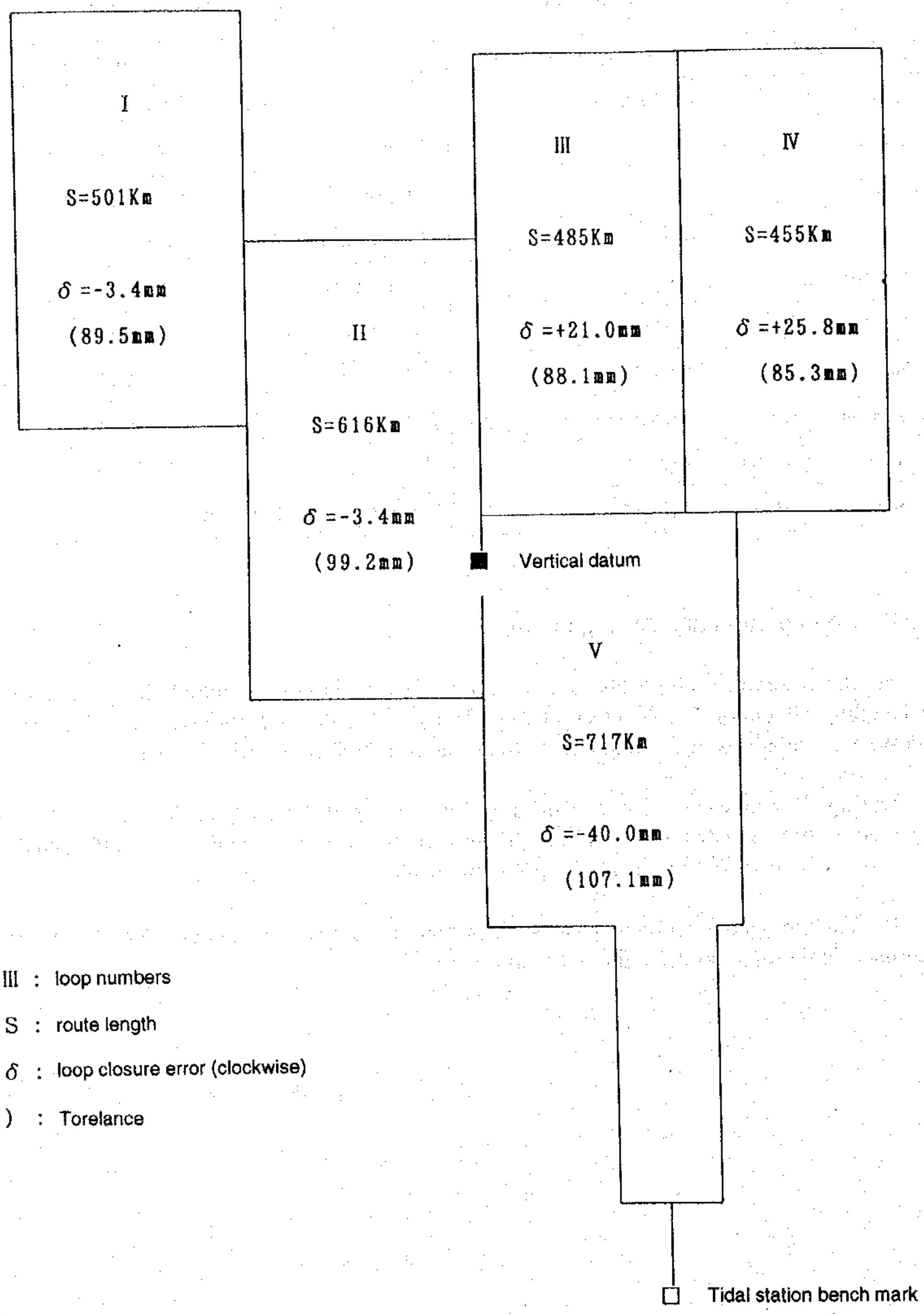
2-3-2 Computation and Arrangement

For the observations for each team, the bar code staff values were automatically read, stored in the levelling calculator (Data Collector LC-12 Tamaya Calculating Systems Co.), and, after various necessary calculations, were output from the printer in the form of print-out slips.

During the work carried out in the field, we arranged the data on the basis of the output observations of height difference and carried out various corrective calculations, confirmed the loop closure error, and verified whether the surveyed results were good or not.

We also arranged other related data such as materials prepared or gathered locally, observation records, calculation records, and point notation.

1) State of loop closure



I II III : loop numbers

S : route length

δ : loop closure error (clockwise)

( ) : Torelance

2-3-3 Network Adjustment

1) To prepare analysis computations and results tables for levelling we used a mainframe computer (NEC ACOS-610) with a levelling network adjustment program.

   (i) We computed the levelling network adjustment by fixing the tidal station bench mark on the basis of the mean sea level (as determined via tide level observation), and thus determined the elevation of the vertical datum.

   Then we fixed the vertical datum (more or less in the centre of the levelling network), and computed the standard deviation in the elevation of the bench marks. (See 4. Bench mark data list)

   (ii) We computed the network adjustment using the observation equation, weighted by the inverse of the distance.

   (iii) We prepared results tables etc. based on the outcome of the adjustment.

A) Observation equation

$$v_{ij} = -x_i + x_j - (h_i - h_j + \Delta h_{ij})$$

where $h_i,\ h_j$: hypothetical elevation of bench marks i and j

$x_i,\ x_j$: correction of hypothetical elevation of bench marks i and j

Δh_{ij} : observed height difference between bench marks i and j

v_{ij} : residual between bench marks i and j

This can be summed up by

$V = AX - L$ Weight P

where the respective matrices and vectors are as follows.

$$\underset{(m,1)}{V} = \begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ v_m \end{bmatrix}, \quad \underset{(m,n)}{A} = \begin{bmatrix} a_{11} & a_{12} \cdots\cdots & a_{1n} \\ a_{21} & a_{22} \cdots\cdots & a_{2n} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} \cdots\cdots & a_{mn} \end{bmatrix}$$

$$\underset{(n,1)}{X} = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{bmatrix}, \quad \underset{(m,1)}{L} = \begin{bmatrix} l_1 \\ l_2 \\ \cdot \\ \cdot \\ l_m \end{bmatrix}, \quad \underset{(m,m)}{P} = \begin{bmatrix} p_1 & & & 0 \\ & p_2 & & \\ & & \ddots & \\ 0 & & & p_m \end{bmatrix}$$

where v_r : v_{ij} for the rth number

l_r : l_{ij} for the rth number ($h_i - h_j + \Delta h_{ij}$)

$$P_{ij} = \frac{1}{S_{ij}}$$ S_{ij} : route length between bench marks i and j

B) Normal equation

$$(A^t P A)\ X = A^t P L$$

$$\therefore\ X = (A^t P A)^{-1} A^t P L$$

C) Results of averaging

(a) Standard deviation in observations per unit weight

$$m_0 = \pm\sqrt{\frac{V^t P V}{(m-n)}}$$

where m : number from the observation equation

n : number of unknown points

(b) Standard deviation in average elevation of unknown points

$$M_1 = M_0\sqrt{Q_{11}},\quad M_2 = M_0\sqrt{Q_{22}},\quad \cdots \quad M_n = M_0\sqrt{Q_{nn}}$$

where

$$\underset{(n,\,n)}{Q} = (A^t P A)^{-1} = \begin{pmatrix} Q_{11} & Q_{12} & \cdots & Q_{1n} \\ Q_{21} & Q_{22} & \cdots & Q_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ Q_{n1} & Q_{n2} & \cdots & Q_{nn} \end{pmatrix}$$

D) Standard deviation in levelling observations

$$m = \pm\sqrt{\frac{1}{4}\cdot\left[\frac{U_i^2}{S_i}\right]\cdot\frac{1}{n}}$$

where m : standard deviation in observations per kilometre (unit: mm)

U_i : discrepancy of double running (unit: mm)

S_i : distance between links (unit: km)

n : number of links

2-3-4 Determination of the Vertical Datum Value

The numerical value of the vertical datum is shown as the height from mean sea level. Mean sea level was an arithmetric means of tide level observations taken over about two years in the tidal observation station built in Chittagong.

Determination of the Vertical Datum Value

1) Mean sea level (from tidal observation data)
   An arithmetric means of observations taken over 22 months from Jan. 1993 to Nov. 1994 was set as the zero point (datum) for elevation.

2) Determining the elevation of the tidal station bench mark (TBM)
   H = 7.5766m (by direct levelling between the standard mark of the tidal observation station and the tidal station bench mark)

3) Determining the elevation of the vertical datum
   We fixed the elevation of the tidal station bench mark (TBM), computed the network adjustment from observed values on all levelling routes, and thus determined the elevation of the vertical datum.
   Next, we fixed the elevation of the vertical datum positioned more or less in the centre of the levelling network, again computed the network adjustment, and computed the standard deviation in elevation at each bench mark.

4) Accuracy of vertical datum elevation
   We carried out surveying with a limit of $\pm 4.0\sqrt{S}$ (S:km) for double-running observation and loop closure.
   The result was that very good closure was achieved in the 5 levelling loops that were formed.

5) Standard deviation throughout the levelling network
   Standard deviation (m) = ± 0.9mm (by levelling network average adjustment)
   Standard deviation per kilometre = ± 0.8mm (double run descrepancy)

6) Elevation of the vertical datum
   H = 6.4292 m (by levelling network adjustment)

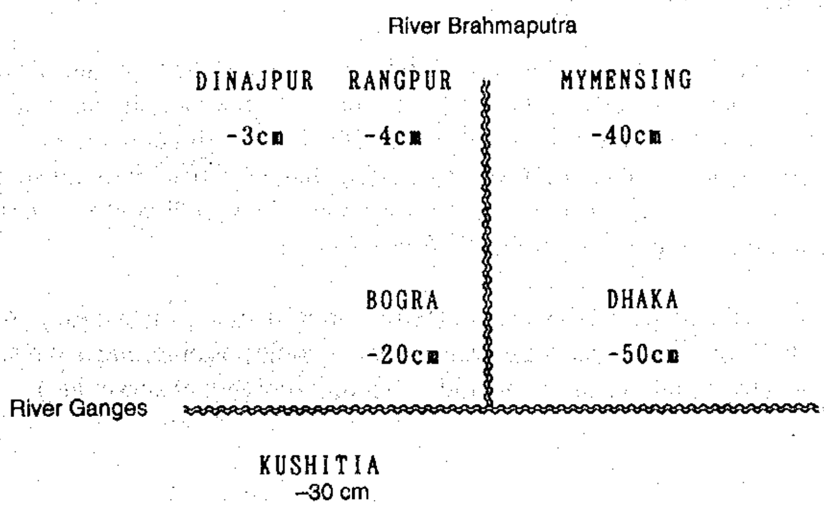
2-3-5 Comparison with Elevation of Existing Bench Marks

From the results of levelling network average computation, we computed the elevation of each bench mark, taking mean sea level as the zero datum. As this levelling network included four existing bench marks, we were thus able to compare elevations.

We also made comparisons with bench marks (FM) installed for flood action plan with aid from Finland.

Bench mark no.	New elevation (m)	Old elevation (m)	Difference in elevation (m)	Location	Installation
540	7.8789	8.3457	-0.4668	DHAKA	SOB
541	6.9231	7.3980	-0.4749	DHAKA	SOI
734	34.5055	34.5329	-0.0274	DINAJPUR	SOI
---	14.7760	15.2220	-0.4460	MYMENSINGH	SOI
GPS 303	8.5344	9.0354	-0.5010	DHAKA	SOB
FM 8032	30.2612	30.3015	-0.0403	RANGPUR	FINN MAP
FM 8025	18.4695	18.6008	-0.1313	BOGURA	FINN MAP
FM8030	15.6500	15.8213	-0.1713	BOGURA	FINN MAP
FM 5102	12.5010	12.7423	-0.2413	TANGAIL	FINN MAP
FM 7914	8.6346	8.8891	-0.2545	GAZIPUR	FINN MAP
FM 8133	7.3865	7.6607	-0.2742	MANIKGANJ	FINN MAP
FM 6412	9.0871	9.4326	-0.3455	FARIDPUR	FINN MAP
FM 8228	13.0527	13.3918	-0.3391	KUSHTIA	FINN MAP

Overall, the elevation levels tended to be lower by 3-50 cm, which is surmised to be attributable to the unclear control point level and the systematic values were indistinguishable.



2-4 Control Point Surveying

Geodesy is one of man's oldest sciences, and is used for calculating the shape and size of the earth. "Geodetic surveying" means measuring the geometric relationship between points on the earth's surface and working out coordinates, while localized measurement of fixed indicators (control points) on the earth's surface is called "control point surveying".

Latitude, longitude, and elevation calculated by geodetic surveying are given to control points on the earth's surface to create a basis upon which the development of national land (e.g. flood control plans, farmland development) can be carried out efficiently and maps can be drawn up.

1) The history of geodetic surveying

Eratosthenes (276-194 BC) was the first to measure the size of the earth using astronomical geodetic methods, based on the hypothesis that the earth was round. With his method, taking the distance between two points north and south on the same meridian, he set the arc length (circumference) as 1 and the angle at the centre of the earth as (υ), and was thus able to calculate the radius of the earth (R) using $1 : 2\pi R = \upsilon° : 360°$.

For (υ), he observed the difference in altitude as heavenly bodies passed over the meridian at both points, while for the arc length (circumference) he estimated the travelling speed of a caravan of camels as 100 stadia per day, and thus calculated the radius of the earth.

Poseidonius (135-50 BC) measured the radius of the earth by using the time it took a boat to travel. And in the 9th century there are records of Arabians who measured arc length with wooden poles.

(1) Triangulation

At the end of the 16th century, the Danish astronomer Tycho Brahe first hit upon the concept of triangulation and used it for surveying between islands. Meanwhile, Jean Picard (1629-82) carried out triangulation from 13 triangles spread over about 120 km to the north of Paris, and made base line measurements using a wooden measuring pole.

(2) Length surveying

In 1936 the State Engineering Research Institute of the USSR produced an electro-optical distance meter using modulated "light". In 1948, the Geodimeter was developed and became widely used for use in surveying. The first Geodimeter was heavy apparatus that could measure distances of up to about 70km, but was difficult to transport. It was highly efficient, with a measuring accuracy of about 1-2 x 10^{-6}, and was used for base line measurement at the Cape Kennedy Space Center.

Thanks to the diffusion of smaller electro-optical distance meters from the second half of the 1960's direct distance measurement has become possible, and as a result triangulation has developed into trilateration and traversing (polygonal surveying).

(3) Surveying by satellite

With the Navy Navigation Satellite System (NNSS) developed in 1959 by the US navy in its Transit Program, the frequency of radio waves transmitted from satellites is used to measure the Doppler effect on earth from the movement of the satellite, and the position of the receiving point on earth is calculated from information on the orbit of the satellite.

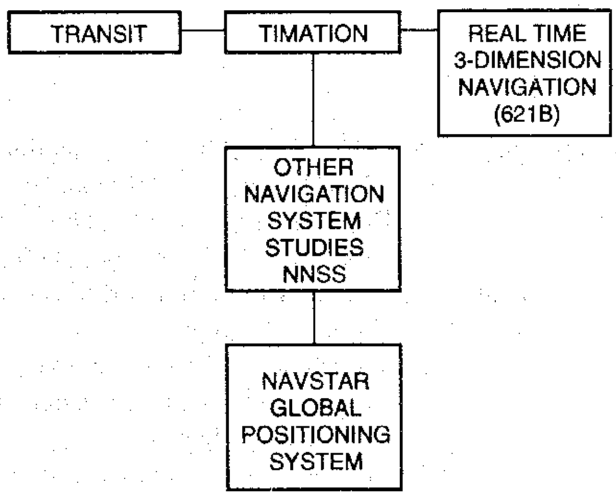
The US navy started operating this system in 1964. In 1967 it became available for civilian use and came to be applied for determining geodetic positions as well.

(4) GPS (Global Positioning System)

This is a navigational system that was developed by the US navy and air force early in 1960, after a plan was launched to develop a system that could give navigational and positional information using signals transmitted from satellites.

In 1973 a GPS:JPO (Joint Program Office) was set up, a concept for a global position measuring system was proposed, and the program was promoted with the aim of full deployment in 1985.

(GPS PROGRAM DEVELOPMENT)



GPS satellites orbit at an altitude of about 20,000 km on six orbits inclined at an angle of approximately 55° to the equator. They have a period of about 11 hours 58 minutes, or exactly 0.5 sidereal days.

The number of satellites operating in the second half of 1993 was 24 for Block I and Block II combined. With this the system is complete, and it is now being used widely for navigation by planes, ships, cars, and so on.

Since this system has good accuracy and can be used for geodetic surveying, it has also come to be applied to general surveying, such as in this study.

2) An outline of GPS

GPS (Global Positioning System) is an all-weather navigational and position measurement system that uses the US Navigation Satellites for Timing and Ranging (NAVSTAR). It receives radio waves, modulated for positioning, that are transmitted by a total of 24 satellites, and enables us to work out the distance between satellites and observation points.

Since the orbital position of each satellite is already known via the World Geodetic System 1984 (WGS 1984), by receiving radio waves from 4 satellites simultaneously we can instantly work out the three-dimensional coordinates and time (UTC) of the observation point (point positioning). Accuracy is about 100 metres and 1μ.

Meanwhile, by observing the carrier phase at several points for 0.5 - 24 hours and combining them for analysis (static relative positioning), we can easily achieve a high relative accuracy (0.01-1ppm), and thus this method can be used for geodetic surveying.

There are other positioning methods such as the kinematic, pseudo kinematic, and rapid static methods that aim for an accuracy of centimetres in a surveying time from several seconds to several minutes, by skilfully processing phase ambiguity (Leick, 1990; Hofmann-Wellenhof et al., 1992).

(1) Composition of GPS

A) The three elements of GPS

- Space segment — NAVSTAR satellites (1-24 satellites)
  - Block I: experimental modules (SVN 1-11)
  - Block II: commercial modules (SVN 13-21)
  - Block IIA: improved modules (SVN 22-40)
- Control segment
  - Master Control Station: orbit calculation, satellite control
    Colorado Springs
  - Monitor Stations: satellite tracking
    Hawaii, Kwajalein, Ascension Island, Diego Garcia, Colorado Springs
  - Ground Antennas: communication with satellites
    Ascension Island, Diego Garcia, Colorado Springs
- User segment — GPS receivers, software

B) Orbit of satellites

A total of 24 satellites have been launched, with 4 satellites on each of six different orbits that diverge from each other by 60° in right ascending node (Ω). The angle of incline of orbits (i) is 55° (63° in the case of Block I).

The semi-major axis of the satellite orbit ellipse (a) is 26,560 km, thus the satellites circle the globe in cycles of exactly 12 hours (0.5 sidereal days). Since one solar day is 23 hours 56 minutes long, the appearance of individual satellites advances by 4 minutes each day.

Orbit eccentricity (e) is 0-0.01, thus almost depicting a circular orbit, and the satellites are so distributed that at least four of them can be seen from anywhere on earth at any one time.

C) Satellite numbers

Satellites are numbered according to two systems, PRN and SVN. PRN (Pseudo Random Noise) is the week number of the P-code that has been allocated to the satellite, while SVN (Space Vehicle Number) is the number of the satellite module itself.

D) Frequency standard

The satellites are fitted with highly accurate atomic clocks representing frequency standards (Cs and Rb clocks).

E) Distribution

Between 1978 and 1985 a total of 11 Block I satellites were launched. Their design life was 4.5 years, but actually some of them have been operating for about ten years. Distribution of Block II commercial modules started in 1989 and that of Block IIA improved modules in 1990. Their design life is 7.5 years. By December 8th, 1993 a total of 24 satellites from Block I, II, and IIA had been made available for use, and the USA declared a state of Initial Operational Capability (IOC). When 24 satellites become available in Block II alone, a state of Full Operational Capability (FOC) will be declared.

As of May 1994, one Block I satellite (PRN12/SVN10) and 24 Block II/IIA satellites were in use. The following table (GPS B2 file) is a list of Block II/IIA satellites as issued by the US Naval Observatory (USNO). This list is available on "Internet".

List for block II/IIA satellites

Order of launch	PRN	SVN (GPS)	Launch date	Frequency index	Orbit
II-1	14	14	14 FEB 89	Cs	E1
II-2	02	13	10 JUN 89	Cs	B3
II-3	16	16	18 AUG 89	Cs	E3
II-4	19	19	21 OCT 89	Cs	A4
II-5	17	17	11 DEC 89	Cs	D3
II-6	18	18	24 JAN 90	Cs	F3
II-7	20	20	26 MAR 90	Cs	B2
II-8	21	21	02 AUG 90	Cs	E2
II-9	15	15	01 OCT 90	Cs	D2
II A-10	23	23	26 NOV 90	Cs	E4
II A-11	24	24	04 JUL 91	Rb	D1
II A-12	25	25	23 FEB 92	Cs	A2
II A-13	28	28	10 APR 92	Cs	C2
II A-14	26	26	07 JUL 92	Cs	F2
II A-15	27	27	09 SEP 92	Cs	A3
II A-16	01	32	22 NOV 92	Cs	F1
II A-17	29	29	18 DEC 92	Cs	F4
II A-18	22	22	03 FEB 93	Cs	B1
II A-19	31	31	30 MAR 93	Cs	C3
II A-20	07	37	13 MAY 93	Cs	C4
II A-21	09	39	26 JUN 93	Cs	A1
II A-22	05	35	30 AUG 93	Cs	B4
II A-23	04	34	26 OCT 93	Cs	D4
II A-24	06	36	10 MAR 94	Rb	C1

F) Positioning signal

The basic frequency f_0 (=10.23MHz*) of the atomic clock fitted in the satellites is magnified by 154 and 120 times, carrier wave frequencies of L1 and L2 are created, and phase shift keying modulation is received in two types of PRN code transmitted from the satellites.

On the receiver side, the same PRN code is reproduced and the time axis of the reproduced PRN code is shifted until the maximum correlation with the received PRN signal has been obtained. Multiplying the amount of shift by the speed of light gives the distance between the satellite and the receiver. Since this distance includes clock errors on either side, it is called a pseudo-range. PRN codes include the generally available Coarse/Acquisition (C/A) code and the Protected (P) code for military use. The codes are in a pseudo-random series made up of 0's and 1's. The repeat frequency of C/A codes is 1ms while that of P codes is 37 weeks. P codes are assigned to each satellite for one week at a time. When the USA switches GPS to Anti-Spoofing (A-S) mode, P codes are encoded and become Y codes. Since the IOC declaration, Block II/IIA satellites have basically been operating on A-S modes.

The pre-modulation carrier wave (L_i) expressed in terms of a_i (amplitude) and f_i (frequency) is $L_i(t)=a_i \cos(f_i\, t)$.

The signal transmitted from GPS satellites is modulated in such a way that

$$L_1(t)=a_1\, P(t)D(t)\cos(f_1 t)+a_1\, C/A(t)D(t)\sin(f_1\, t)$$
$$L_2(t)=a_2\, P(t)D(t)\cos(f_2\, t)$$

where P(t): P code, C(t): C/A code, and D(t): navigation message (Spilker, 1980).

The following table shows the frequency and wavelength of signals emitted from GPS.

