

## 2.2.2 Control survey with traverses or triangulation

### 1) General

#### (Purpose)

"Control Survey" in this chapter are to establish the newly fundamental points based on national geodetic datum or existing control points.

#### (Methods of Survey)

Survey shall be carried out by the following method:

- (1) Method of Conjugated traverses
- (2) Method of Closed traverses (Net adjustment)
- (3) Method of Simple traverse
- (4) Method of triangulation

As a rule, (1) use for 1st and 2nd control point survey, (3) use for 3rd and 4th control point survey.

#### (Classes of Control Points)

Established control points are classified according to the accuracy of connected known points as follows:

Class	Kinds of known points	Standard length between known points	Standard length between newly established points
1st		4,000 m	1,000 m
2nd		2,000 m	500 m
3rd		1,500 m	200 m
4th		500 m	50 m

Note: In case of standard length between known point are more than 4,000 m, the 1st or 2nd control survey must be adopted.

#### (Accuracy of Control Point)

Accuracy of control point for traverse or triangulation shall be as follows:

Class	Relative accuracy
1st	higher than 1/
2nd	higher than 1/
3rd	higher than 1/

(under consideration)

(Main Instruments to be Used)

Main instruments to be used for this work are theodolite and electromagnetic distance measurement equipment (EDM).

The instruments to be used carefully after examining according to the inspection rules.

2) Planning for Control Survey

(Location of Control Point)

Newly control points shall be established at equal intervals.

The place of newly location shall be selected in good place for the utilization and conservation of points.

Planning shall be performed in consideration of the following condition:

(1) Traverses

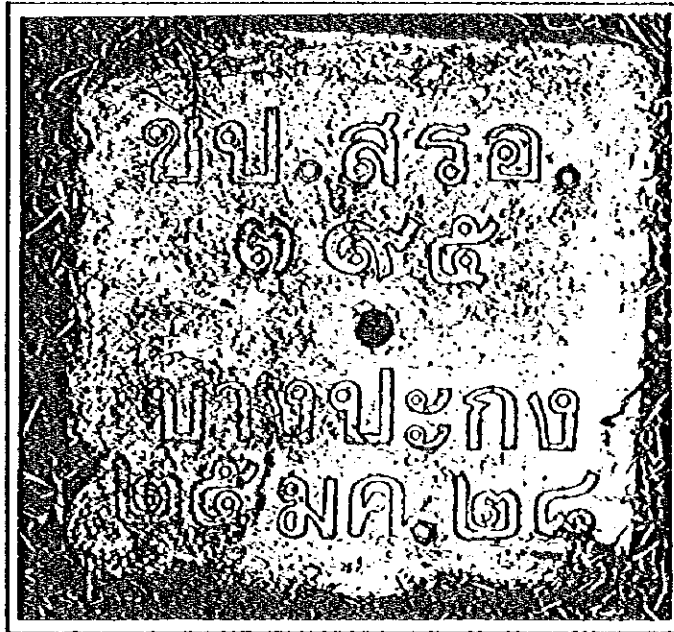
Item\Class	1st	2nd	3rd	4th
Number of known coordinate points	more than $2 + \frac{NP}{3}$	more than $2 + \frac{NP}{3}$	more than 2 points	
Number of known azimuth point				
Number of nodal point	less than 10 points	less than 12 points	less than 7 points	less than 10 points
Length between nodal points	more than 250 m	more than 150 m	more than 70 m	more than 20 m
Total length	less than 3000 m	less than 2000 m	less than 1000 m	less than 500 m

Note: NP = Total Model Points

(2) Triangulation

Item	1ST
Figure of triangle	<p>Angle of a triangle shall be more than 25 in principle.</p> <p>A regular triangle is desirable.</p> <p>Braced quadrilateral may be desirable for strengthening the figure in this case, an included angle may be more than 30 .</p> <p>Control point forming the triangles network shall be the intersection point where more than 3 sides intersect.</p>

SOME EXAMPLES OF CONTROL MONUMENT



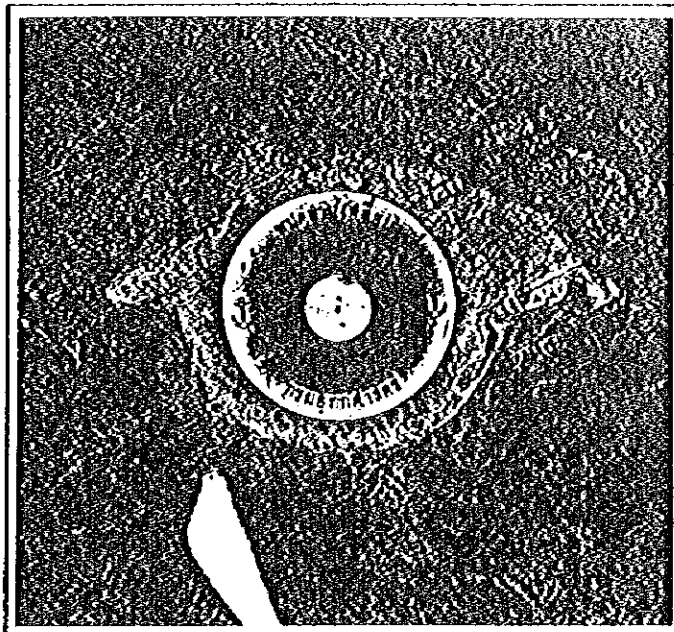
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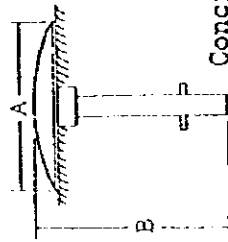
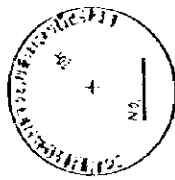
THE ROYAL IRRIGATION DEPARTMENT



HYDROGRAPHIC DEPARTMENT OF ROYAL THAI NAVY

Fig. 2-2.1 CONTROL POINT MONUMENTS

Form of Metal Monument



A = 8 CM  
B = 9 CM