Carrier wave		C/A code	P code
L1	1575.42MHz (=19.0cm)	1.023MHz (=293m)	10.23MHz (=29.3m)
L2	1227.60MHz (=24.4cm)		10.23MHz (=29.3m)

In parentheses: wavelengths

In static relative positioning the distance from the satellite is measured using the phase of the carrier wave itself rather than the PRN code. The carrier wave can be obtained by decoding the PRN code. And even if the code pattern is unknown the phase can be reproduced using techniques such as square-law detection or cross-correlation.

* The actual frequency has been set 0.00455Hz lower than this, in order to make a relativistic correction due to the difference in gravity between the satellite altitude and the earth's surface.

G) Navigation messages

The orbit of satellites and the behaviour of the satellite clocks are mainly calculated by the Master Control Station on the basis of satellite tracking data from monitor stations. This information is uploaded to the satellites via ground antennae and is then transmitted by the satellites to users in the form of navigation messages. The orbit of the satellites is given in the form of Kepler orbit elements supplemented by a quantity based on time, and on the basis of this the position of a satellite at any given time can be calculated by the WGS-84 system. These orbit elements are called broadcast ephemeris.

H) SA

For military reasons, the USA controls the frequency of satellite clocks, and in addition to manipulator degrade in broadcast ephemeris this leads to a diminished accuracy of independent positioning. We call this Selective Availability (SA). It has hardly any effect on relative positioning.

3) A mathematical model for GPS

To analyze the GPS observational data, we need a mathematical model to make the link between the amount of observable data and the unknown parameters that we wish to discover. Here we shall introduce a mathematical model that gives the satellite position and coordinates of observation points (known points) that form a basis, and estimates the coordinates of observation points other than known points (unknown points).

(i) GPS observation

We install a number of GPS receivers at each observation point and record the carrier beat phase from each satellite simultaneously for every sampling interval determined in advance. Each observation time is called an "epoch". Although the observation period differs according to the required accuracy and the distance between observation points, normally it can be anything between 30 minutes and 24 hours. A series of observations is called a "session".

(ii) Carrier beat phase

The carrier beat phase (or one-way phase) from a satellite (j) that is received by a receiver (i) at a certain time (t) can be represented in the model as

$$\Phi^{j}_{i}(t) = \rho^{j}_{i}(t)/\lambda + N^{j}_{i} + f\delta^{j}(t) - f\delta_{i}(t) - \Delta ion/\lambda + \Delta trop/\lambda$$

The unit is frequency (cycles).

$\rho^{j}_{i}(t)$: the distance that the carrier wave received by receiver (i) at time (t) has travelled from satellite (j) (propagation distance)

λ : length of carrier wave

N^{i}_{j} : phase ambiguity or integer bias

f : frequency of carrier wave

$\delta^{j}(t)$: clock error of satellite (j)

$\delta_{i}(t)$: clock error of receiver (i)

Δion : ionospheric delay

$\Delta trop$: tropospheric delay

The normal practice is to indicate satellites (in space) with superscript figures and receivers (on earth) with subscripts. If there is no cycle slip, the phase ambiguity of a series of carrier beat phases for a specific satellite and receiver in one session is fixed.

When selecting the carrier beat phase for the observed quantity, the clock errors of the satellite and the receiver are estimated for each epoch and the tropospheric delay is given in a model, a suitable angle of elevation dependence is hypothesized and the amount of vertical delay is estimated at appropriate time intervals for each point.

Ionospheric delay is removed through linear combination of dual frequency data (to be discussed later). The parameters connected with the positions of satellites and observation points are included in the propagation distance $\rho^{j}_{i}(t)$.

(iii) Propagation distance

When the satellite position (or to be more exact the phase centre of the transmission antenna) at time (t) in an inertial system (J2000 system) is given as $x^{j}(t)$ and the observation point position (phase centre of the receiving antenna) as $x_{i}(t)$, the distance of propagation $\rho^{j}_{i}(t)$ from satellite (j) of the carrier wave received by receiver (i) at time (t) is obtained by solving the following light-time equation through iteration.

$$\rho^{j}_{i}(t) = |X^{j}(t - \rho^{j}_{i}(t)/c) - X_{i}(t)|$$

When the position of the satellite and receiver are given in terms of a terrestrial system (e.g. WGS-84 system) the special relativistic effect (Sagnac effect) arising from the rotational coordinate series is corrected. In any case, coordinates occasionally have to be switched between inertial and terrestrial systems. The earth's rotational parameters such as polar motion, precession, and nutation are included in this.

The partial differential coefficient relating to the observation point coordinates for the propagation distance is needed whenever setting up the observation equation. If we assume in rectangular coordinates $x^j(t-\rho/c)=(x^j, y^j, z^j)$, and $x_i(t)=(x_i, y_i, z_i)$, then

$$\rho^j{}_i(t)=\{(x^j-x_i)^2+(y^j-y_i)^2+(z^j-z_i)^2\}^{1/2}$$

Thus, if we ignore infinitesimal terms, we obtain

$$\partial\rho^j{}_i/\partial x_i=-(x^j-x_i)/\rho^j{}_i,\quad \partial\rho^j{}_i/\partial y_i=-(y^j-y_i)/\rho^j{}_i,$$

$$\partial\rho^j{}_i/\partial z_i=-(z^j-z_i)/\rho^j{}_i$$

In other words, the partial differential for the constituents of observation point coordinates of ρ is the same as that of the constituents of unit vectors from the satellite position at time $(t-\rho/c)$ that move towards the observation point position at time (t).

(iv) Phase difference observed quantity

By taking the difference in the carrier beat phase obtained from different receivers and satellites we can simplify models in which common errors cancel each other out. These phase differences are sometimes regarded as the new observed quantity.

A) Single difference

Here we consider the carrier beat phases from a satellite (j) received by two receivers (A) and (B) at time (t). When the difference between the two is subtracted we are left with the "single difference", shown as $\Phi^j{}_{AB}(t)$. Clock error shared by satellites can be cancelled out in this way.

$$\Phi^j{}_{AB}(t) \equiv \Phi^j{}_B(t)-\Phi^j{}_A(t)$$

$$= \{\rho^j{}_B(t)-\rho^j{}_A(t)\}/\lambda+N^j{}_{AB}-f\delta_{AB}(t)$$

where $N^j{}_{AB} \equiv N^j{}_B-N^j{}_A$, $\delta_{AB}(t) \equiv \delta_B(t)-\delta_A(t)$

B) Double difference

Here we shall consider the carrier beat phases from satellites (j) and (k) received by receivers (A) and (B) at time (t). Single differences can be defined for each of the two satellites. The product after subtracting the difference between these single differences is called "double difference", and is expressed as $\Phi^{jk}{}_{AB}(t)$. Receiver clock errors can also be cancelled out against each other in this way. In normal analysis, the use of double difference is preferred.

$$\Phi^{jk}{}_{AB}(t) \equiv \Phi^k{}_{AB}(t)-\Phi^j{}_{AB}(t)$$

$$= \{\rho^k{}_B(t)-\rho^j{}_B(t)-\rho^k{}_A(t)+\rho^j{}_A(t)\}/\lambda+N^{jk}{}_{AB}$$

where $N^{jk}{}_{AB} \equiv N^k{}_B-N^j{}_B-N^k{}_A+N^j{}_A$

C) Triple difference

Receivers (A) and (B) receive signals from satellites (j) and (k) at times (t_1) and (t_2). The double differences for times (t_1) and (t_2) can be defined. The product after subtracting the difference between these double phase differences is called "triple difference", and is expressed as $\Phi^{jk}{}_{AB}(t_1; t_2)$. Phase ambiguity can be cancelled out in this way.

$$
\begin{aligned}
\Phi^{jk}{}_{AB}(t_1;t_2) &\equiv \Phi^{jk}{}_{AB}(t_2)-\Phi^{jk}{}_{AB}(t_1)\\
&= [\{\rho^{k}{}_{B}(t_2)-\rho^{j}{}_{B}(t_2)-\rho^{k}{}_{A}(t_2)+\rho^{j}{}_{A}(t_2)\}\\
&\quad -\{\rho^{k}{}_{B}(t_1)-\rho^{j}{}_{B}(t_1)-\rho^{k}{}_{A}(t_1)+\rho^{j}{}_{A}(t_1)\}]/\lambda
\end{aligned}
$$

(v) Correlation between observed quantities

Although carrier beat phases obtained from the same session can involve some kind of correlation in physical terms, normally calculations are made on the assumption that there is no correlation. Even when the carrier beat phase is independent, a mathematical correlation arises between the phase differences within the same epoch that has been made by combining them. When seeking the weight of the observed quantity we need to consider the variance-covariance of the observed quantity.

(vi) Observation equation for static relative positioning

Let us now show an observation equation for cases in which the coordinates of unknown points are sought from single cycle observation data via double difference. We shall ignore ionospheric and tropospheric delay. We have obtained data across (n_k) epochs from (n_j) satellites at (n_i) observation points. For simplicity we shall assume that there are no lost observations. From these observations we determine the observed value of $\Phi^{j}{}_{i}(k)$ for observation point (i) = 1, 2, ... n_i, satellite (j) = 1, 2, ... n_j, and epoch (k) = 1, 2, ... n_k. In order to calculate the unknown point coordinates by the least square method, we need observed quantity vectors, the observed quantity variance-covariance matrix, approximate values of unknown parameters, approximate values of observed quantities via the mathematical model, and the planned matrix.

A) Observed quantity vectors

Taking satellite (1) and observation point (1) as standard, the linearly independent double difference can be calculated from the observed values of carrier beat phase, thus:

```
do k=1,nk
  do j=2,nj
    do i=2,ni
        Φ1,j1,i(k)=Φj1(k)-Φ11(k)-Φji(k)+Φ1i(k)
    enddo
  enddo
enddo
```

The total number is $(n_i-1)(n_j-1)n_k$. We can form an observed quantity vector by aligning these in sequence vertically, thus:

$$
\begin{aligned}
L_b = [\,&\Phi^{12}{}_{12}(1), \Phi^{12}{}_{13}(1), \ldots, \Phi^{12}{}_{1,ni}(1), \Phi^{13}{}_{12}(1), \ldots, \Phi^{13}{}_{1,ni}(1), \ldots \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad , \Phi^{1,nj}{}_{1,ni}(1), \\
&\Phi^{12}{}_{12}(2), \Phi^{12}{}_{13}(2), \ldots, \Phi^{12}{}_{1,ni}(2), \Phi^{13}{}_{12}(2), \ldots, \Phi^{13}{}_{1,ni}(2), \ldots \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad , \Phi^{1,nj}{}_{1,ni}(2), \\
&\ldots \\
&\Phi^{12}{}_{12}(n_k), \Phi^{12}{}_{13}(n_k), \ldots \qquad\qquad\qquad\qquad\qquad\qquad , \Phi^{1,nj}{}_{1,ni}(n_k)\,]^T
\end{aligned}
$$

(T) is a symbol indicating transposition of rows. Since in actual observations there are some lost observations and switches between satellites, for each epoch we carefully select linearly independent double differences. For algorithms in this respect refer to Remondi (1984).

B) Variance-covariance of observed quantity

If we take the observation noise in the carrier beat phase as (σ), the correlation between the double differences $\Phi^{1,j1}{}_{1,i1}(k)$ and $\Phi^{1,j2}{}_{1,i2}(k)$ for a given epoch (K) will be

$4\sigma^2$ when $i1=i2$ and $j1=j2$,
$2\sigma^2$ when $i1=i2$ and $j1\neq j2$,
$2\sigma^2$ when $i1\neq i2$ and $j1=j2$, and
σ^2 when $i1\neq i2$ and $j1\neq j2$.

The variance-covariance matrix (C(k)) can be produced by vertically and horizontally aligning this correlation in order so that it corresponds to the observed quantity vector. Thus:

$$
C(k) = \sigma^2
\begin{array}{|llll|l}
422\ldots2 & 211\ldots1 & 211\ldots1\ldots & 211\ldots1 & \leftarrow \Phi^{12}{}_{12} \\
242\ldots2 & 121\ldots1 & 121\ldots1\ldots & 121\ldots1 & \leftarrow \Phi^{12}{}_{13} \\
\ldots & & & & \ldots \\
222\ldots4 & 111\ldots2 & 111\ldots2\ldots & 111\ldots2 & \leftarrow \Phi^{12}{}_{1,ni} \\
211\ldots1 & 422\ldots2 & 211\ldots1\ldots & 211\ldots1 & \leftarrow \Phi^{13}{}_{12} \\
121\ldots1 & 242\ldots2 & 121\ldots1\ldots & 121\ldots1 & \leftarrow \Phi^{13}{}_{13} \\
\ldots & & & & \ldots \\
111\ldots2 & 222\ldots4 & 111\ldots2\ldots & 111\ldots2 & \leftarrow \Phi^{13}{}_{1,ni} \\
\ldots & & & & \ldots \\
211\ldots1 & 211\ldots1 & 211\ldots1\ldots & 422\ldots2 & \leftarrow \Phi^{1,nj}{}_{12} \\
121\ldots1 & 121\ldots1 & 121\ldots1\ldots & 242\ldots2 & \leftarrow \Phi^{1,nj}{}_{13} \\
\ldots & & & & \ldots \\
111\ldots2 & 111\ldots2 & 111\ldots2\ldots & 222\ldots4 & \leftarrow \Phi^{1,nj}{}_{1,ni} \\
\multicolumn{1}{l}{\uparrow} & \uparrow & \uparrow & \multicolumn{1}{l}{\uparrow} & \\
\multicolumn{1}{l}{\Phi^{12}{}_{12}} & \Phi^{13}{}_{12} & \Phi^{14}{}_{12}\;\ldots & \multicolumn{1}{l}{\Phi^{1,nj}{}_{12}} &
\end{array}
$$

The size of (C(k)) is the square matrix of $(n_i-1)(n_j-1)$.

Since the correlation between double differences is "0" if the epochs are different, the variance-covariance in relation to the total observed quantity will be:

$$\Sigma_L = \begin{vmatrix} C(1) & & & & 0 \\ & C(2) & & & \\ & & C(3) & & \\ & & & \cdots & \\ 0 & & & & C(k) \end{vmatrix}$$

C) Unknown parameters

Since we are dealing here with relative positioning, we fix the coordinates of observation point (1) to work out the coordinates of observation points (2, 3, ..., n_i). Assuming that we know the respective approximate coordinates, we estimate corrections to the approximate values as parameters. We show these, for example, in a three-dimensional orthogonal coordinate series ($X_2=(x_2,y_2,z_2)$). The number of unknown coordinates is ($3x\,(n_i-1)$) in total. Since the double difference includes an integer bias, this is also worked out as an unknown number. Since no approximate value can be found for the integer bias this is given as "0". The number of the integer bias is the same as that of the linearly independent double difference in the first epoch, and is thus $(n_i-1)\,(n_j-1)$.

To summarize the above, the unknown parameter vector is:

$$X = [x_2, y_2, z_2, x_3, y_3, z_3, \ldots, x_{ni}, y_{ni}, z_{ni}, N^{12}{}_{12}, N^{12}{}_{13}, \ldots, N^{1,nj}{}_{1,ni}]^T$$

D) Approximate values of observed quantities via the mathematical model

The mathematical model F(X) for the double difference involves aligning the following equation vertically so that it corresponds to the observed quantity vector:

$$\Phi^{i,j}{}_{1,i}(t) = \{\rho^j{}_i(k) - \rho^i{}_i(k) - \rho^j{}_1(k) + \rho^i{}_1(k)\}/\lambda + N^{ij}{}_{1i}$$

The satellite position is already known via broadcast ephemeris or precise ephemeris. When the coordinates of the observation points and the approximate value of the integer bias are introduced, $F(X_0)$ can be calculated. ($L=L_b-F(X_0)$) produces values for (0-C).

E) Planned matrix

Since the partial differentiation of propagation distance for the unknown point coordinates was

$$\partial\rho^j{}_i/\partial x_i = -(x^j-x_i)/\rho^j{}_i, \quad \partial\rho^j{}_i/\partial y_i = -(y^j-y_i)/\rho^j{}_i$$

$$\partial\rho^j{}_i/\partial z_i = -(z^j-z_i)/\rho^j{}_i$$

the partial differentiation for the unknown coordinates of the double difference will be:

$$\partial \Phi^{1,j}{}_{1,i}(k)/\partial x_i = [-(x^j - x_i)/\rho^j{}_i + (x^1 - x_i)/\rho^j{}_i]/\lambda$$

$$\partial \Phi^{1,j}{}_{1,i}(k)/\partial y_i = [-(y^j - y_i)/\rho^j{}_i + (y^1 - y_i)/\rho^j{}_i]/\lambda$$

$$\partial \Phi^{1,j}{}_{1,i}(k)/\partial z_i = [-(z^j - z_i)/\rho^j{}_i + (z^1 - z_i)/\rho^j{}_i]/\lambda$$

In addition, the partial differentiation for the integer bias will be:

$$\partial \Phi^{1,j}{}_{1,i}(k)/\partial N^{1,j'}{}_{1,i'} = \begin{cases} 1 \ldots\ldots i = i' \text{ and } j = j' \\ 0 \ldots\ldots i \neq i' \text{ or } \ j \neq j' \end{cases}$$

If we align the above partial differential coefficients in order as follows and substitute the approximate value (X_0) of the parameter we can work out the value of A.

$$A = (\partial F/\partial X)_{X_0}$$

$$= \begin{vmatrix} \partial \Phi^{12}{}_{12}(1)/\partial x_2 & \partial \Phi^{12}{}_{12}(1)/\partial y_2 & \partial \Phi^{12}{}_{12}(1)/\partial z_2 & \ldots\ 1000\ldots0 \\ \partial \Phi^{12}{}_{13}(1)/\partial x_2 & \partial \Phi^{12}{}_{13}(1)/\partial y_2 & \partial \Phi^{12}{}_{13}(1)/\partial z_2 & \ldots\ 0100\ldots0 \\ \ldots & & & \\ \partial \Phi^{1,nj}{}_{1,ni}(1)/\partial x_2 & \partial \Phi^{1,nj}{}_{1,ni}(1)/\partial y_2 & \partial \Phi^{1,nj}{}_{1,ni}(1)/\partial z_2 & \ldots\ 1000\ldots1 \\ & & & \\ \partial \Phi^{12}{}_{12}(2)/\partial x_2 & \partial \Phi^{12}{}_{12}(2)/\partial y_2 & \partial \Phi^{12}{}_{12}(2)/\partial z_2 & \ldots\ 1000\ldots0 \\ \ldots & & & \\ \partial \Phi^{1,nj}{}_{1,ni}(n_k)/\partial x_2 & \partial \Phi^{1,nj}{}_{1,ni}(n_k)/\partial y_2 & \partial \Phi^{1nj}{}_{1ni}(n_k)/\partial z_2 & \ldots\ 1000\ldots1 \end{vmatrix}$$

In this way the various terms for the observation equation are deduced. If then the observation equation (V=AX-L) is solved by a suitable method we can estimate the unknown parameters. When we include those that are not linearly independent in the observed quantity, ranking defects arise in matrix A and the solution becomes indeterminate.

(vii) Linear combination of dual frequency data

In order to correct ionospheric delay, we may choose the observed quantity of linear combinations of carrier beat phase (Φ_1), (Φ_2) in bands (L1) and (L2). If we express (Φ_1), (Φ_2) as frequency (cycles) and set (n_1), (n_2) at any real number, we can express the linear combination (Φ) (cycles) of dual frequency data in the following terms:

$$\begin{aligned} \Phi &\equiv n_1\Phi_1 + n_2\Phi_2 \\ &= n_1 f_1 t + n_2 f_2 t \\ &= (n_1 f_1 + n_2 f_2)t \end{aligned}$$

Therefore, we can define the frequency (f) and wave length (λ) of the linearly combined phase with:

$$\begin{aligned} f &= n_1 f_1 + n_2 f_2, \\ \lambda &= c/f \end{aligned}$$

Commonly used linear combinations are shown in the following table.

Symbol	Name of observed quantity	n1	n2	Wave length (cm)
L1	L1 carrier wave	1	0	19.0
L2	L2 carrier wave	0	1	24.4
L1+L2	Narrow-lane	1	1	10.7
L1-L2(L5)	Wide-lane	1	-1	86.2
LC(L3)	Ionospheric free	1	$-f_2/f_1$	48.4*
LG(L4)	Ionospheric delay	1	$-f_1/f_2$	∞

* As defined by Hofmann-Wellenhof et al. (1992).

According to the relevant literature, (LC) is defined by multiplying this (n1), (n2) by $(f_1^2/(f_1^2-f_2^2))$ (King et al., 1985). In this case the wave length would be 19.0cm, the same as (L1).

Since phase delay due to the ionosphere is in inverse proportion to frequency, (LC) is an observed quantity that is not susceptible to ionospheric effects. Therefore we use (LC) to analyze long base lines of around 10km or more. When using (LC) for narrow- and wide-lanes, we use the phase ambiguity inherent in (L1) and (L2) for resolution.

The carrier beat phase of (L1) and (L2) may be shown as the dimensions (m) of the length, and the ionospheric free observed quantity (LC)m may be defined as:

$$LC = f_1^2/(f_1^2-f_2^2)L_1 - f_2^2/(f_1^2-f_2^2)L_2$$

(Beutler et al., 1989). In this case the wave length is not defined.

(viii) Resolution of phase ambiguity

Using the fact that the phase ambiguity of double difference of (L1) and (L2) is an integer, the phase ambiguity estimated as a real numerical value is sometimes rounded up into an integer and the other parameters re-estimated. This is known as ambiguity resolution. The accuracy of coordinate estimation may be increased in this way. When using (LC) with dual frequency, the phase ambiguity of (LC) itself is not an integer, but the phase ambiguity of (L1) and (L2) may be resolved by using wide- and narrow-lanes.

(ix) Physical model

To analyze data, a concrete physical model for the earth and satellites is needed in addition to the structure of a mathematical model. The International Earth Rotation Service (IERS) has compiled physical models necessary for data analysis by space geodesy technology (e.g. crustal movement, earth gravity field, earth tide, ocean-tide loading displacement, tropospheric delay models, and solar radiation pressure models) in the form of IERS standards (McCarthy, 1992).

(x) Transformation between terrestrial and inertial systems

To transform between a conventional inertial system (X_{CIS}; e.g. the J2000 system) and a conventional terrestrial system (X_{CTS}; e.g. the WGS-84 system):

X_{CTS} = R (polar motion) R (earth's rotation) R (precession) R (nutation) X_{CIS}.

Here, R is a rotational matrix taken as an argument for the earth's rotational parameters as named inside the parentheses, and is a function of time. For specific formulae see for example Hofmann-Wellenhof et al. (1992).

The above details are quoted from "Geodetic Formulae" issued by the Geodetic Society of Japan.

2-4-1 Control Point Surveying Using GPS

Control point surveying was carried out in accordance with the "Work Specification for Primary Control Point Surveying in Precision Geodetic Networks using GPS" and the "Guideline of Data Recording" of the same (Geographical Survey Institute, Ministry of Construction), as well as "Overseas Surveying Specifications" (for base maps).

1) Reconnaissance of control points

(i) Reconnaissance area

The area is located in a range from 22°15' to 26°40' north latitude and from 88°03' to 92°40' east longitude, and has a land area of 95,000 square kilometres (about seven-tenths of the total land area of Bangladesh = 144,000 square kilometres).

(ii) Number of points explored and reconnoitred

Existing points	24
New control points:	115 (26 type A, 89 type B)
Geodetic datum:	1
(Total)	140 points

(iii) Team composition

1 or 2 Japanese surveyars, 1 or 2 SOB counterparts

(iv) Reconnaissance method

As we were not permitted to use radios, the greatest priority was placed on the safety of the reconnaissance work, while the control point team and the bench mark team worked together in view of the efficiency of the work.

We used handy GPS and maps (1:250,000, 1:50,000) in order to ensure an efficient access to the places scheduled for the installation of control points.

For reconnaissance, we

- set the intervals between control points at around 30km;
- avoided natural objects and vegetation that could obstruct radio waves;
- chose places that were good for use and security (public land); and
- chose places in which an angle of elevation of at least 15° for visibility from horizon could be secured.

At the installation points we decided on sites for driving wooden posts. In our "Description of control points locationing" we entered information that would be necessary for the future continuation of the work.

2) Observation of control point surveying

The "Control point monumentation" in Phase II was completed successfully. Observations by GPS were made for 60 points in the western district.

In Phase III, we made observations at 81 points in the eastern district, totalling 141 points in all. These include one point at which observations were made to determine the geographical coordinate of the standard tidal observation station.

The majority of the land consists of a delta zone formed from the confluence of vast rivers, and the flat topography is dotted with large trees. Since these would obstruct GPS signal reception, for our surveying we used specially-ordered 10m and 15m GPS antenna poles. Thanks to these, GPS antennae could be installed accurately in positions beyond the height of trees, reception was unobstructed and so we were able to collect good data.

The reproducibility of the data was very good and a high degree of accuracy was maintained. These data were recorded on floppy disk and brought back to Japan for analysis/computation. (See Figure 14)

Meanwhile, in order to assess geoidal undulation in the geodetic network, we carried out elevation linking survey in consideration of the network as a whole. Elevation linking survey was carried out via direct levelling (3rd order levels) in a total of 51 control points (4 in Phase II, 23 in Phase III, and 24 in Phase IV).

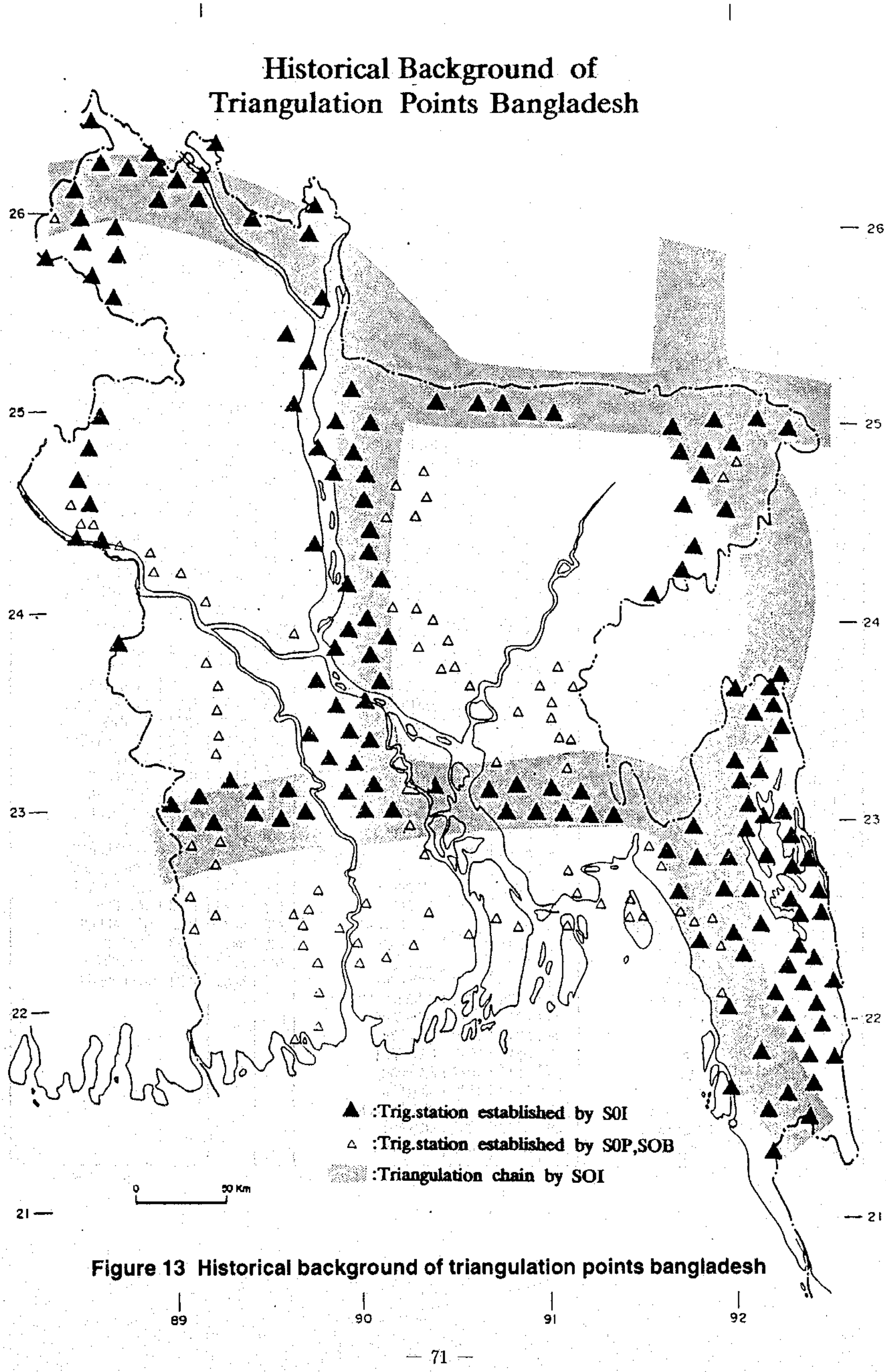


Figure 13 Historical background of triangulation points bangladesh

Table 1 Symmetry table of ID. No. Name

Symmetry table of ID. No. NAME

No	ID	No	NAME	No	ID	No	NAME	No	ID	No	NAME
1	EPF8	303	GULSHAN	51	NPL8	3469	MUNSHIGANJ	101	NP36	1477	JAGAIBER
2	EP10	2200	MAUCHAK	52	EPL9	3479	DAUDKANDI	102	NP20	1126	RANSHIA
3	EPL4	3398	SAVAR	53	NPI5	242	N. KASHADAHA	103	NP22	1180	NILPHAMARI
4	NPL6	3442	PALAS	54	NPH1	2019	BANI	104	NP37	1495	CHANDRAPUR
5	NPH6	2110	MIRZAPUR	55	NPK8	2903	SONPACHA	105	NP04	288	GOIBARI
6	EPL1	3327	GAZIKHALI	56	NPL2	3344	BRAHMANGAON	106	NP40	1558	MAHENDRANAGAR
7	NPF1	131	GOFFARGAON	57	NPL3	3354	BHAGYAKUL	107	NP43	1649	KURIGRAM
8	NPI1	2227	ATHARABARI	58	NPF4	199	BHATURIA	108	NP46	1685	MADARGANJ
9	NPI4	2291	KISHORGANJ	59	NPG4	2876	BABUPUR	109	NP06	350	KASHDAH
10	NPI5	2300	KATIADI	60	NPK0	2615	SABDARPUR	110	NP44	1658	NARSINGBANJ
11	EPF2	181	KALI KACHCHA	61	NPK1	2624	JIBANNAGAR	111	NP45	1668	KANCHIPARA
12	NPH0	3533	NABINAGAR	62	EPK2	2661	PIPRAGACHI	112	NP47	1695	RAHUMARI
13	NPH9	2181	SRIPUR	63	EPK3	2694	NALDANGA	113	NPG7	1713	DIGRIRCHAR
14	NPI3	2281	TARAIL	64	NPK7	2867	BHABANIPUR	114	NPG8	1722	BAKSHIGONJ
15	NPI6	2308	BAJITPUR	65	NPK4	2759	MAGURA	115	NPH2	2037	HALUAGHAT
16	NPI7	2317	BELABA	66	NPK6	2849	NOHATA	116	NPG2	456	POLR BARI
17	NPG1	395	BANIACHUNG	67	NPK9	2930	BANAHALIPUR	117	EPG9	1992	POILSA
18	NPJ0	2381	KAMDARPUR	68	NPL0	2957	BANIARI	118	NPH3	2064	NOKLA
19	NPJ2	2427	DIRAI	69	NP03	280	GRAGONJ	119	NPH4	2073	ARANKHOLA
20	NPJ6	2481	NABIGANJ	70	EP50	1821	PABNA	120	EPH7	2127	RUPCHANDPUR
21	NPI2	2272	AIPARA	71	NP58	2597	KALIDASPUR	121	EPH8	2145	PHULBARIA
22	NPF6	261	JARIA	72	NP07	355	CHAIMOHAR	122	NPG3	481	ICHAPUR
23	NPI8	2337	BALIJURI	73	EP49	1803	LALPUR	123	NPH0	2010	BELIA
24	NPF9	343	SUNAMGONJ	74	NP35	1468	BHERAMARA	124	NPH5	2091	KACHUA
25	NPI9	2373	KANDIGAON	75	NP57	2561	BARADI	125	NP59	2722	SAIBARIA
26	NPJ4	2454	GOBINDAGONJ	76	EP05	333	KAKONHAT	126	NP60	2795	KASINATHPUR
27	NPJ5	2471	GEAHPUR	77	EP31	1369	PABA	127	NPK5	2813	RAHDIA
28	NPF7	295	PAKIBAR TILA	78	NP13	1776	DIGHAPATIA	128	NP08	380	DUPCHANCHIA
29	NPH7	4317	MOKITALA	79	NP52	1857	TARAS	129	NP51	1830	BOGRA
30	NPH8	4334	JURI	80	NP56	1956	SAIBARIA	130	EP32	1387	SANTAHAR
31	NPH9	4344	HARARGAJ	81	NP33	1406	BAGHARA	131	NP14	1911	DHANGORA
32	NPG0	369	AGRABAD	82	EP34	1433	ARANI	132	EP48	1759	RANBAGHA
33	NPJ9	2525	LAURAGA TILA	83	NP27	1279	ROHANPUR	133	NP39	1549	JOYPURHAT
34	NPF3	193	DUPITILA	84	NP28	1298	CHAPAINA WABGANJ	134	NP42	1612	GOBINDAGANJ
35	NPJ8	2507	BARUINI TILA	85	EP30	1333	PARBATIPURADA	135	NP53	1884	SHARIAKANDI
36	NPN0	4352	KHARACHARA	86	NP09	388	DINAJPUR	136	NP38	1504	RAMACHANDI
37	NPJ3	2435	HABIGANJ	87	NP10	430	NIIPUR	137	NP23	1190	SAIDPUR
38	EPJ7	2498	CHUNARUGHAT	88	NP25	1234	DHAMUIRHAT	138	NP41	1585	MITHAPUKUR
39	NPJ1	2400	MITAMAIN	89	NP11	1522	NAWABGANJ	139	NP12	1568	KOBARU
40	NPG5	3891	MIRPUR	90	NP24	1217	BAJITPUR	140	NP54	1902	KAZIPUR
41	NPG6	3908	COMILLA	91	NP26	1252	BHOLAHAT	141	NP55	1921	SIRAJGANJ
42	EPH2	3899	JAFARGANJ	92	EP29	1325	MOHADEBPUR				
43	NPH6	3980	FULGAZI	93	EP01	117	LOHAGARA				
44	NPH1	3882	AKHAURA	94	NP15	1009	TITALYA				
45	EPH3	3926	LAKSAM	95	NP16	1055	BOALMARI				
46	NPH4	3953	MOIABI	96	NP17	1072	SONAHAR				
47	NPH5	3972	CHAUDDAGRAM	97	EP19	1109	HARIPUR				
48	ISN1	ISN1	TIDAL STA.	98	NP21	1144	MUKANDAPUR				
49	NPL5	3407	SERAJDIKHAN	99	NP02	231	CHAPANI				
50	NPL7	3452	GOPALDI	100	NP18	1099	BALAPARA				

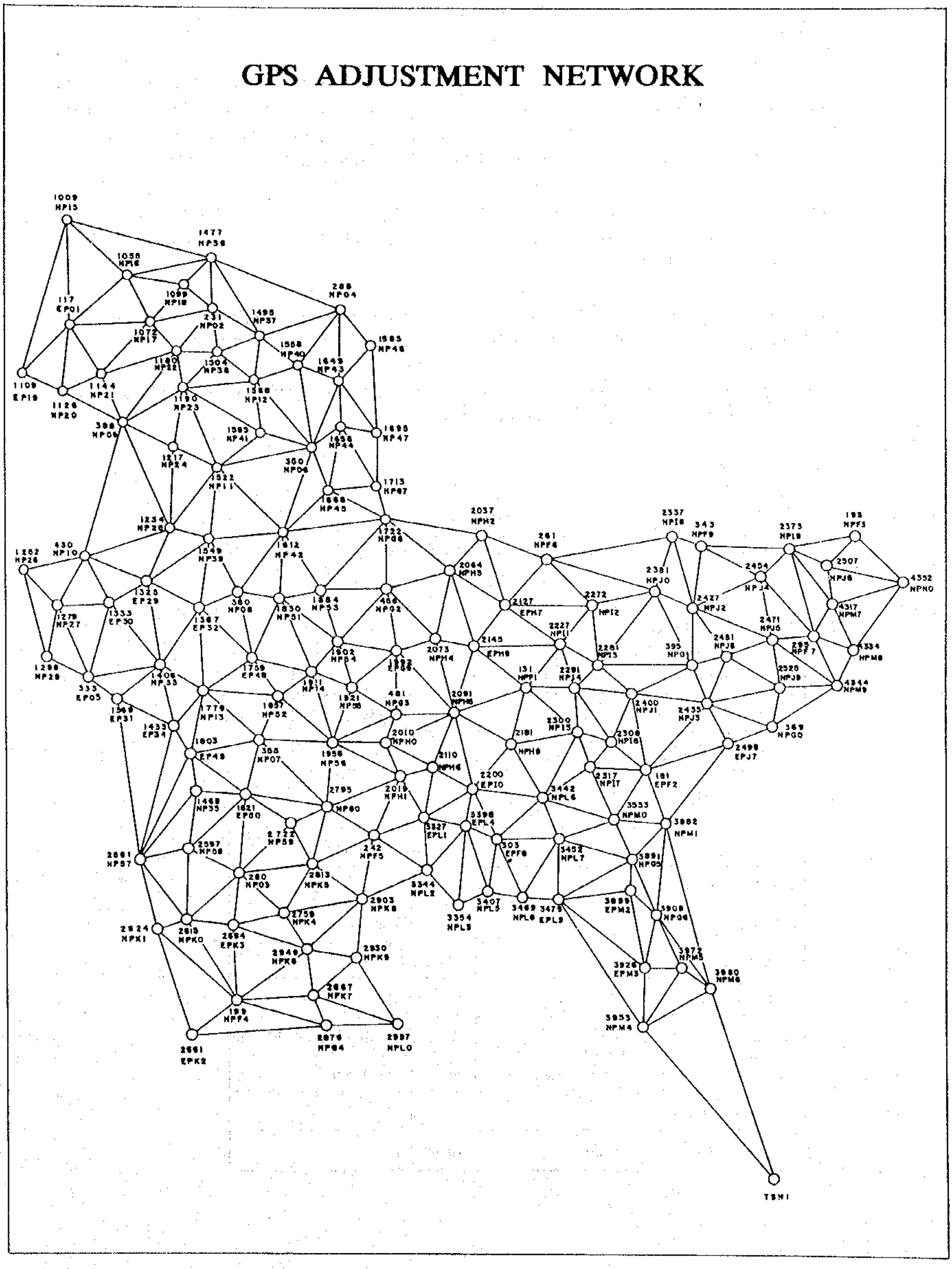


Figure 14 GPS adjustment network

3) GPS observation

The plan for observation of the control point network was implemented via the GPS interference positioning method on the basis of control point surveying network maps. Several GPS receivers (between 6 and 13) occupied to each observation point at the same time, and satellite signals were received from at least four satellites simultaneously.

We set a standard of 3 hours as the time needed for each session of observations and achieved a relative accuracy of at least 1/100,000. We maintained elevation angle of at least 15° for the space above the antenna centre.

We carried out elevation linking survey via direct levelling from the levelling route (discrepancy of double running $\pm 10\text{mm}\sqrt{S}$ (S:km)). (See Figure 15 and Table 2)

(Session: observations carried out continuously in segments of acquisition of fixed amounts of data, using several receivers simultaneously)

(i) Characteristics of GPS receivers

Reception band	Characteristics	Remarks
L1, L2 (dual frequency)	$\pm$ (5mm + 1 x 10^{-6} x D) D : distance	Trimble 4000 SSE

(ii) Units of observation

Item	Unit	Reading
Base line vector	m	0.001
Antenna height	m	0.01

(iii) Standard observation time etc.

Item	Standard time/standard value
Number of sessions	2
Interval between sessions	At least 5 hours
Observation time per session	At least 3 hours
Data collection rate	15 seconds
Descrepancy between sessions	Distance: 30mm Height: 50mm
Simultaneous observation per session	At least two station
Satellite altitude	At least 15°
Number of satellites received	At least 4

(iv) Total number of control points

Category	Control point surveying
Average distance	30 km
Total number of control points	141
Brake down:	
Geodetic datum	1
Tidal observation station	1
Type A	26
Type B	89
Existing points	24

(v) Equipment used for surveying

Category	Control point surveying	Number
GPS receivers	Trimble 4000 SSE	13
Antenna poles	Denki Kogyo PA-7-90	13
Computers	Toshiba J3100GT-XD	4
Handy GPS	Sony Pyxis	15

(vi) Checks of observed values

The observed values were checked by referring to the length of the closure vector (ds). The permissible range was ds < 1.5ppm x $\sum D$, where $\sum D$: route length. The locally computed results yielded good reproducibility and were within the permissible range.

(vii) Formation of Observation Teams for Control Point Survey

The formation of observation teams for the control point survey was made by taking into consideration accuracy, safety and technology transfer.

A Japanese engineer was assigned to the handling of a GPS receiver as it requires experience in handling or operating the equipment. A locally hired assistant was procured from a local surveying company. Also, locally hired attendants were placed to check crowds of onlookers when necessary.

The 12 observation teams were formed in the observation work conducted in the second year and third year. Each team was assigned with the SOB counterparts who were responsible for negotiating with local people and lodging.

The GPS observation at the standard datum of geographical coordinates was conducted under the leadership of the chief engineer, who was also responsible for the observation work at the control points and the headquarters staffs assisted him. Also, in the latter part of the third year study the SOB counterparts participated in the GPS observation with the intention of knowing base-line analyses or technology transfer.

A schematic diagram of Work Assignment is given in Fig. 16.

Geodetic Control Network in Bangladesh

Geoidal Undulation Control GPS Stations

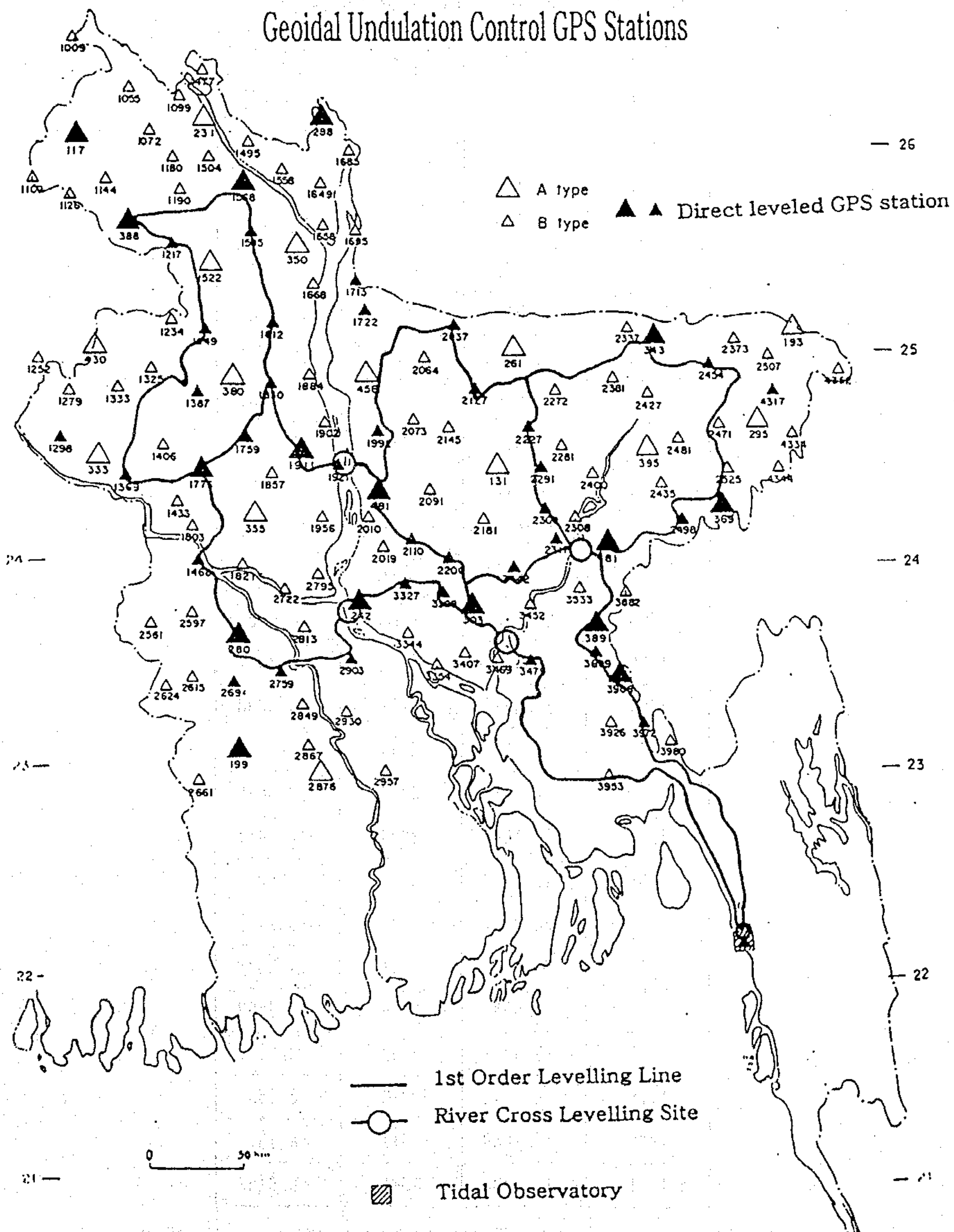
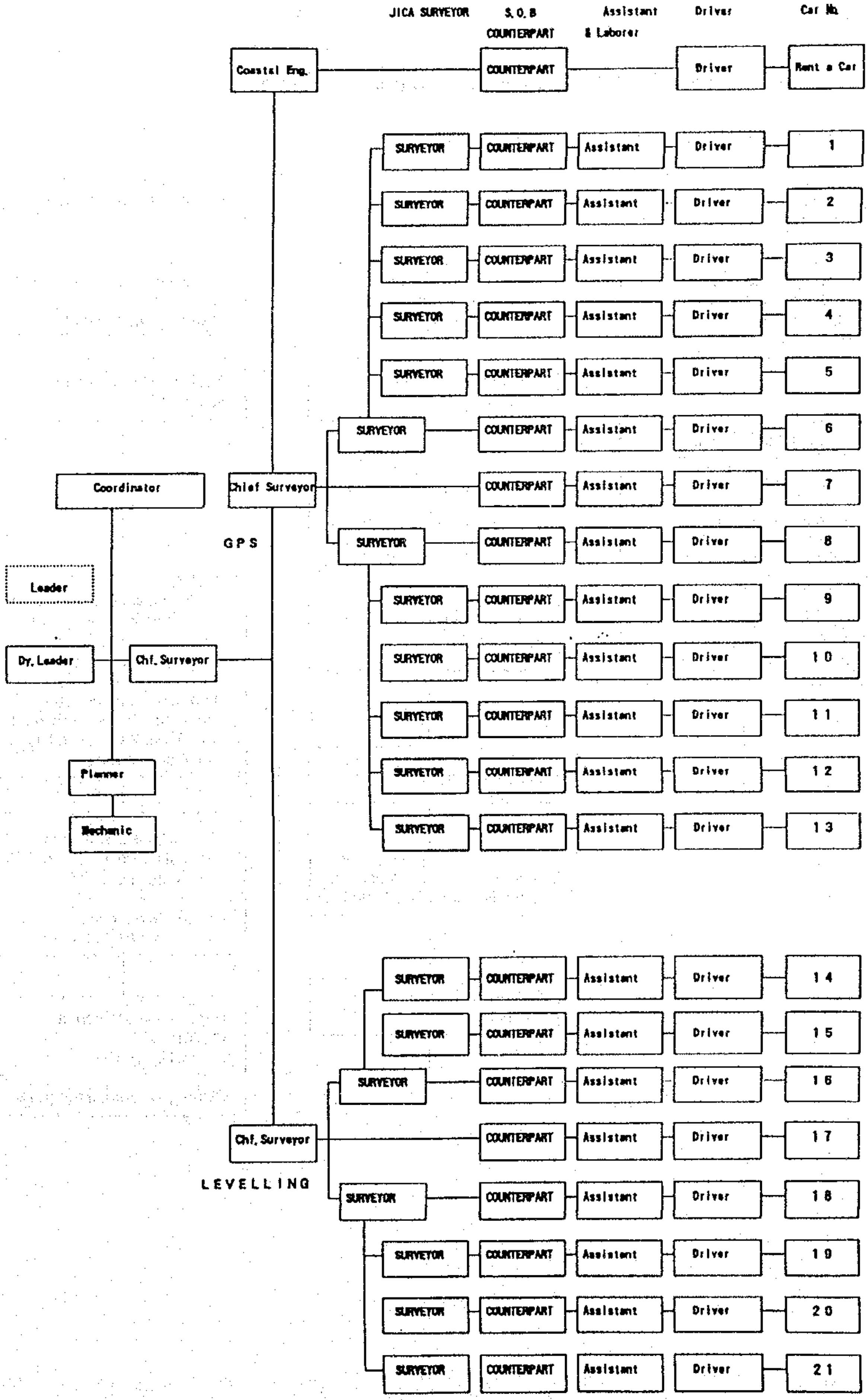


Figure 15 Geoidal undulation control GPS stations

Table 2 Orthometric Height of GPS stations

GPS station	Height	GPS station	Height
117	54.644	1921	13.8213
181	6.824	1992	14.401
199	5.141	2037	13.1614
242	8.6924	2110	9.4878
280	10.558	2127	12.3447
288	32.900	2200	12.7642
303	8.5344	2227	8.6030
343	8.9698	2291	9.7681
369	24.975	2300	9.5180
388	36.0160	2317	7.597
481	11.2468	2454	10.1201
1217	32.0478	2498	15.6964
1298	21.606	2525	13.059
1369	17.1686	2694	7.834
1387	14.528	2759	6.5398
1468	15.218	2903	7.5537
1549	20.3131	3327	8.6679
1568	35.160	3398	10.0411
1585	28.7147	3442	7.901
1612	20.8514	3479	5.276
1713	21.115	3891	7.5678
1722	20.958	3899	7.8552
1759	13.5711	3908	9.776
1776	14.4244	3972	8.2532
1830	18.7859	4317	18.737
1911	13.4733	TSN1	6.6153

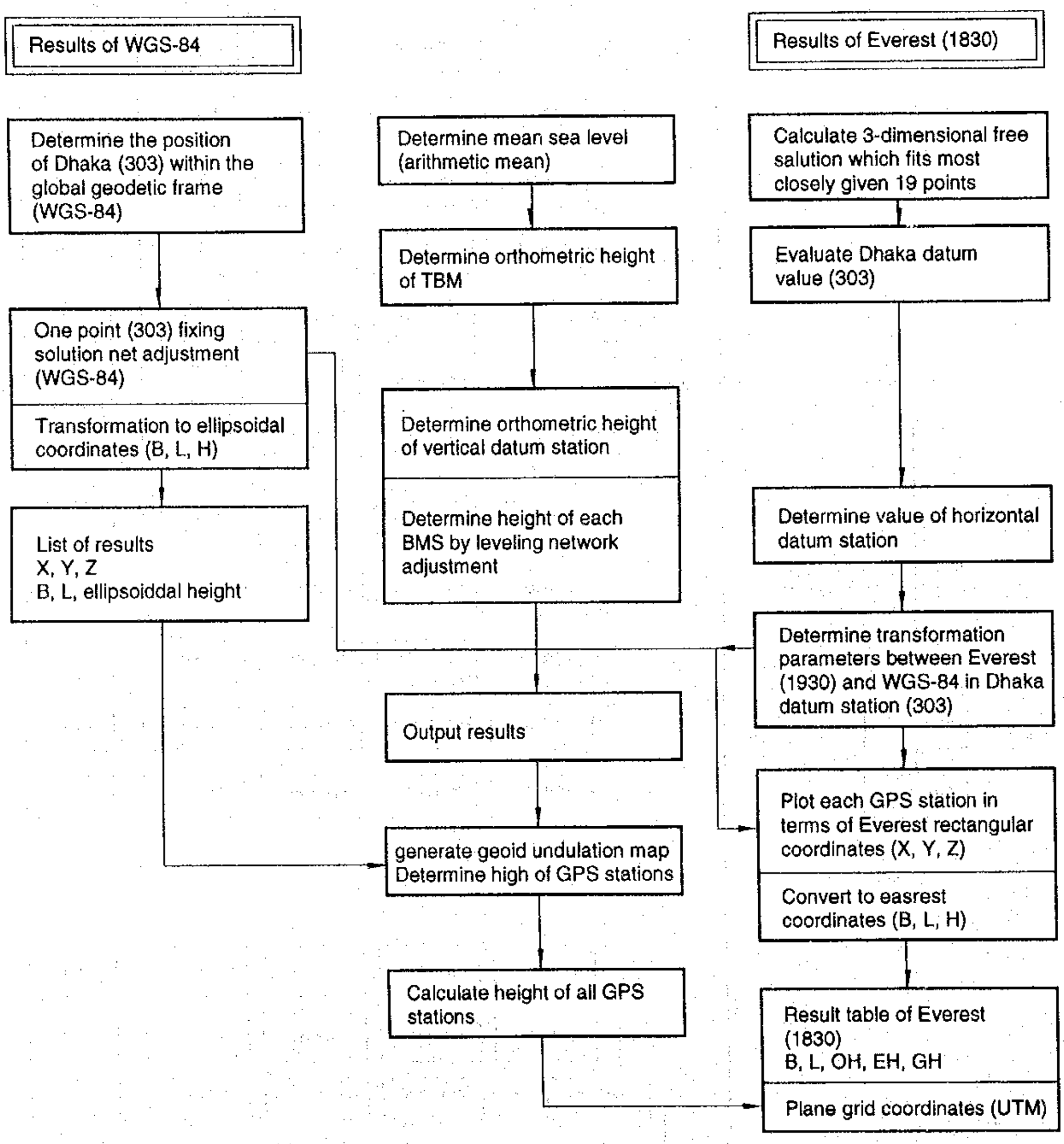
- Note that the above heights represent the height above sea level of the GPS stations indicated as mesured using the direct levelling method.



Figuer 16 A schematic diagram of local workforce assignment

2-4-2 Computations of datum stations of geographic coordinates

1) Results of geodetic survey of Bangladesh



2) Background of the determination of values for the standard geographic coordinates

Geographical coordinate of horizontal datum station (303) in Dhahc serve as the standards for the creation of the Bangladesh geodetic network, and this coordinate serve to define the origin from which the position of all GPS control points (i.e., latitude and longitude) are determined.

With a view towards allowing the results of the current geodetic survey to be used in the future, continuous GPS observations were made at the horizontal datum station (303), to determine the exact position in the world geodetic frame using high-precision global geodetic system set in accordance with the IGS almanac.

Note that separate results of control points were calculated for WGS-84 and Everest (1830).

(1) IGS observation system

Created with the objective of providing GPS data to researchers in all parts of the world based on a framework of cooperation between related research organizations throughout the world as to promote research in geodesics and geophysics, the International GPS Service for Geodynamics is an organization which was formed in 1991 as a result of a proposal made by the International Association of Geodesy, and after conducted a worldwide GPS campaign and offering a range of pilot services, the IGS began regular operations on January 1, 1994.

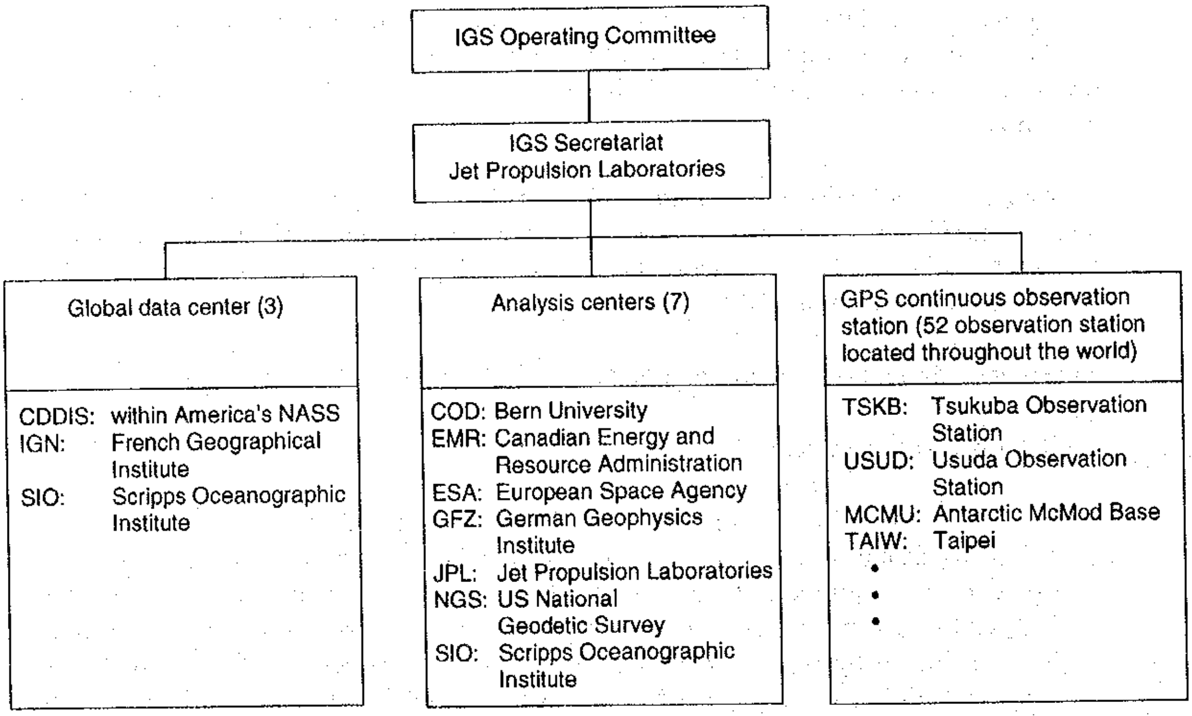
The main work performed by the IGS is as shown below:

A. To construct a GPS satellite tracking network throughout the world and to use this network to perform continuous observations.

B. To centrally maintain data from observations at its data centers and to provide this data to interested researchers.

C. To use this observation data to produce analysis data such as high-precision orbital data (precision ephemeris), data on the earth's rotation, and data on movements in the earth's crust, and to provide this data to users in the earth sciences.

IGS Organizational Chart



(2) Values of geographical coordinates using the ITRF (92) system

The horizontal datum for Dhaka (303) were calculated on an ellipsoid using the IERS Terrestrial Reference Frame (92).

GPS observations were made from Dhaka horizontal datum station through a series of continuous observations performed at 30-second data collection rate over a seven-day period from September 17–23, 1994.

The four IGS monitoring stations at Tsukuba (Japan), Wettzell (Germany), Yaragadee (Australia), and Hartebeesthoek (South Africa) were used to perform a base-line analysis. (See Figure 17)

The GAMIT baseline analysis program, a comprehensive GPS analysis software developed by the Massachusetts Institute of Technology and University of California, and the GLOBK network adjustment program, a GPS network adjustment software developed by the Massachusetts Institute of Technology, University of California, and Harvard University, were used to perform computrone and analyses.

The computation of network adjustment were performed using the daily 3-dimensional coordinates obtained using the above baseline analysis, with the four IGS stations noted above used as fixed points from which to determine the final positions.

The data used in performing baseline analyses were taken from 2,000 epochs (approximately 17 hours worth) of data deemed to be reliable, and the IGS ephemeride was used to perform computations.

The precision of coordinates obtained using network adjustment is estimated to be ±0.191 meters along the X-axis, ±0.304 meters along the Y-axis, and ±0.165 meters along the Z-axis.

Note that calculations and analyses were performed by the Geodetic Department of the Geographical Survey Institute of the Ministry of Construction.

(3) Values of control points obtained using WGS-84 system

Coordinates of horizontal datum station (B, L, H) determined within the ITRF (92) system were fixed, and 141 positions were calculated as (B, L, H, X, Y, Z) WGS-84 system results using 3-dimensional network adjustment.

Parameters of WGS-84 and Dhaka (303) Coordinate

Semi-major axis	6378137.000 m	Coordinate on WGS-84	
Semi-minor axis	6356752.314 m	Latitude B	23°47'52"02714
Flattening (inverse)	298.257223563	Longitude L	90°24'56"34024

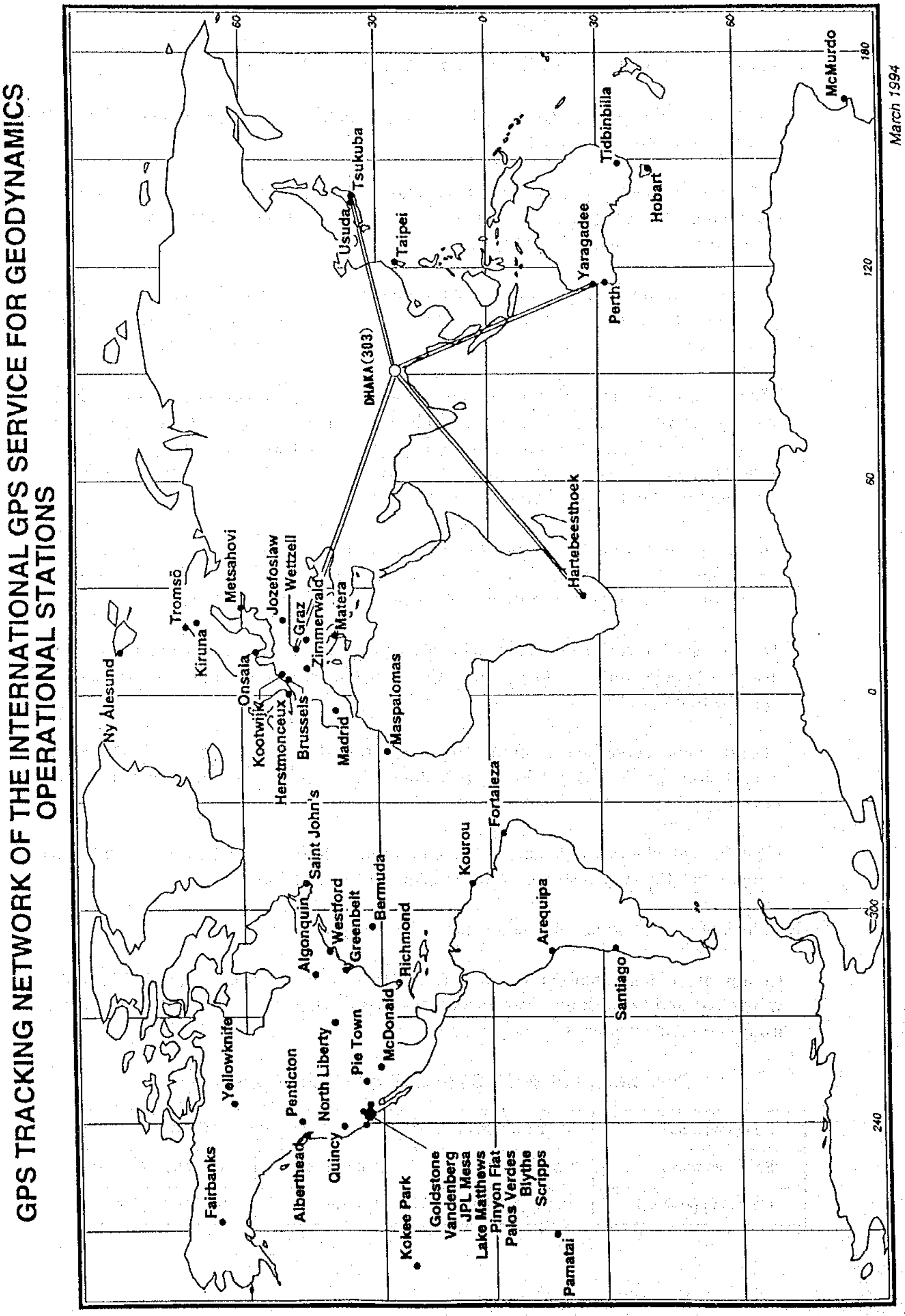


Figure 17 Computation of network adjustment using 4 IGS monitor stations

(4) Results of Computations of absolute position on WGS-84 ellipsoid

DETERMINATION OF ABSOLUTE POSITION OF DHAKA(303) STATION ON WGS-84 ELLIPSOID

In GPS Tracking network of the International GPS Service for Geodynamics

GLOBK Ver 3.1 Global Solution

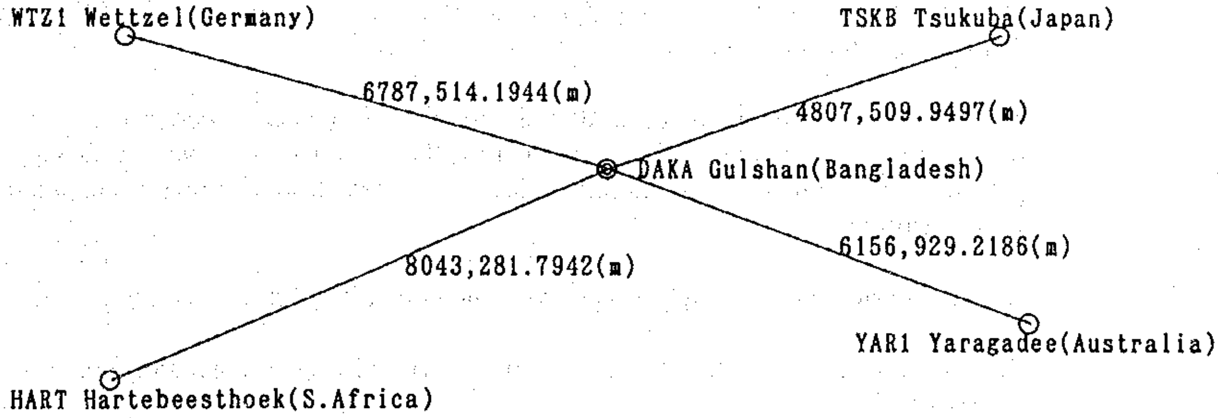
```
Solution commenced with :1994/ 9/17 15:19 (1994. 7109)
Solution ended with     :1994/ 9/23 16:19 (1994. 7274)
Solution refer to       :1994/ 9/23 16:19 (1994. 7274)
Satellite IC epoch      :1994/ 9/23 11:59 50.00
Run time                :1994/12/15 16:08 42.00

There were      7 exps from   7 global files in the solution
There were  91679 data used.  0 data not used and  91679 data total.
There were     15 global parameters estimated
There were      5 station.    0 radio sources.  and 25 satellites
```

Parameter Estimates From GLOBK Vers 3.1

	Estimate	Adjustment	Sigma
DAKA GPS X coordinate (m)	-42358.1522	0.0389	0.0022
DAKA GPS Y coordinate (m)	5838826.2338	0.0171	0.0035
DAKA GPS Z coordinate (m)	2557788.4491	-0.2349	0.00194
DAKA GPS N coordinate (m)	2649156.7947	-0.2219	0.00094
DAKA GPS E coordinate (m)	9209261.7213	-0.0390	0.0022
DAKA GPS U coordinate (m)	-44.8288	-0.0794	0.00397
NE NU EU position correlation	0.5440	0.1028	0.02031

GLOBK:BASELINE LENGTH

Base Line	Length (m)	Adjust(m)	Sigma(m)
TSKB GPS to DAKA GPS	4807509.9497	0.0983	0.0025
YAR1 GPS to DAKA GPS	6156929.2186	-0.1980	0.0021
DAKA GPS to HART GPS	8043281.7942	-0.1736	0.0030
DAKA GPS to WTZ1 GPS	6787514.1944	0.0664	0.0023



GPS Universal Computation Program(Without Geoid Model)
Vers J1.0 1994-12-7

Dhaka Gulshan(303)		IERS-92	WGS-84	estimation accuracy
Latitude	B =		23 47 52.02714	
Longitude	L =		90 24 56.34024	
Ellipsoid Height	H(m)=		-45.4494	
3D Coordinate	X(m)=	-42358.1522	-42358.2817	± 0.191
3D Coordinate	Y(m)=	5838826.2338	5838825.4441	± 0.304
3D Coordinate	Z(m)=	2557788.4491	2557788.6983	± 0.165

		Everest 1830(Kalianpur Datum)	- WGS-84	Difference
Latitude	B =	23 47 49.54	23 47 52.02714	-2.49
Longitude	L =	90 25 06.48	90 24 56.34024	10.14

(5) Values obtained using Everest (1830) system

In order to ensure that the values for the Everest (1830) system were as close as possible to the currently existing 19 control points, these values were determined using 3-dimensional free net averaging.

In consideration of the facts that existing maps and the results of prior surveys will continue to be used in Bangladesh for some time, 3-dimensional free net averaging was used to study and verify existing results from prior geodesic surveys, and then old and preliminary origin points were modified to a degree deemed necessary to prevent them from presenting problems in actual use, and these modified values were then used to determine the values of the origin.

Parameter of Everest 1830 and Coordinate of Dhaka (303)

Semi-major axis	6377276.345 m	Coordinate (Everest 1830) H=8.534	
Semi-minor axis	6356075.413 m	Latitude B	23°47'49"4850
Flattening (inverse)	300.801700000	Longitude L	90°25'06"5527

The survey results currently being published by the Bangladesh government make use of surveys performed in the 19th and 20th centuries. This results were based on Kalianpur in India as the origin and surveyed approximately 1,300 kilometers apart from the origin using triangulation chain method.

The existing triangulation points scattered throughout Bangladesh were set using traditional surveying methods, and for this reason their relative positions are lacking in precision. To evaluation the reliability of these survey results, the following computation were performed.

A. 3-dimensional network adjustment with one point (303) fixing (See Figure 18)

B. 3-dimensional network adjustment with all 19 existing triangulation points (See Figure 19)

C. This was then checked using 3-dimensional network adjusting with 11 triangulation points regecting 8 points with extreme vector descrepancies. (See Figure 20)

D. With respect to the determination of the values for the horizontal datum (303), discussion were held with the Survey of Bangladesh and the JICA Study Team, and decided the 19 existing triangulation points solution in (B), 3-dimensional network adjustment was performed and the results of this computation were then used as the values for the national Horizontal Datum.

E. The transformation constants used to convert between WGS-84 and Everest (1830) at the Horizontal Datum are as follows:

$\Delta X = -283.729$ m $\Delta Y = -735.942$ m $\Delta Z = -261.143$ m

Using these parameters, the Everest (1830) results were computed. (Table 3)

F. The relative position of the geodetic Datum (in horizontal and vertical terms) was then surveyed. (See Figure21)

Final values of Horizontal Datum of Bangladesh

Name Number	GULSHAN 303
Latitude Longitude Height above sea level Location of measured point	23°47'49.48502"N 90°25'6.55270"E 8.5344 m Gulshan Tank Park, DHAKA

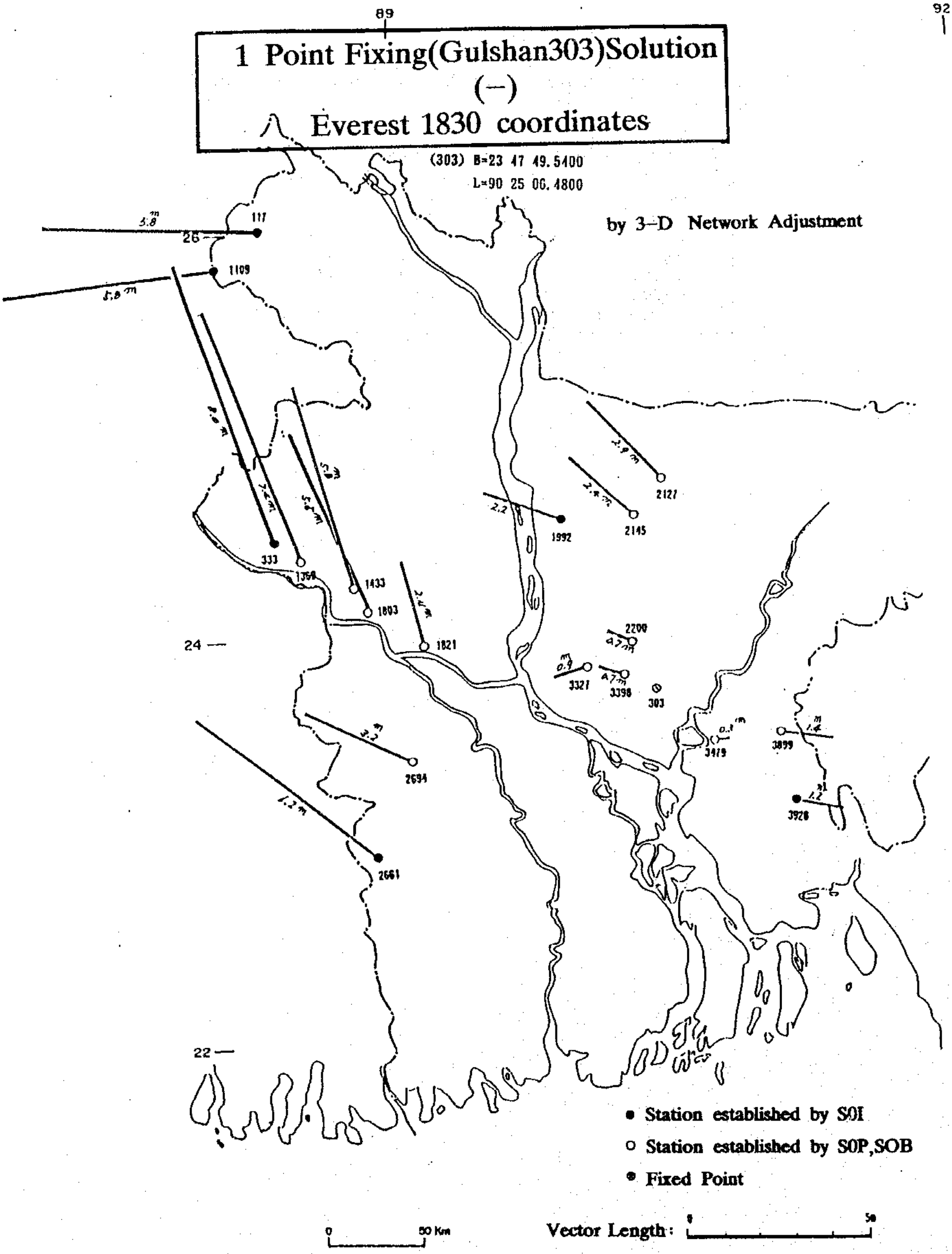


Figure 18 1 point fixing (Gulshan 303) solution (-) everest 1830 coordinates

89

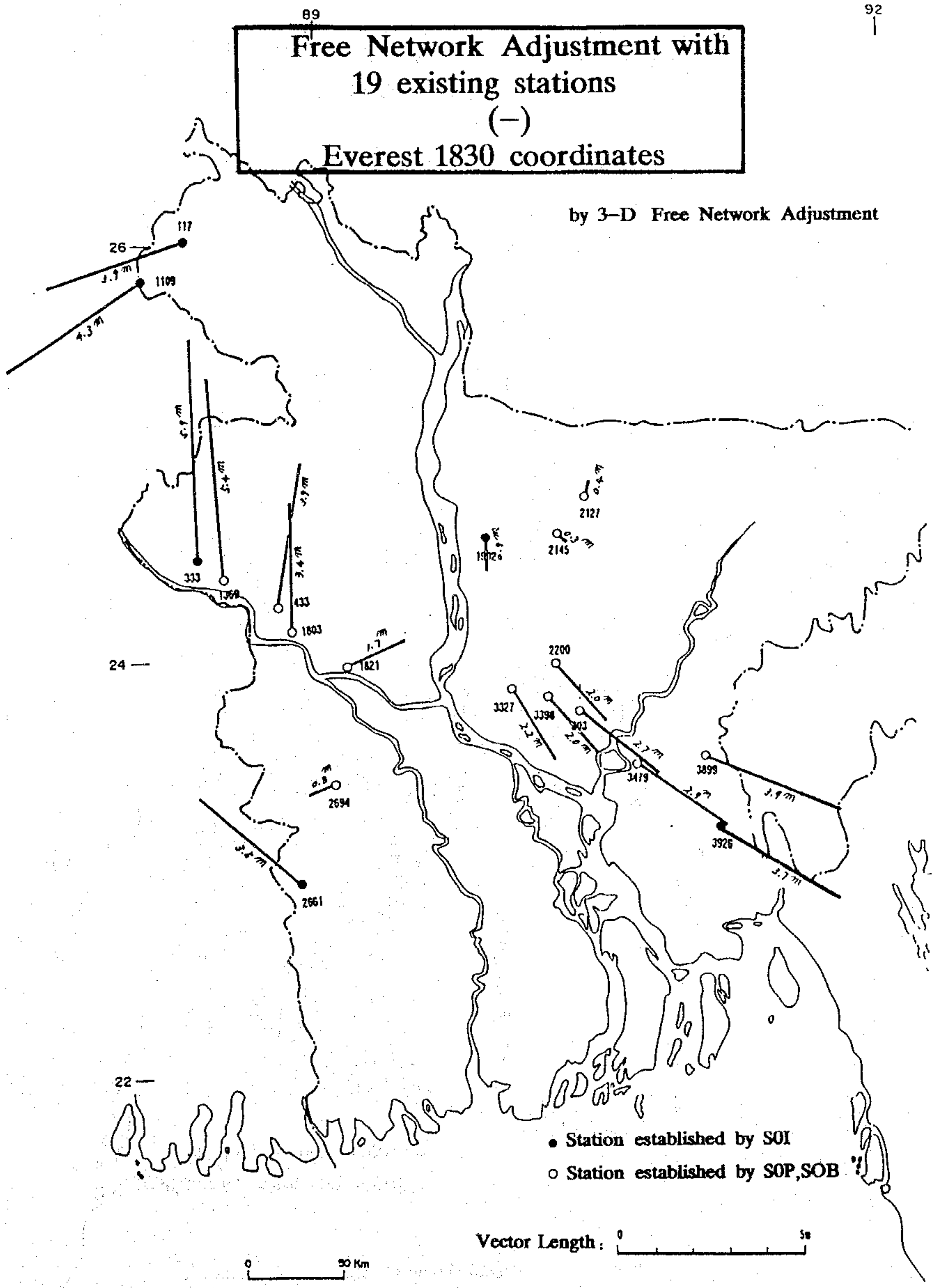


Figure 19 Free network adjustment with 19 existing stations (-) everest 1830 coordinates

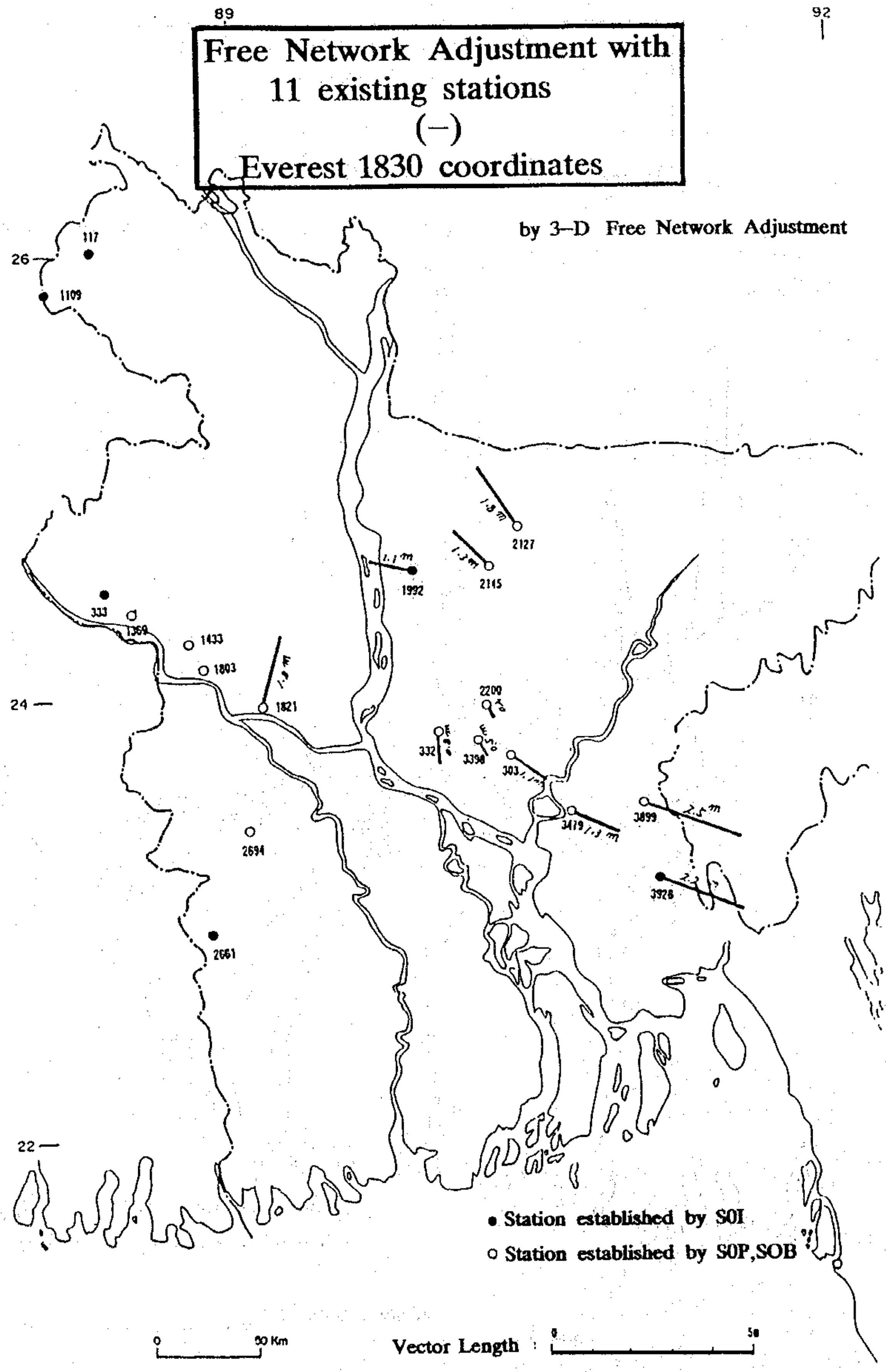


Figure 20 Free network adjustment with 11 existing stations (-) everest 1830 coordinates

89 92

Table 3 Parameters for converting between WGS-84 and Everest (1830)

TRANSFORMATION CONSTANT
(WGS-84 → EVEREST-1830)

「303 GULSHAN」

(BANGLADESH Origin of Longitude and Latitude)

WGS-84	EVEREST-1830	CONSTANT
m	m	m
X= − 42 358. 282	X= − 42 642. 018	ΔX=−283. 736
Y= 5 838 825. 444	Y= 5 838 090. 537	ΔY=−734. 907
Z= 2 557 788. 698	Z= 2 557 528. 011	ΔZ=−260. 687

(1995)

Relation Plan of Origin Point

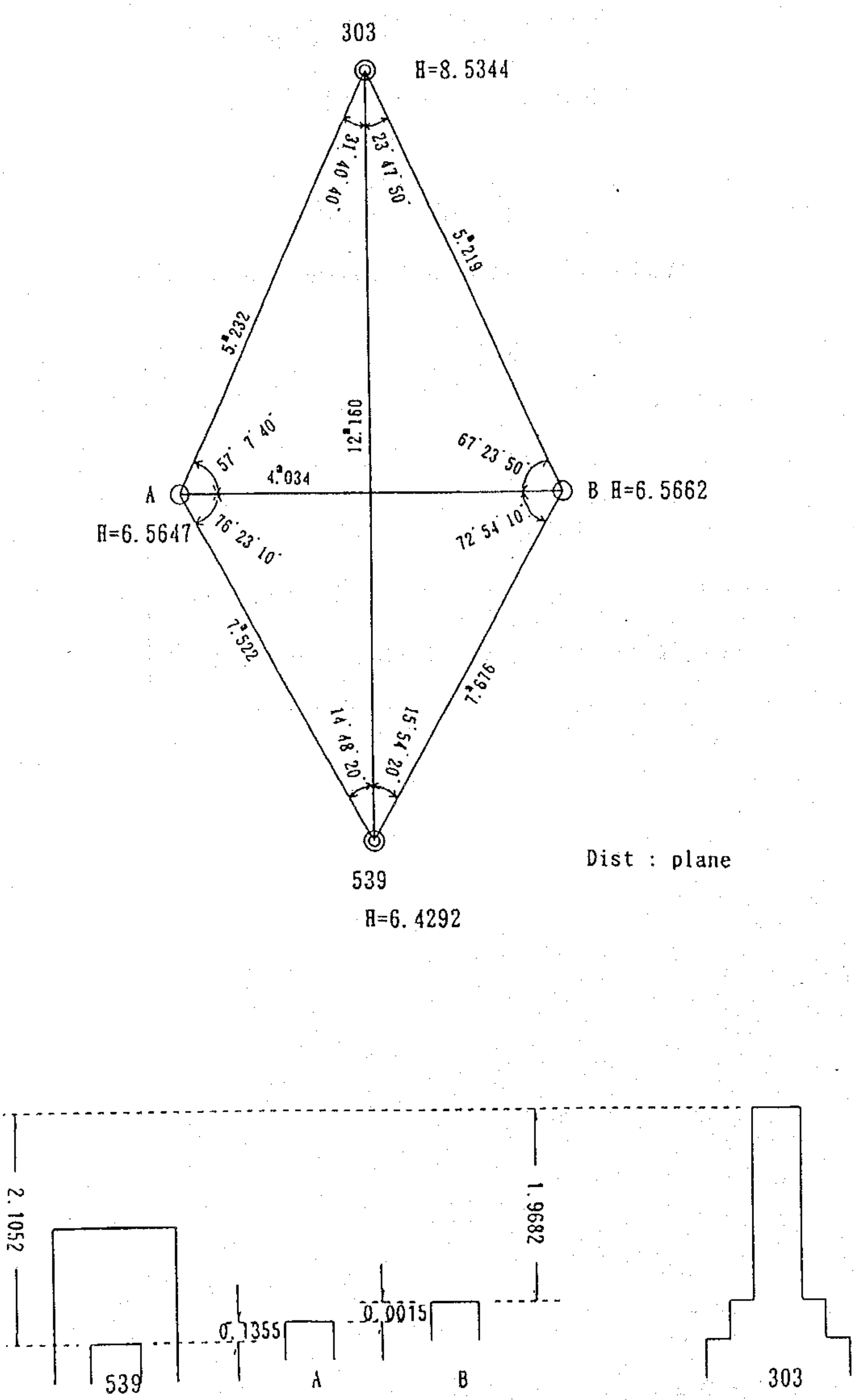


Figure 21 Diagram showing relative positions of origin points

G. 3-dimensional free net adjustment (Hermart's Transformation)

If the X, Y, and Z axes and the degree of rotation are assumed to be represent the amount of movement of the origin points κ, φ, and ω as represented by t_x, t_y, and t_z, then the 3-dimensional Hermart's transformation may be expressed as follows:

$$\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = R\omega R\phi R\kappa \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} \quad \cdots (1)$$

$$R\omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{pmatrix}$$

$$R\phi = \begin{pmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{pmatrix}$$

$$R\kappa = \begin{pmatrix} \cos\kappa & -\sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The minimization of the sum of the squares of the residual left after the 3-dimensional Hermart's transformation in the computation of the 3-dimensional free net adjustment may be computed by taking the coordinates of each position as determined using the single point fixing net adjustment computation to be P_{0i} (X_{0i}, Y_{0i}, Z_{0i}) and then calculating the amounts by which the origin points have moved as represented by tx, ty, and tz together with those values for the 3-dimensional degree of rotation as represented by κ, φ, and w for which the sum of the squares of the distances between point P_{1i} as calculated using the conversion in (1) and point P_i for the existing point or points.

$$\Sigma \left| \begin{pmatrix} X_i \\ Y_i \\ Z_i \end{pmatrix} - \left\{ R\omega R\phi R\kappa \begin{pmatrix} X_{0i} \\ Y_{0i} \\ Z_{0i} \end{pmatrix} + \begin{pmatrix} tx \\ ty \\ tz \end{pmatrix} \right\} \right| \rightarrow \text{minimum}$$

H. Evaluation of old results

The vectors of the differences between the old and new results show a tendency to group together in blocks A, B, C, D, and E. If the positions of the old points are taken as controls for the new points, then the vectors of the new points are as follows:

Block A	4 meters westward
Block B	3-6 meters north
Block C	1 meter or less
Block D	2-4 meters east
Block E	3 meters north by northwest

The cause for this is believed to lie in the fact that the former surveying framework (triangulation chain) was constructed using traditional surveying methods in surveys implemented over the period from the 19th to the 20th century. (See Figure 22)

Comparison of old and new results using existing control points

Block	New results	Old results	N − O	N − O ／ N
A BLOCK 1109 ～ B BLOCK 333	150,387.^{m}16	94.^{m}47	-7.^{m}31	1/ 20,000
A 1109 ～ C 1992	224,980.42	78.39	2.03	1/110,000
A 1109 ～ D 3926	417,100.44	95.92	4.52	1/ 92,000
A 1109 ～ E 2661	327,158.91	63.13	-4.22	1/ 78,000
A 117 ～ A 1109	33,033.52	32.66	0.86	1/ 38,000
B 333 ～ B 1821	94,522.06	17.41	4.65	1/ 20,000
C 1992 ～ C 2127	56,442.74	2.28	0.46	1/122,000
D 303 ～ D 3899	69,606.52	5.08	1.44	1/ 48,000
E 2694 ～ E 2661	54,268.12	9.73	-1.61	1/ 34,000

The relative accuracy of the positions of existing control points within Bangladesh come to 1/75,000 between blocks and 1/52,000 within single blocks.

The average overall accuracy falls somewhere between 1/50,000 and 1/60,000.

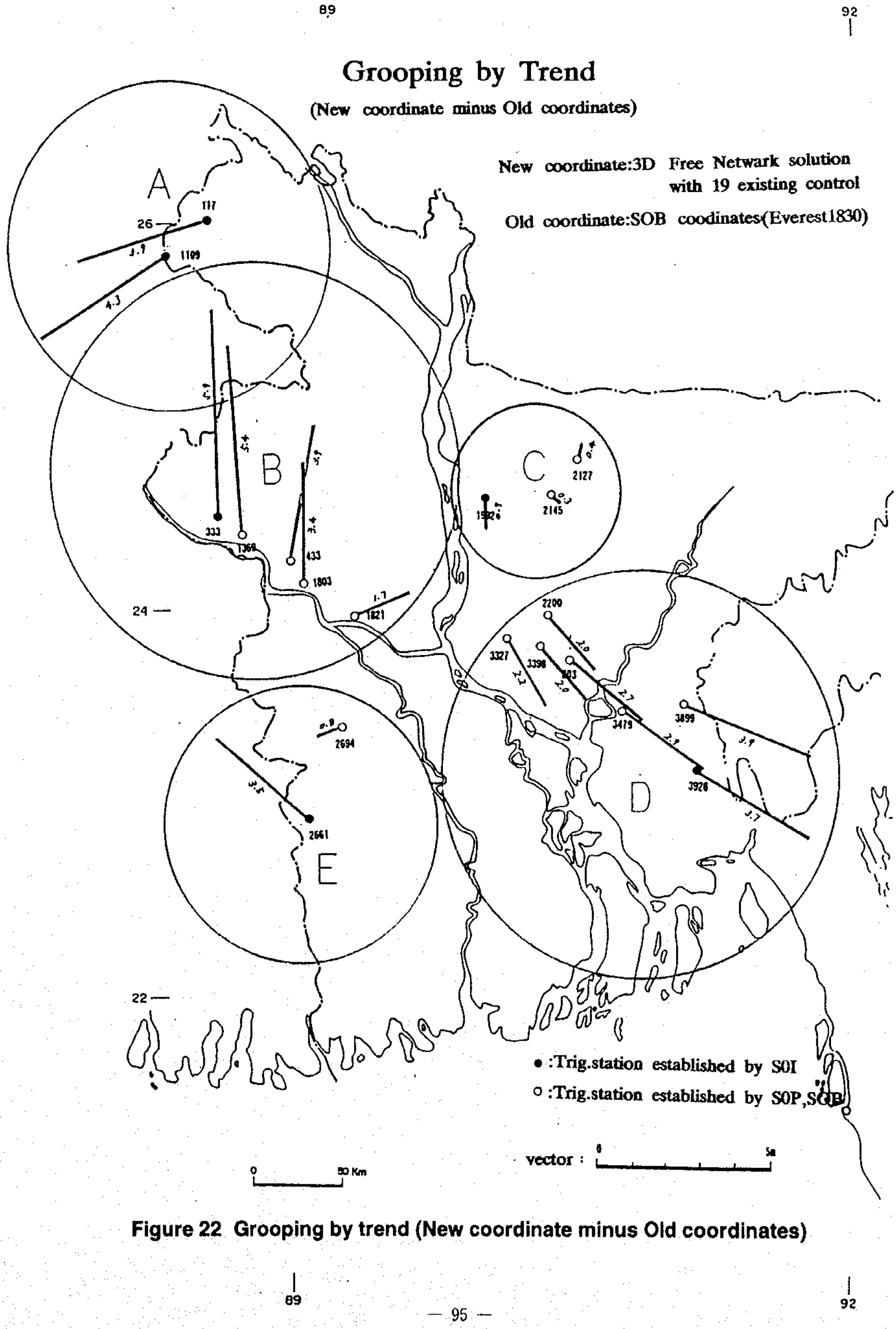


Figure 22 Grooping by trend (New coordinate minus Old coordinates)

I. Determining the values of standard geographical coordinates

Geographical coordinates may be uniquely represented by giving the latitude and longitude of these coordinates as represented upon the ellipsoid defined by the surface of the earth, and once corrected for any errors, the origins of these geographical coordinates may be used as a standard for use in surveying the land of Bangladesh, and they may then be used as the starting points from which to determine the position (in terms of latitude and longitude) of the control points scattered throughout the country.

Background behind the determination of the values of standard geographical coordinates

1) Value for origin points of old results, Everest 1830 (using Kalianpur, India geodetic system)

Latitude B = 23 47 49.54
Longitude L = 90 25 06.48
calculated using the triangulation method using theodolites

2) Values for origin points in the WGS-84 system using GPS

Latitude	B = 23 47 52.02714	
Longitude	L = 90 25 56.34024	
Ellipsoidal height	h = -45.4494	Estimated accuracy
	X = -42,358.2817	±0.191
	Y = 5,838,825.4441	±0.304
	Z = 2,447,788.6983	±0.165

IGS (net, ephemerisis) computations using GPS

3) Value for datum point of new results, Everest 1830 (using Dhaka, Bangladesh geodetic system)

3-dimension free net adjustment was used to determine the datum station in relation to the 19 existing control points.

Latitude B = 23 47 49.48502 N
Longitude L = 90 25 06.55270 E

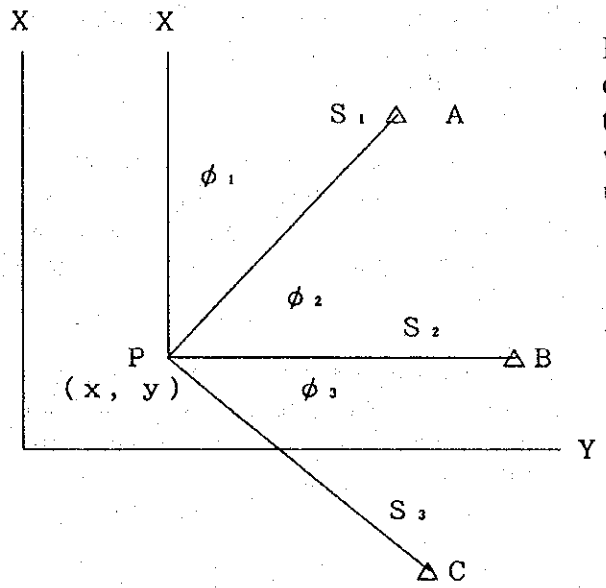
Transformation between WGS-84 and Everest (1830) coordinate systems

Transformation formula

$$\begin{pmatrix} X_B \\ Y_B \\ Z_B \end{pmatrix} = \begin{pmatrix} \Delta X_0 \\ \Delta Y_0 \\ \Delta Z_0 \end{pmatrix} + \begin{pmatrix} X_A \\ Y_A \\ Z_A \end{pmatrix} = \begin{pmatrix} -283.736\text{m} \\ -734.907\text{m} \\ -260.687\text{m} \end{pmatrix} + \begin{pmatrix} X_A \\ Y_A \\ Z_A \end{pmatrix}$$

2-4-3 GPS survey computations

1) Verification of strength of surveying network



If one considers the case where point P is calculated using given points A, B, and C, then the observation equation to be used when these three points have been measured are as follows:

$$v_1 = a_1x + b_1y - l_1$$
$$v_2 = a_2x + b_2y - l_2$$
$$v_3 = a_3x + b_3y - l_3$$

where a_i and b_i are

$$a_i = \frac{\rho''}{S_i} \sin\phi_i$$

$$b_i = \frac{\rho''}{S_i} \cos\phi_i$$

and where the degree of error of point P is calculated as

$$m^2_P = \left\{ \frac{([aa] + [bb])}{D} \right\} \sigma^2_0$$

where D is calculated as $D = [aa]\,[bb] - [ab]^2$

Note that the numerator and denominator of Q_p are calculated as follows

$$Q_p = \left\{ \frac{([aa] + [bb])}{D} \right\}$$

$$[aa] + [bb] = \rho^2 \left(\frac{1}{S^2_1} + \frac{1}{S^2_2} + \frac{1}{S^2_3} \right)$$

$$D = (a_1b_2 - a_2b_1)^2 + (a_1b_3 - a_3b_1)^2 + (a_2b_3 - a_3b_2)^2$$

$$= \rho^4 \left\{ \frac{\sin^2(\phi_2 - \phi_1)}{S^2_1 \cdot S^2_2} + \frac{\sin^2(\phi_3 - \phi_1)}{S^2_1 \cdot S^2_3} + \frac{\sin^2(\phi_3 - \phi_2)}{S^2_2 \cdot S^2_3} \right\}$$

The above equation shows that the size of the interior angle and the length of the sides affect the value of Q_p in a complex manner.

In this example, if we assume that $S_1 = S_2 = S_3$ and if we assume that $\phi_2 - \phi_1 = \phi_3 - \phi_2 = 60°$ (i.e., if we assume equilateral triangles),
furthermore, if we assume that $Q_p = 1.33S^2/\rho^2$ and if we assume both that $S_1 = S_2 = S_3$ and that $\phi_2 - \phi_1 = \phi_3 - \phi_2 = 20°$, then
$Q_p = 4.64\ S^2/\rho^2$, thus allowing us to see the strength of the triangular and thereby proving that an equilateral triangle is best suited for use in these computation. (See Figure 23)

Then, in the selection of points, an effort was made to select these points so that they would comprise a set of points describing equilateral triangles, and in consideration of the fact that the size of Q, as noted above, would be subject to complex influences, with respect to the surveying network as a whole, a net adjustment computation was performed using free net releasing before performing GPS observations, and the value of Q was calculated so as to perform an overall evaluation of the strength of the network.

As a result of this evaluation, the square root of Q was found to lie somewhere in a range between 0.09×10^{-6} to 0.12×10^{-6}, thus enabling us to conclude that the network was a good one. Note, however, that the TSN-1 GPS station for the standard tidal observation station set for use in determining the position and of the height above sea level was not used in computation of Q. (See Figure 23)

The observation network of control points was formed in consideration of the geometrical strength of its design and in accordance with an observation network designed for use with the trilatration survey method. Coordinates of the geodetic coordinates were computer in accordance with observations by using 3-dimensional net adjustment.

Figure 23 Graph of square root of Q

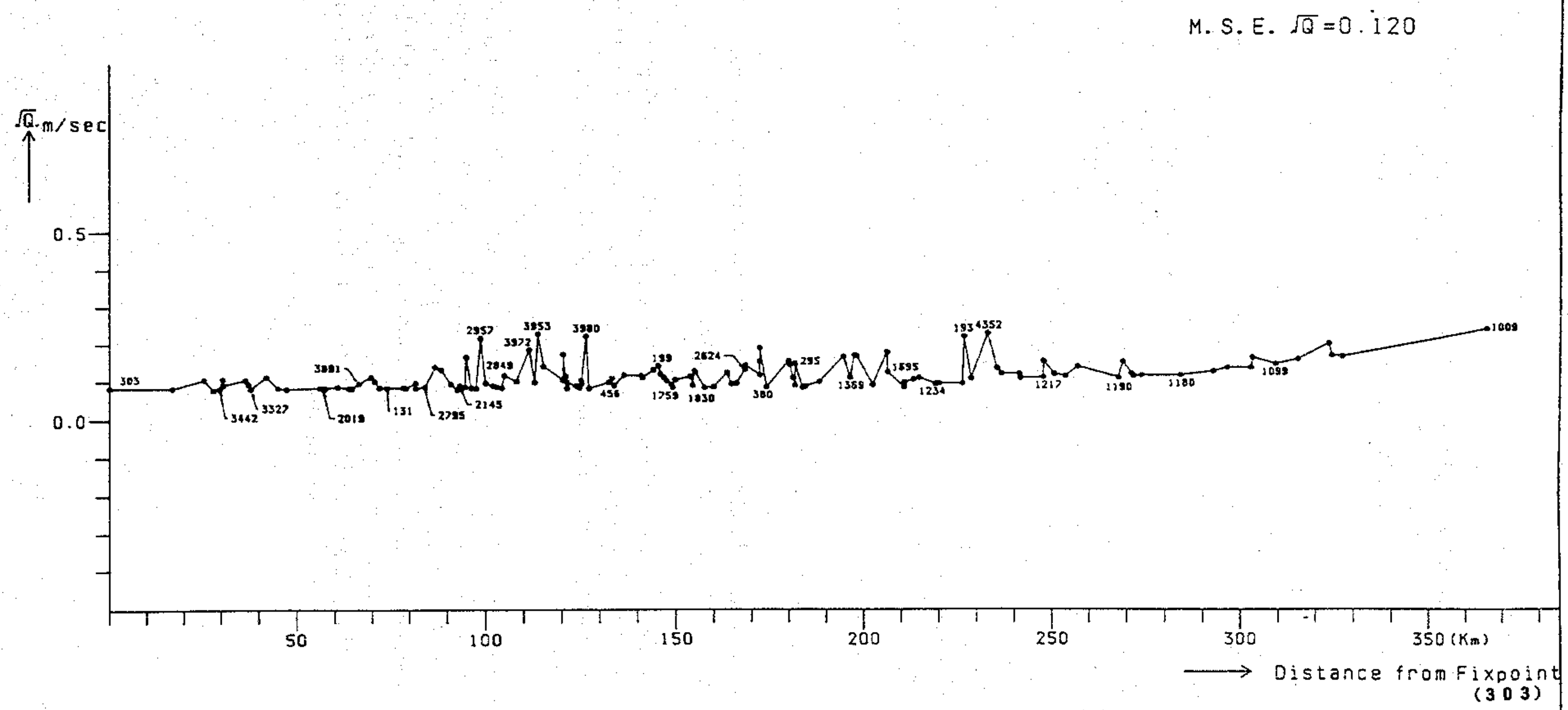


Figure 23 Square root of Q

2) GPS baseline analysis

A total of 756 baselines determined from observations made over the period from November 1992 to February 1993 and over the period from November 1993 to January 1994 were subjected to analysis using baseline analysis software from Trimble Incorporated of the U.S.A.

(1) Baseline analysis

Personal computers used	3 Toshiba J-3100 GT/XD computers CPU: Intel 80486/33M Main memory: 8 MB Disk capacity: 200 MB Operating system: MS-DOS Ver. 3.T3
Software	Trimvec Version 92.030MBP
Orbital data	Broadcast ephemerisis
Ellipsoid	WGS-84
Carrier waves used	L1/L2
Baseline analysis method	MBP-Single

(2) Analysis procedure

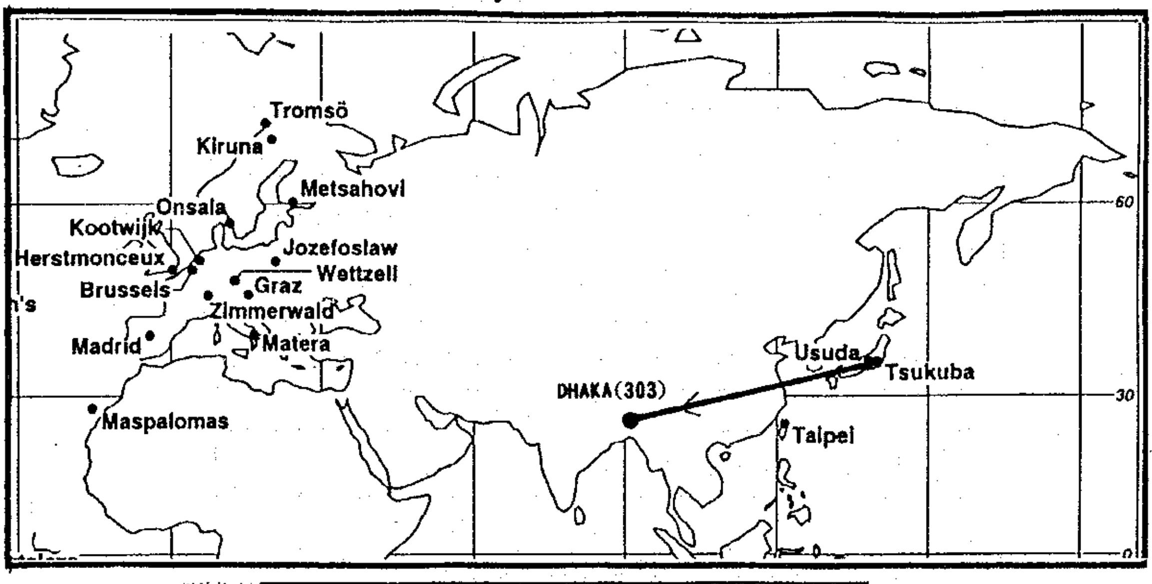
A) On-site preliminary analysis

The value of the control station (117) required as starting points for the analysis were computed using single point fixing.

At the survey headquarters in Dhaka, the latitudes, longitudes, and heights of this points (117) with respect to the ellipsoid were used as a standard by which to perform baseline analyses during the phase II (60 points) and the phase III (81 points) of the survey.

B) Using the data from observations performed over three-hour periods over a five-day period (for a total of 5 observation periods) in February of 1993 and January of 1993, the IGS point at Tsukuba, Japan was used as the known point from which to compute the latitude, longitude and height above ellipsoid (WGS-84) of the Dhaka datum point (303). This origin point was then used as a starting point from which to perform a baseline analysis, and further baseline analyses were performed in sequence starting from this point. (See Table 4)

Table 4 Baseline analysis between Tsukuba and Dhaka



BASELINE SOLUTION BETWEEN TSUKUBA AND DHAKA

*Adj. Software : WAVE 1.19b
*Orbit Infomation : Broadcast Ephemeris
*Ellipsoid : WGS84

LINE	DATE	LATITUDE LONGITUDE ELLIP. HGT.	SLOPE DISTANCE (Std. Dev)	δ	δ δ	RME.
GS105 →303	11JAN93	° ′ ″ 23 47 52.0423N 90 24 56.2951E - 45.691m	m 4807514.708 (0.004)	0.0007 -0.0237 0.720	0.00000049 0.00056169 0.5184	
GS105 →303	12JAN93	23 47 52.0512N 90 24 56.2594E - 46.063m	4807515.276 (0.003)	-0.0082 0.0120 1.092	0.00006724 0.00014400 1.192464	
GS105 →303	13JAN93	23 47 52.0438N 90 24 56.2684E - 45.336m	4807515.439 (0.004)	-0.0008 0.0030 0.365	0.00000064 0.00000900 0.133225	
GS100 →303	03FEB94	23 47 52.0376N 90 24 56.2693E - 44.296m	4807519.559 (0.004)	0.0054 0.0021 -0.675	0.00002916 0.00000441 0.455625	
GS100 →303	04FEB94	23 47 52.0403N 90 24 56.2646E - 43.471m	4807519.941 (0.005)	0.0027 0.0068 -1.500	0.00000729 0.00004624 2.25	
	AVERAGE	23 47 52.0430N 90 24 56.2714E - 44.97 m			ME= 0.0023″ ME= 0.0062″ ME= 0.477 m	

C) Procedure used in performing analysis

Download data

↓

Start up analysis program

↓

Specify processing method: stationary/kinematic, automatic/manual

↓

Enter data•Output results Specify directory

↓

Specify analysis mode: single baseline/multiple baselines
Specify analysis vectors: names of fixed point/names of new points

↓

Enter data from observation points: coordinates/approximate coordinates, height of antennas, meteorological data

↓

Specify block of data used: hh:mm—hh:mm
Specify epoch intervals: 15/30/45/... seconds

↓

Specify other parameters used in computation
Specify other parameters used to control format of display and output

↓

Begin computation

↓

Perform computation while displaying intermediate results

↓

Terminate computation and output results

D) Results of baseline calculations

The error of closure between sessions in which different GPS networks are used fell within a range of 2×10^{-7} to 0.5×10^{-7}, thus enabling us to obtain a baseline accuracy far better than specified tolerance limited 1/100,000.

The errors of closure resulting from a rotation of the perimeter of the GPS network are as follows:

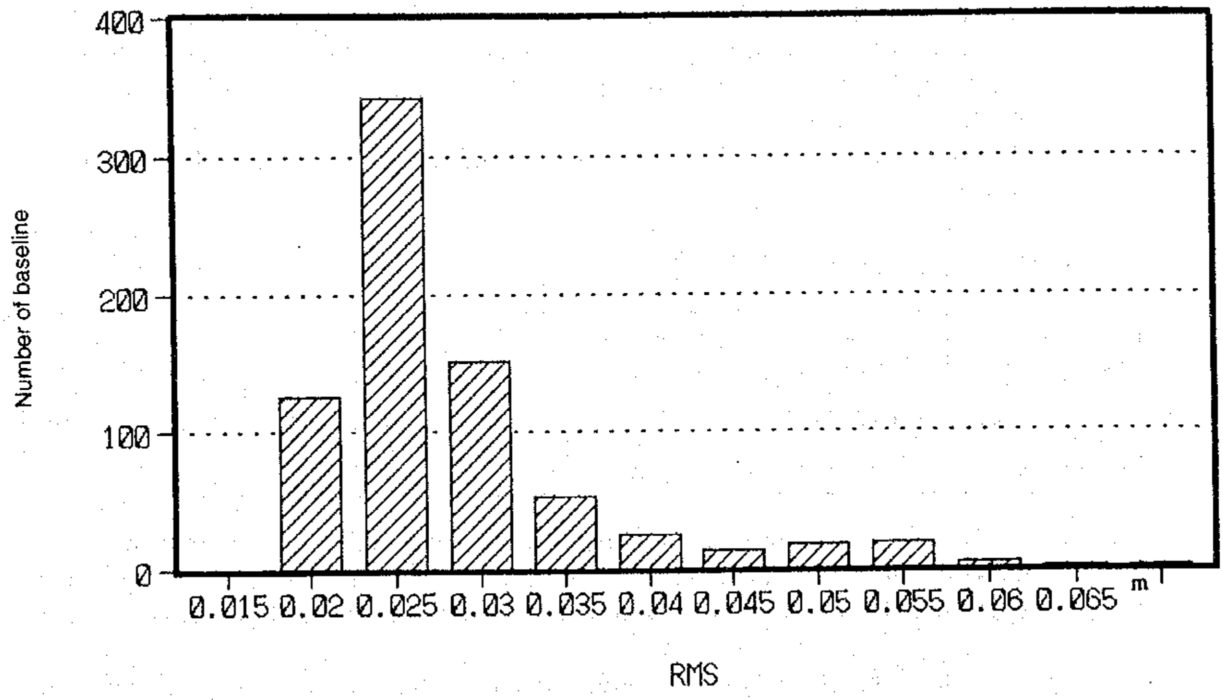
Session	Distance	Relative height error	DX	DY	DZ	DS	Accuracy
	m	m	m	m	m	m	
First session	1,795,394.237	-0.316	-0.175	-0.312	-0.078	0.366	0.20 ppm
Second session	1,795,394.213	-0.018	-0.008	0.003	-0.052	0.053	0.03 ppm
Descrepancy	0.024	-0.298					

The relationship between the number of baselines and RMS in baseline analysis is shown in Figure 24. Note that the percentage of baselines falling within a range of 0.015 to 0.030 meters with respect to the total number of baselines was 85 percent.

Since these baselines have an average length of 30 kilometers, the precision of the baseline analysis is approximately 1/1,000,000.

The percentage of invalid data amongst the data used in baseline analyses is shown in Figure 24.

R.M.S Diagram of baseline analysis



Rate of rejection data in Baseline analysis

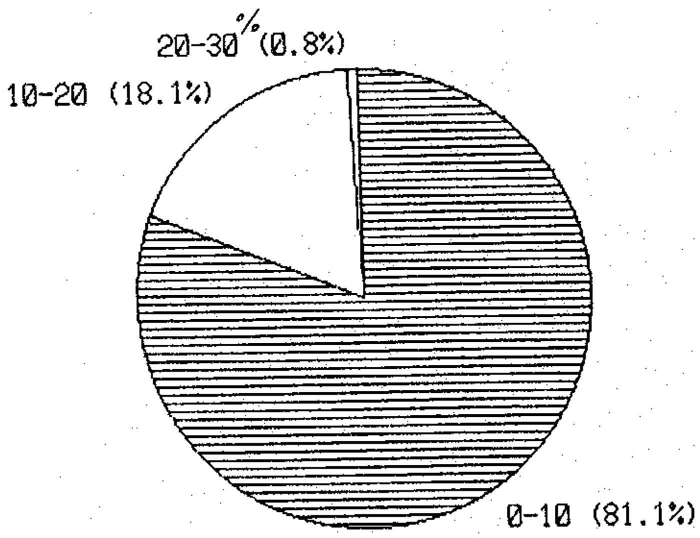


Figure 24 Baseline analysis (percentage of invalid RMS data)

2-4-4 GPS 3-dimensional adjustment computations

NEC ACOS-610 and Toshiba J3100GT-JX computers were used to perform analysis computations associated with control points and to create output of the results, with the following computations being performed.

1) Ellipsoid transformation

(1) Parameters of ellipsoid

A) Values used in Everest (Everest 1830)

Semi-major axis	$a_e = 6377276.345\text{m}$
Flattening	$f_e = 1/300.801700000$

B) Values used in GPS (WGS-84)

Semi-major axis	$a_w = 6378137\text{m}$
Flattening	$f_w = 1/298.257223563$

(2) Transformation to 3-dimensional rectangular coordinate system using values for latitude, longitude, and height

$$X = (N + H)\cos\phi \cdot \cos\lambda$$

$$Y = (N + H)\cos\phi \cdot \sin\lambda$$

$$Z = \{N(1 - e^2) + H\}\sin\phi$$

$$N = a / \sqrt{1 - e^2 \cdot \sin^2\phi}$$

$$e^2 = f(2 - f)$$

where

ϕ : latitude

H : height above ellipsoid

a : semi-major axis

f : flattening

λ : longitude

N : radius of curvature of latitudinal lines (parallel)

e : primary eccentricity

(3) Transformation from 3-dimensional rectangular coordinates to latitudinal, longitudinal, and height coordinates

$$\phi = \tan^{-1}\{Z / (P - e^2 \cdot N_{i-1} \cdot \cos\phi_{i-1})\}$$ (calculated iteratively)

$$\lambda = \tan^{-1}(Y / X)$$

$$H = P / \cos\phi - N$$

$$P = \sqrt{X^2 + Y^2}$$

$$N_{i-1} = a / \sqrt{1 - e^2 \cdot \sin^2\phi_{i-1}}$$

where the condition of convergence for ϕ is: $|\phi_i - \phi_{i-1}| \leqq 10^{-12}$ (rad)
where ϕ_i represents the *i*th value of ϕ
and where ϕ_0 is $\tan^{-1}(Z/P)$

(4) Transformation of coordinates

The following formula may be used to convert WGS-84 coordinates in those coordinates used for the Bangladesh geodetic coordinate system:

$$\begin{pmatrix} X_B \\ Y_B \\ Z_B \end{pmatrix} = \begin{pmatrix} \Delta X_0 \\ \Delta Y_0 \\ \Delta Z_0 \end{pmatrix} + \begin{pmatrix} X_A \\ Y_A \\ Z_A \end{pmatrix} = \begin{pmatrix} -283.736\mathrm{m} \\ -734.907\mathrm{m} \\ -260.687\mathrm{m} \end{pmatrix} + \begin{pmatrix} X_A \\ Y_A \\ Z_A \end{pmatrix}$$

where

X_A, Y_A, and Z_A represent rectangular coordinates in the WGS-84 coordinate system

X_B, Y_B, and Z_B represent rectangular coordinates in the Bangladesh survey coordinate system

and ΔX_0, ΔY_0, and ΔZ_0 represent the degree of horizontal displacement between coordinates in the WGS-84 and Bangladesh coordinate systems.

2) Computation of 3-dimensional network adjustment

(1) GPS baseline vectors

$$\begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} = \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} - \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix}$$

$$\begin{pmatrix} X_i \\ Y_i \\ Y_i \end{pmatrix} = \begin{pmatrix} (N_i + h_i) \cdot \cos\phi_i \cdot \cos\lambda_i \\ (N_i + h_i) \cdot \cos\phi_i \cdot \sin\lambda_i \\ (N_i \cdot (1 - e_i^2) + h_i) \cdot \sin\phi_i \end{pmatrix} \quad i = 1, 2$$

(2) Computation of network adjustment when computing results

A) Observation equation using geodetic coordinates (i.e., latitude, longitude and height)

$$\underset{\text{(residual)}}{\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}} = m_2 \underset{\text{(unknown quantity)}}{\begin{pmatrix} \delta\phi_2 \\ \delta\lambda_2 \\ \delta h_2 \end{pmatrix}} - m_1 \cdot \underset{\text{(unknown quantity)}}{\begin{pmatrix} \delta\phi_1 \\ \delta\lambda_1 \\ \delta h_1 \end{pmatrix}} + M\xi \cdot \begin{pmatrix} \Delta X^0 \\ \Delta Y^0 \\ \Delta Z^0 \end{pmatrix} \cdot \xi$$

$$+ M\eta \cdot \begin{pmatrix} \Delta X^0 \\ \Delta Y^0 \\ \Delta Z^0 \end{pmatrix} \cdot \eta + \underset{\text{(approximation)}}{\begin{pmatrix} \Delta X^0 \\ \Delta Y^0 \\ \Delta Z^0 \end{pmatrix}} - \underset{\text{(observed value)}}{\begin{pmatrix} \Delta X_{ob} \\ \Delta Y_{ob} \\ \Delta Z_{ob} \end{pmatrix}}$$

$$m_i = \begin{pmatrix} -(M_i + h_i) \cdot \sin\phi_i \cdot \cos\lambda_i & -(N_i + h_i) \cdot \cos\phi_i \cdot \sin\lambda_i & \cos\phi_i \cdot \cos\lambda_i \\ -(M_i + h_i) \cdot \sin\phi_i \cdot \cos\lambda_i & -(N_i + h_i) \cdot \cos\phi_i \cdot \sin\lambda_i & \cos\phi_i \cdot \cos\lambda_i \\ (M_i + h_i) \cdot \cos\phi_i & 0 & \sin\phi_i \end{pmatrix} \quad i=1,2$$

$$M\xi = \begin{pmatrix} 0 & 0 & -\cos\lambda_0 \\ 0 & 0 & -\sin\lambda_0 \\ \cos\lambda_0 & -\sin\lambda_0 & 0 \end{pmatrix} \qquad \text{(estimated value)}$$

$$M\eta = \begin{pmatrix} 0 & -\cos\phi_0 & -\sin\phi_0 \cdot \sin\lambda_0 \\ \cos\phi_0 & 0 & -\sin\phi_0 \cdot \sin\lambda_0 \\ \sin\phi_0 \cdot \sin\lambda_0 & -\sin\phi_0 \cdot \sin\lambda_0 & 0 \end{pmatrix}$$

where η and ξ represent parameters used to estimated the average geoid slope, thus corresponding to the average vertical deflection in the area being surveyed. Note, however, that parameters η and ξ are not used when calculating preliminary averages.

$$N_i = a / \sqrt{1 - e^2 \sin^2 \phi_i} \quad (i = 1, 2)$$

$$M_i = a \cdot (1 - e^2) / \sqrt{(1 - e^2 \sin^2 \phi_i)^3} \quad (i = 1, 2)$$

where ϕ_0 and λ_0 represent the latitude and longitude of arbitrary known points.

B) Observation equation using a three-dimensional graph (X, Y, Z)

$$\underset{\text{(remainder)}}{\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}} = \underset{\text{(unknown quantity)}}{\begin{pmatrix} \delta X_2 \\ \delta Y_2 \\ \delta Z_2 \end{pmatrix}} - \underset{\text{(unknown quantity)}}{\begin{pmatrix} \delta X_1 \\ \delta Y_1 \\ \delta Z_1 \end{pmatrix}} + M\xi \cdot \begin{pmatrix} \Delta X^0 \\ \Delta Y^0 \\ \Delta Z^0 \end{pmatrix} \cdot \xi$$

$$+ M\eta \cdot \begin{pmatrix} \Delta X^0 \\ \Delta Y^0 \\ \Delta Z^0 \end{pmatrix} \cdot \eta + \underset{\text{(estimated value)}}{\begin{pmatrix} \Delta X^0 \\ \Delta Y^0 \\ \Delta Z^0 \end{pmatrix}} - \underset{\text{(observed value)}}{\begin{pmatrix} \Delta X_{ob} \\ \Delta Y_{ob} \\ \Delta Z_{ob} \end{pmatrix}}$$

C) Observation weights

$$P = \sigma_0^2 (\Sigma_{\Delta X, \Delta Y, \Delta Z})^{-1}$$

where
σ_0^2 represents the variance of unit weights
and where $\Sigma_{\Delta X, \Delta Y, \Delta Z}$ represents the variance and covariance matrix of ΔX, ΔY, and ΔZ.

D) Calculation of averages

$$V = AX - L$$

$$(A^T P A) X - (A^T P L) = 0$$

$$X = (A^T P A)^{-1} A^T P L$$

$$\mu^2 = (V^T P V) / (n - m)$$

where
V: residual vector
X: unknown vector
P: weight calculated according to equation in (C)
μ: unit weight of standard deviation of observations
n: number of observation equations
A: a matrix of unknown
L: vector of constant term
m: number of unknowns

3) Variance and covariance matrix and correlative matrix

(1) Variance and covariance matrix

A) Variance and covariance matrix for rectangular coordinates (X, Y, and Z)

ΔX, ΔY, and ΔZ variance and covariance matrix:

$$\Sigma_{\Delta X,\Delta Y,\Delta Z} = \begin{bmatrix} \sigma_{\Delta X\Delta X} & \sigma_{\Delta X\Delta Y} & \sigma_{\Delta X\Delta Z} \\ \sigma_{\Delta Y\Delta X} & \sigma_{\Delta Y\Delta Y} & \sigma_{\Delta Y\Delta Z} \\ \sigma_{\Delta Z\Delta X} & \sigma_{\Delta Z\Delta Y} & \sigma_{\Delta Z\Delta Z} \end{bmatrix}$$

B) Variance and covariance matrix for geographic coordinates (i.e., latitude, longitude, and height)

$$\Sigma_{\phi,\lambda,h} = R \cdot \Sigma_{\Delta X,\Delta Y,\Delta Z} \cdot R^{T}$$

where

$$R = \begin{bmatrix} -\sin\phi \cdot \cos\lambda / (M+h) & -\sin\phi \cdot \sin\lambda / (M+h) & \cos\phi / (M+h) \\ -\sin\lambda / \{(N+h)\cdot\cos\phi\} & \cos\lambda \ \{(N+h)\cdot\cos\phi\} & 0 \\ \cos\phi \cdot \cos\lambda & \cos\phi \cdot \sin\lambda & \sin\phi \end{bmatrix}$$

(2) Relationship between correlative matrix and variance and covariance matrix

$$\rho_{ij} = \sigma_{ij} / \sqrt{(\sigma_{ii} \cdot \sigma_{jj})}$$

(3) Correlative matrix

$$\text{correlative matrix: } C = \begin{bmatrix} 1 & \rho_{\Delta X\Delta Y} & \rho_{\Delta X\Delta Z} \\ \rho_{\Delta Y\Delta X} & 1 & \rho_{\Delta Y\Delta Z} \\ \rho_{\Delta Z\Delta X} & \rho_{\Delta Z\Delta Y} & 1 \end{bmatrix}$$

(4) Standard deviation

Standard deviation of ΔX: $\sigma_{\Delta X} = \sqrt{\sigma_{\Delta X\Delta X}}$
Standard deviation of ΔY: $\sigma_{\Delta Y} = \sqrt{\sigma_{\Delta Y\Delta X}}$
Standard deviation of ΔZ: $\sigma_{\Delta Z} = \sqrt{\sigma_{\Delta Z\Delta X}}$
Standard deviation of slope distance (D): $\sigma_{D} = \sqrt{\sigma_{D}^{2}}$

where

$$\sigma_{D}^{2} = G \cdot \Sigma_{\Delta X,\Delta Y,\Delta Z} \cdot G^{T}$$
$$G = (\Delta X / D,\ \Delta Y / D,\ \Delta Z / D)$$
$$D = \sqrt{\Delta X^{2} + \Delta Y^{2} + \Delta Z^{2}}$$

4) Equation for calculating GPS survey relationships

(1) Calculations of vector closures

Closed vector: $(\Sigma\Delta X,\ \Sigma\Delta Y,\ \Sigma\Delta Z)$
Scale of closed vector: $\Delta S = \sqrt{(\Sigma\Delta X)^{2} + (\Sigma\Delta Y)^{2} + (\Sigma\Delta Z)^{2}}$
Height component of closed vector: $\Delta H = \cos\phi \cdot \cos\lambda \cdot (\Sigma\Delta X) + \cos\phi \cdot \sin\lambda \cdot (\Sigma\Delta Y) + \sin\phi \cdot (\Sigma\Delta Z)$

Angle closure: $\Delta A = \Delta S / \Sigma D$

where

ΣD: length of route
ϕ: latitude of control point
λ: longitude of control point

(2) Equation for use in calculating angle of elevation from differences in 3-dimensional coordinates

Angle of elevation: $v = \sin^{-1}\{(\cos\phi\cdot\cos\lambda\cdot\Delta X + \cos\phi\cdot\sin\lambda\cdot\Delta Y + \sin\phi\cdot\Delta Z)/D\}$

(3) Equation for use in calculating azimuth angles and length of survey lines between two points from the latitude and longitude of those two points

$$
\begin{aligned}
S\cdot\cos\alpha &= M\cdot\Delta\phi \\
&+ \frac{N}{24}(3\eta^2-6\eta^4+9\eta^6-3t^2\eta^2+21t^2\eta^4-54t^2\eta^6)\cdot\Delta\phi^2 \\
&+ \frac{N\cos^2\phi}{24}(-2-3t^2+3t^2\eta^2-3t^2\eta^4+3t^2\eta^6)\cdot\Delta\phi\Delta\lambda^2 \\
&+ \frac{N}{5760}(-36\eta^2+270\eta^4+36t^2\eta^2-1062t^2\eta^4+135t^4\eta^4)\cdot\Delta\phi^5 \\
&+ \frac{N\cos^2\phi}{5760}(-16-60t^2+4\eta^2-4\eta^4+102t^2\eta^2+48t^2\eta^4+90t^4\eta^2-630t^4\eta^4)\cdot\Delta\phi^3\Delta\lambda^2 \\
&+ \frac{N\cos^4\phi}{5760}(-8-20t^2+15t^4-8\eta^2+96t^2\eta^2-15t^4\eta^2+15t^4\eta^4)\cdot\Delta\phi\Delta\lambda^4 \\
&+ \frac{N\cos^2\phi}{1935360}(-192-2016t^2)\cdot\Delta\phi^5\Delta\lambda^2 \\
&+ \frac{N\cos^4\phi}{1935360}(256+784t^2+4200t^4)\cdot\Delta\phi^3\Delta\lambda^4 \\
&+ \frac{N\cos^6\phi}{1935360}(-64-224t^2+1148t^4-42t^6)\cdot\Delta\phi\Delta\lambda^6
\end{aligned}
$$

$$S \cdot \sin\alpha = N\cos\phi \cdot \Delta\lambda$$

$$+ \frac{N\cos\phi}{24}(1-\eta^2+\eta^4-\eta^6-9t^2\eta^2+18t^2\eta^4-27t^2\eta^6)\cdot\Delta\phi^2\Delta\lambda$$

$$+ \frac{N\cos^3\phi}{24}(-t^2)\cdot\Delta\lambda^3$$

$$+ \frac{N\cos\phi}{5760}(7+10\eta^2-27\eta^4-54t^2\eta^2-642t^2\eta^4+675^4\eta^4)\cdot\Delta\phi^4\Delta\lambda$$

$$+ \frac{N\cos^3\phi}{5760}(-16-70t^2-158t^2\eta^2+158t^2\eta^4+90t^4\eta^2-180t^4\eta^4)\cdot\Delta\phi^2\Delta\lambda^3$$

$$+ \frac{N\cos^5\phi}{5760}(-24t^2+3t^4-24t^2\eta^2)\cdot\Delta\lambda^5$$

$$+ \frac{N\cos\phi}{1935360}\cdot 62\Delta\phi^6\Delta\lambda$$

$$+ \frac{N\cos^3\phi}{1935360}(-416-2954t^2)\cdot\Delta\phi^4\Delta\lambda^3$$

$$+ \frac{N\cos^5\phi}{1935360}(-192-1680t^2+2562t^4)\cdot\Delta\phi^2\Delta\lambda^5$$

$$+ \frac{N\cos^7\phi}{1935360}(-816t^2+528t^4-6t^6)\cdot\Delta\lambda^7$$

$$\Delta\alpha = \cos\phi \cdot t \cdot \Delta\lambda$$

$$+ \frac{\cos\phi\cdot t}{24}(3+2\eta^2-2\eta^4+2\eta^6)\cdot\Delta\phi^2\Delta\lambda$$

$$+ \frac{\cos^3\phi\cdot t}{24}(2+2\eta^2)\cdot\Delta\lambda^3$$

$$+ \frac{\cos\phi\cdot t}{5760}(75-4\eta^2+92\eta^4-120t^2\eta^2+264t^2\eta^4)\cdot\Delta\phi^4\Delta\lambda$$

$$+ \frac{\cos^3\phi\cdot t}{5760}(60-120t^2+52\eta^2-320t^2\eta^2-112t^2\eta^4)\cdot\Delta\phi^2\Delta\lambda^3$$

$$+ \frac{\cos^5\phi\cdot t}{5760}(48-24t^2+96\eta^2-48\eta^4-120t^2\eta^2-96t^2\eta^4)\cdot\Delta\lambda^5$$

$$+ \frac{\cos\phi\cdot t}{967680}\cdot 1281\Delta\phi^6\Delta\lambda$$

$$+ \frac{\cos^3\phi\cdot t}{967680}(1050-5880t^2)\cdot\Delta\phi^4\Delta\lambda^3$$

$$+ \frac{\cos^5\phi\cdot t}{967680}(1008-5544t^2+1008t^4)\cdot\Delta\phi^2\Delta\lambda^5$$

$$+ \frac{\cos^7\phi\cdot t}{967680}(816-1248t^2-96t^4)\cdot\Delta\lambda^7$$

$$S = \sqrt{S^2\cos^2\alpha + S^2\sin^2\alpha} = \frac{S\ \cos\alpha}{\cos\alpha} = \frac{S\ \sin\alpha}{\sin\alpha}$$

$$\alpha = \tan^{-1} \frac{S \sin \alpha}{S \cos \alpha}$$

$$\Delta \alpha = \alpha_{21} - \alpha_{12}$$

$$\alpha_{12} = \alpha - \frac{\Delta \alpha}{2}$$

$$\alpha_{21} = \alpha + \frac{\Delta \alpha}{2}$$

$$\Delta \lambda = \lambda_2 - \lambda_1$$

$$\Delta \phi = \phi_2 - \phi_1$$

$$\phi = (\phi_1 + \phi_2)/2$$

$$\alpha = (\alpha_{12} + \alpha_{21})/2$$

$$t = \tan \phi$$

$$\eta^2 = e'^2 \cos^2 \phi$$

where

S: length of survey line joining points P_1 and P_2
α_{12}: azimuth angle of p_2 as measured at p_1
α_{21}: azimuth angle of p_1 as measured at p_2
λ: longitude of point *i* (i=1,2)
ϕ: latitude of point *i* (i=1,2)
M: radius of curvature of meridian
N: radius of curvature of latitudinal lines
e': secondary eccentricity

5) Data file

Total number of control points	141 (including control tide gauge stations)
Fixed points	1
Number of baselines	756 (378 points calculated in 2 sessions)
Number of observation data items	2,268
Number of baselines per point	5.4

6) Results of adjusted calculations

Distribution	141 mm
Mean square error	12 mm

(1) Degree of compensation for observation values ΔX, ΔY, and ΔZ

Graphs showing the number of baselines for the degree of compensation for ΔX and ΔZ show that this degree of compensation falls into a range from 30 to 10 millimeters, thus matching almost perfectly.

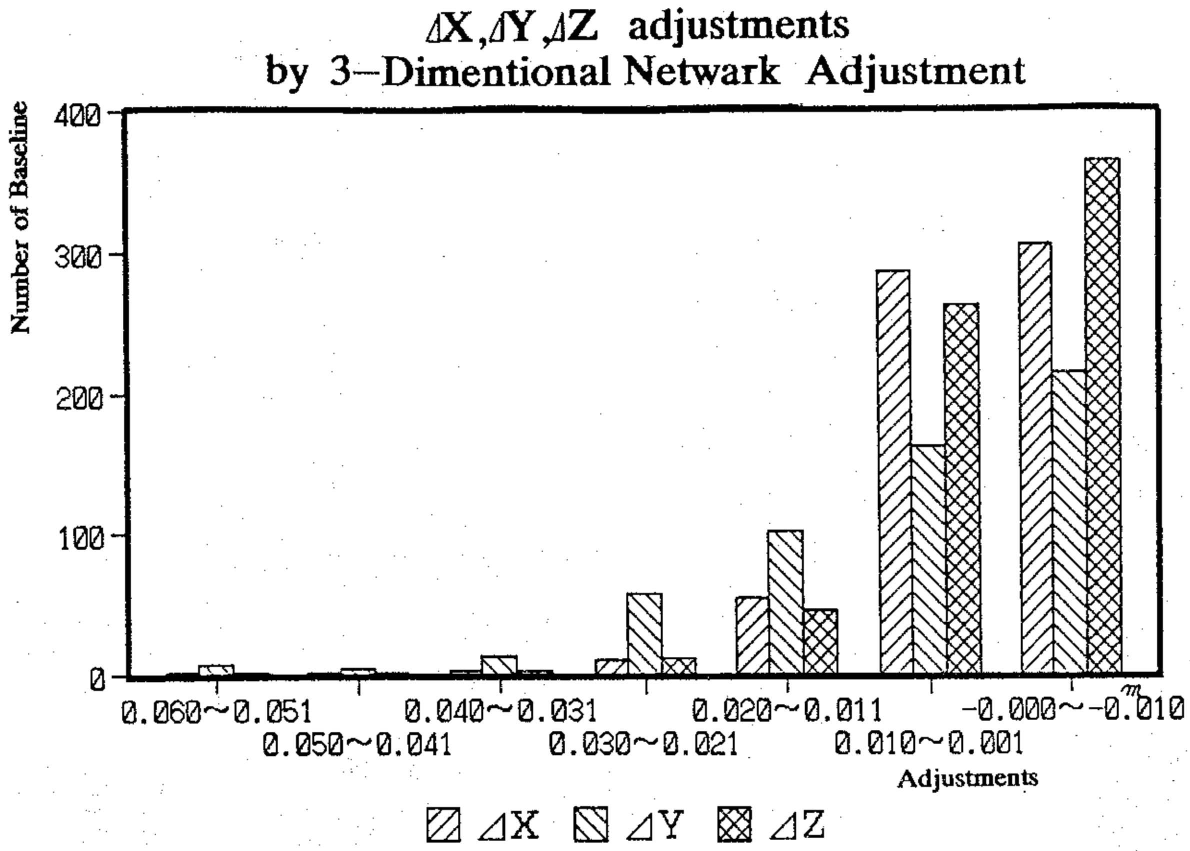
As for ΔY, the degree of compensation is large, and the number of baselines is also large. In other words, the values for Y obtained from observations are scattered over a wide area, and the precision of these observations is inferior to that of those for the values of X and Y. (See Figure 25)

One possible cause of this difference in precision may lie in the fact that the distribution of gravity in the area around India and Bangladesh is believed to be a zone of gravitational abnormaly, and these abnormal levels of gravitation might be believed to affect the orbit of the satellite from which observations are made, thus resulting in a discrepancy between the actual orbit of the satellite and the orbit as recorded in the broadcast almanac.

(2) Error ellipses

Control point positions may be estimated to possess an average accuracy of approximately 10 millimeters along the north-south axis, approximately 15 millimeters along the east-west axis, and approximately 20 millimeters along the vertical axis.

Note that the error spheres for points along the edges of the network are somewhat larger than that for other points, as may be seen in the graph showing the values of the square root of Q. (See Figure 26)



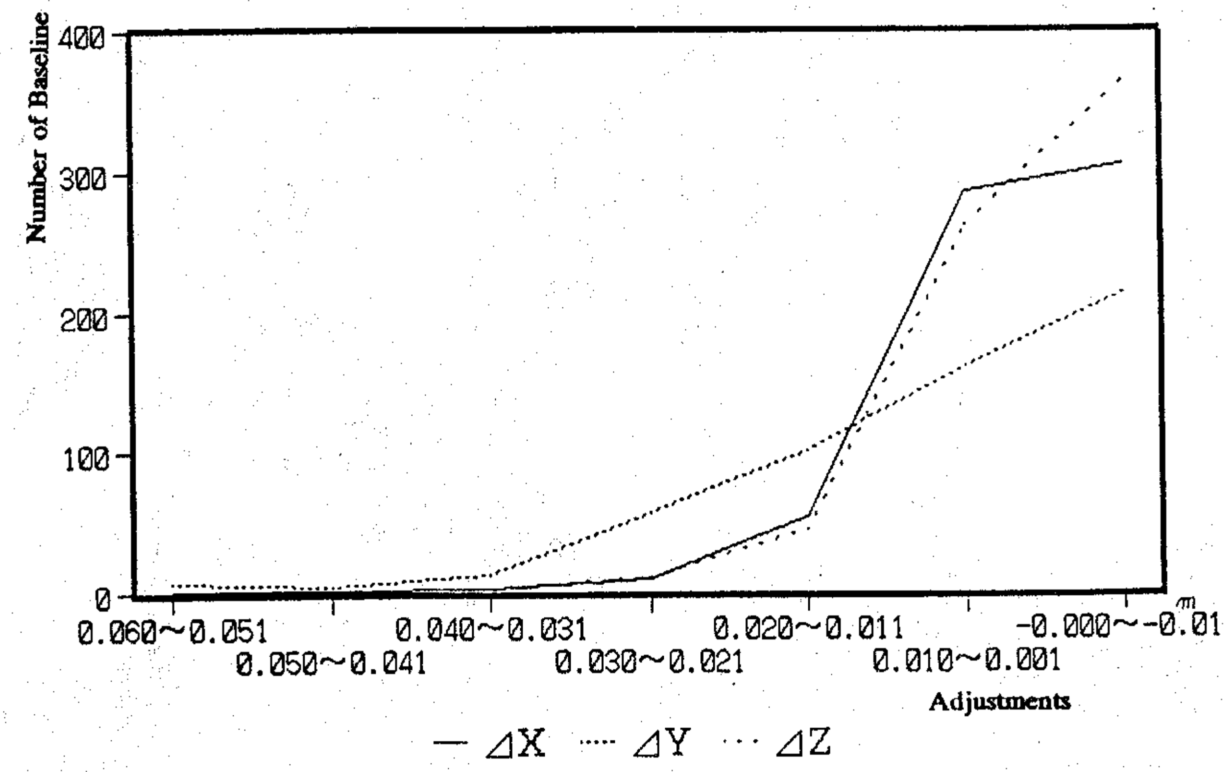


Figure 25 Degree of compensation using 3-dimensional net averages

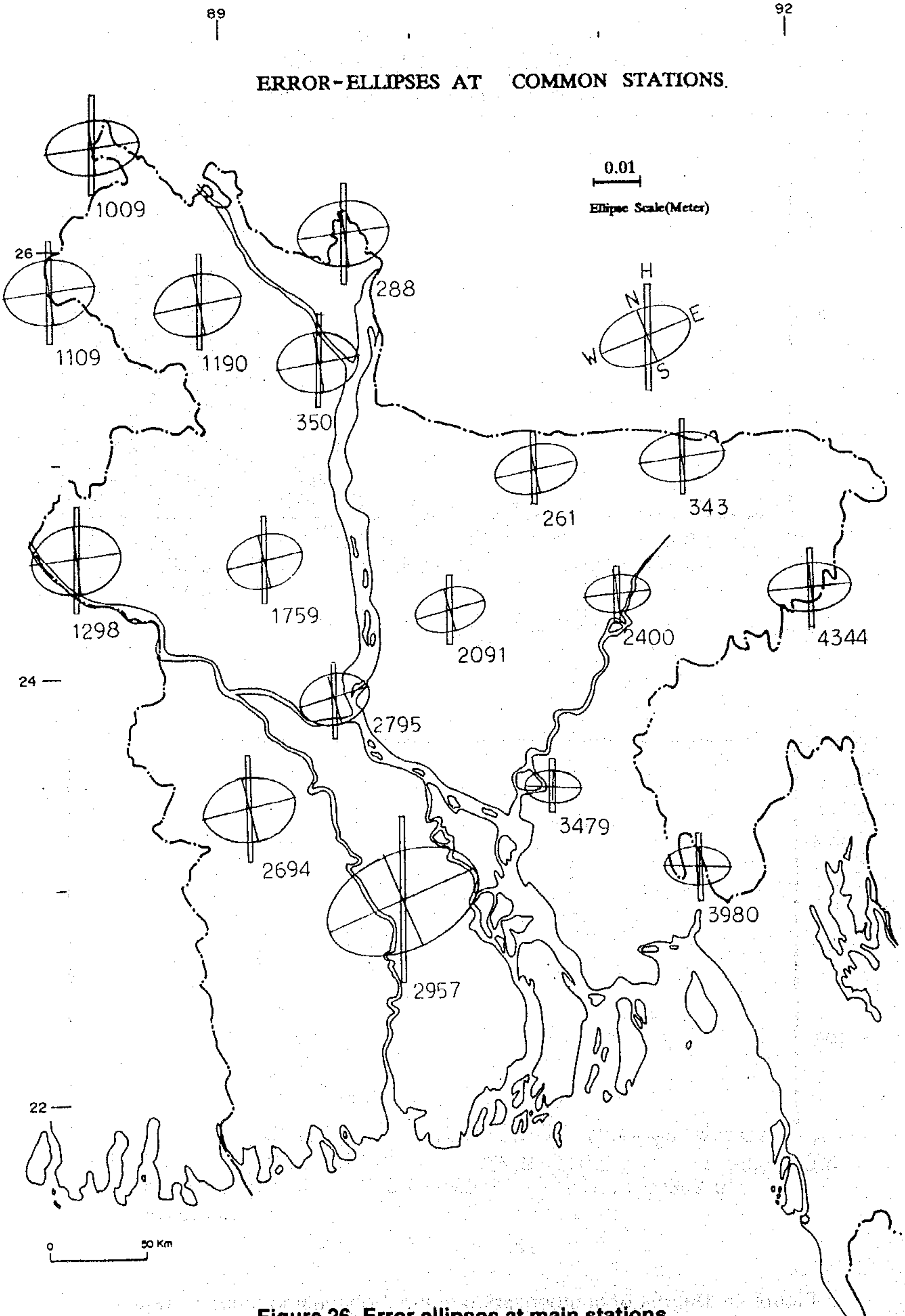


Figure 26 Error ellipses at main stations

7) Generating local geoid models and determining height of GPS stations

In performing this survey, since first-order bench marks were placed only by the shoulders of a portion of the main roads in the area covered in the survey, heights were given for all GPS control points set at approximately 1 point for every 30 kilometers.

It is for this reason that it was necessary to generate a local geoid model of the area being surveyed and to use GPS-derived levelling to calculate the height of those GPS stations for which it was impossible to calculate heights using ordinary levelling.

The relationship between the orthometric height (H), the ellipsoidal height (h), and the geoidal height can be understood in terms of the following equations:

$h = H + N$ or $H = h - N$

where

h: ellipsoidal height
H: orthometric height
N: geoidal height (undulation)

Note that while this relationship was illustrated in Figure 27, although the angle (ε) represents the angle of deflection from the vertical, this angle is of a sufficiently small degree so that the relationship may simply be viewed as being expressed by the equation $h = H + N$.

Figure 28 show the relationship between geoidal undulations in the region in and around Bangladesh and the World Geodetic System.

The area in and around Bangladesh is located in a plane located at a height of approximately 50 meters below that of the WGS-84 ellipsoid.

(1) Generation of local geoid model for Bangladesh

What may be directly surmised from the results of GPS observations is that the WGS-84 ellipsoidal height (h), and the orthometric height (H), and geoidal height (N) are all unknowns. Therefore, in order to obtain the orthometric height (H) of all GPS stations, it is necessary either to combine these values with level routes or to create an accurate geoid model to make it possible to determine the geoidal height (N) of all GPS stations.

In the current survey, 51 of the total of 141 GPS stations (52 if tide gauge stations are included) were joined to first-order benchmark lines using direct levelling, and with these points as givens, the Digital Terrain Model contouring method was employed to create equi-potential lines of the geoid model, and the remaining 89 points were then interpolated. (See Table 2 and Figure 29)

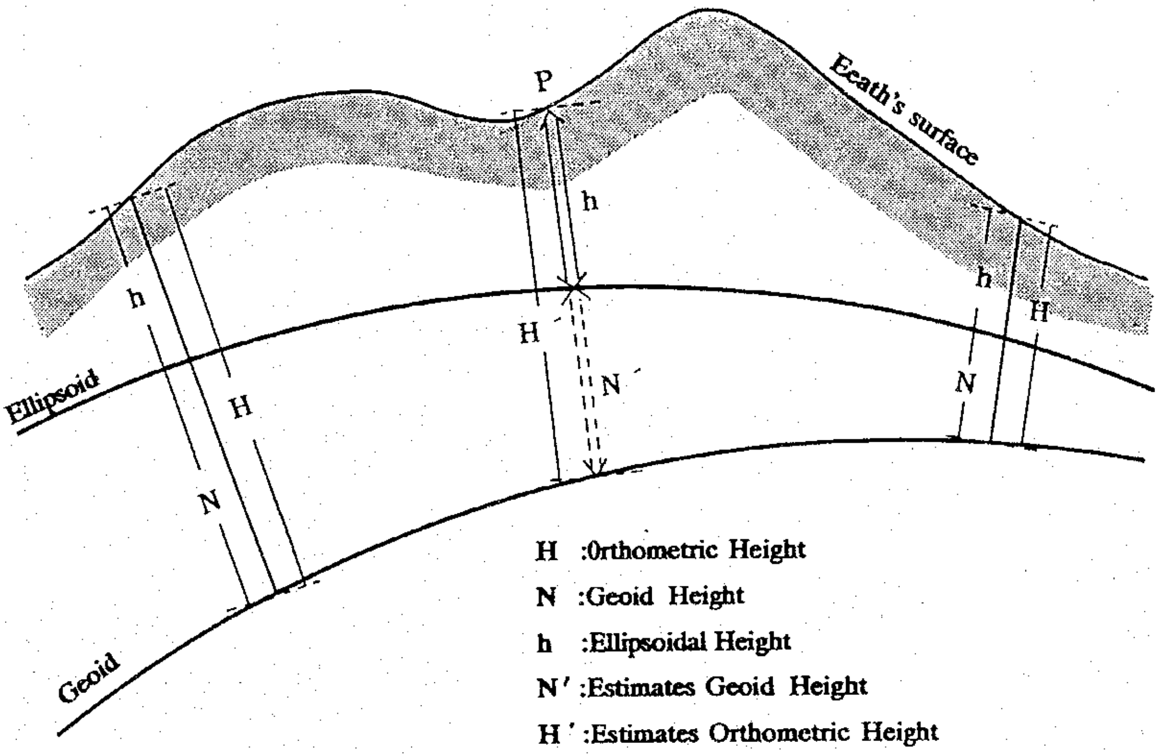
There were a total of 93 triangles used to connect the given points used to create the geoid model, and the equi-potential lines drawn around this set of triangles has been reported by the U.S.D.M.A. This data was then compiled using the values for the WGS-84 Geoid and the Geoid-91 A developed by Prof. Rapp of Ohio State University for reference.

(2) Display of values for orthometric height of GPS stations

While all 140 GPS stations were assigned values for their orthometric heights using the method described above, since the procedures followed in calculating this values differ and since this produces difference in the reliability of the results, the following classifications were used in displaying the results of these calculations.

GPS stations on first-order levelling lines:	0.0001 m (10^{-4} m)
GPS stations linked with BM by 3rd-order levelling:	0.01 m (10^{-3} m)
GPS stations interpolated from local geoid model:	0.1 m (10^{-1} m)

Orthometric Height Estimation
by GPS/Levelling



H :0rthometric Height

N :Geoid Height

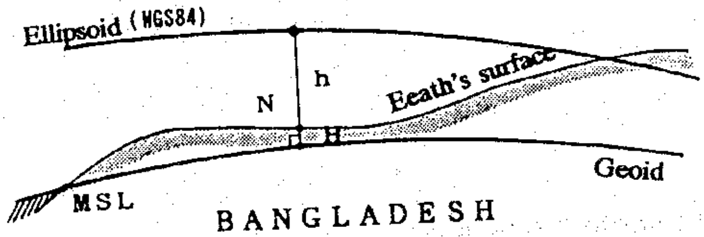
h :Ellipsoidal Height

N′ :Estimates Geoid Height

H′ :Estimates Orthometric Height

N=H−h

Orthometric Height at GPS station" P" H′ =N′+ h



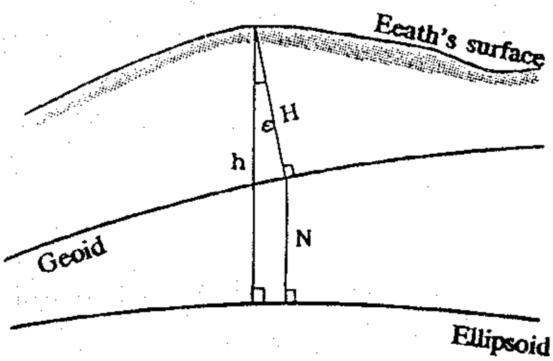


Figure 27 Orthometric hight estimation by GPS/Levelling

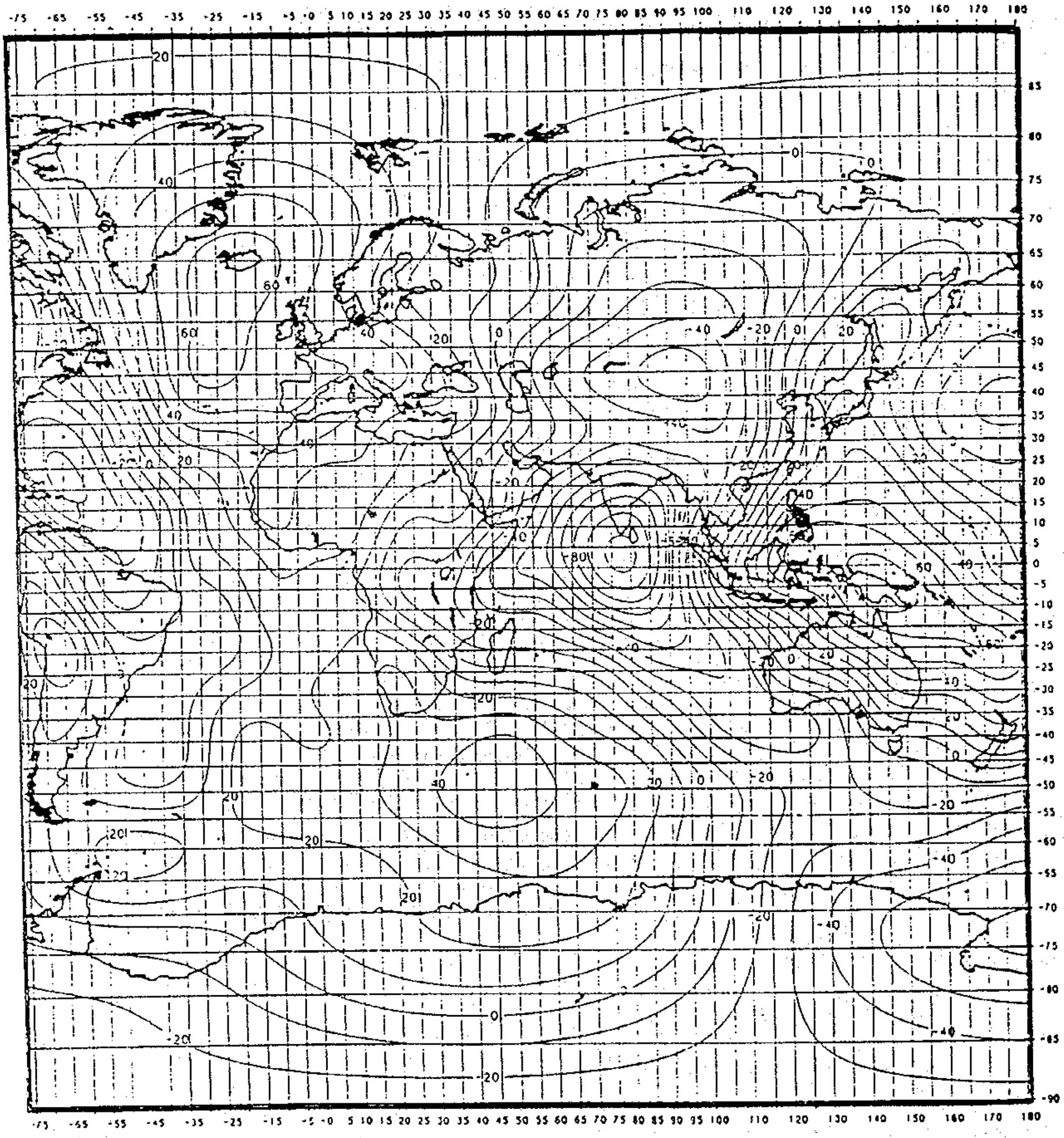


Figure 28 World Geoid Contour Line Map

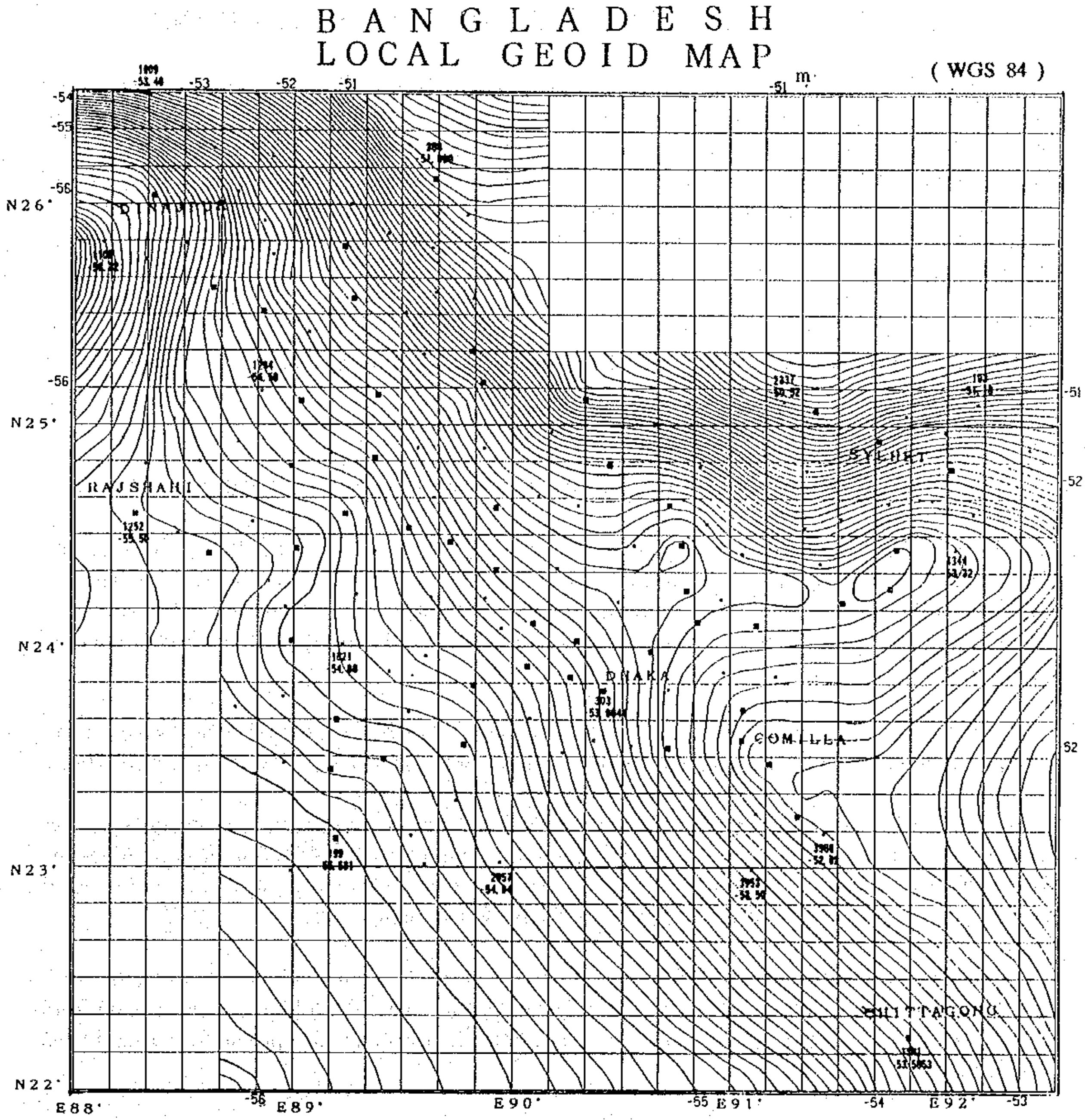


Figure 29 Bangladesh Geoid Map

3. SOME CONSIDERATIONS AND RECOMENDATIONS

The communication between the JICA Study Team and the SOB counterparts went smoothly and satisfactorily all through the study period, including the daily contact, confirmation, reporting on the study operations and technical consultations between the two parties. However, what were least expected occurred in the course of our entire project operations when we had to negotiate with the other government agencies in the matters related to the fulfillment of our duties or when we encountered irresistible difficulties and when we were involved in the employment trouble recuring local drivers.

We would like to refer to what we consider are useful and helpful in carrying out similar projects and what we expect of the SOB after handing over the final results of the study to Bangladesh as follows:

3-1 Point Locationing for Control Points and Bench Marks and Monumentation

Prior to the commencement of the point locationing survey, an official bulletin was issued by SOB to the heads of various governmental agencies such as District Commissioners and Chairmen of the Road & Highway Authority calling for their support and cooperation in the fieldwork of the JICA Study Team, stressing the aims and significance of the study and the importance of monumentation.

The Road & Highway Authority issued a certificate that authorized the JICA Study Mission to conduct surveying operations and copies of the certificate were distributed to regional branch offices.

The actual point locationing work was conducted in accordance with what were described under section: 2-1-4 and 2-1-5 as the precautions to be borne in mind, and the SOB counterparts helped the JICA Study Team obtain permission from the land owner to drive a wooden pile on the spot selected and prepare point records and get things ready for burying bench marks.

However, immediately before the monumentation work started in the Phase II study, the work was held in abeyance at several sites because the land custodians were told to get permission from the upper authority all over again even though they had already obtained approval for the exclusive use of the land.

The fault may be ascribable to the misunderstanding of the land custodians who had taken the size of a control point or a bench mark for a mere point when they were first told about it at the time of the point selection and failed to imagine the physical dimensions of such a structure at the time of its completion.

This gives us a lesson that sufficient information about the spot where a monument is laid, including the physical size and scale, should be provided at the time of the point selection to avoid such a confusion, and the point selection should be made by taking into consideration the status quo of the land in relation to the uses of the surrounding land.

3-2 Tidal Station Site Selection and Construction Work

In the stage of preliminary investigations a point adjacent to the mooring dolphin at the CUFL cargo loading pier was selected as the site for site a tidal station.

Prior to the commencement of the study work at the selected site, JICA Study Team sought approval from CUFL (Managing director) and failed to get it because of his administrative responsibility. We were inevitably obliged to transfer to the spot under the control of Chittagong Port Authority and it was about 60 m away from the mooring dolphin.

The change necessitated a full design modification as seen in the installation of an approach (a cat-walk pier) to the tidal station, which was otherwise unnecessary.

We would like to point out, however, that CUFL had been fully cooperative with us and favorably responded to our requirements except the above incident. CUFL assisted us in many ways on an agreement for providing us with services and conveniences in the installation of a bench mark as an annex facility to the tidal station, the work site office, the dormitory for workers, the supply of fresh water for concrete mixing, the transportation of construction materials, etc.

The auxiliary tidal station installed off Patenga Beach that directly faces the Bay of Bengal was destroyed by the floating substance (assumedly a vessel) when a cyclone hit the area in May of 1993, though it was structurally a temporal one. To continue tidal observation, the broken station has been reconstructed in the iron reinforced concrete and steel pipe structure surrounded by the three protective piles.

3-3 Levelling on the Railway Bridges

On the planned route for bench marks there are two bridges, Hardings Bridge and Ghorashal Bridge. The domestic laws of Bangladesh have designated these two bridges as key points for reasons of security and restrict the surveying by foreigners. Permission for surveying the railway bridges under regulations must be obtained first from the Ministry of Communication, National Railway, local police stations and then special permission from the Security Police of the Ministry of Home.

All of these proceedings were made by the SOB, but it was necessary to submit an application for permission about two months prior to the surveying work by designating the week when the work was slated to start.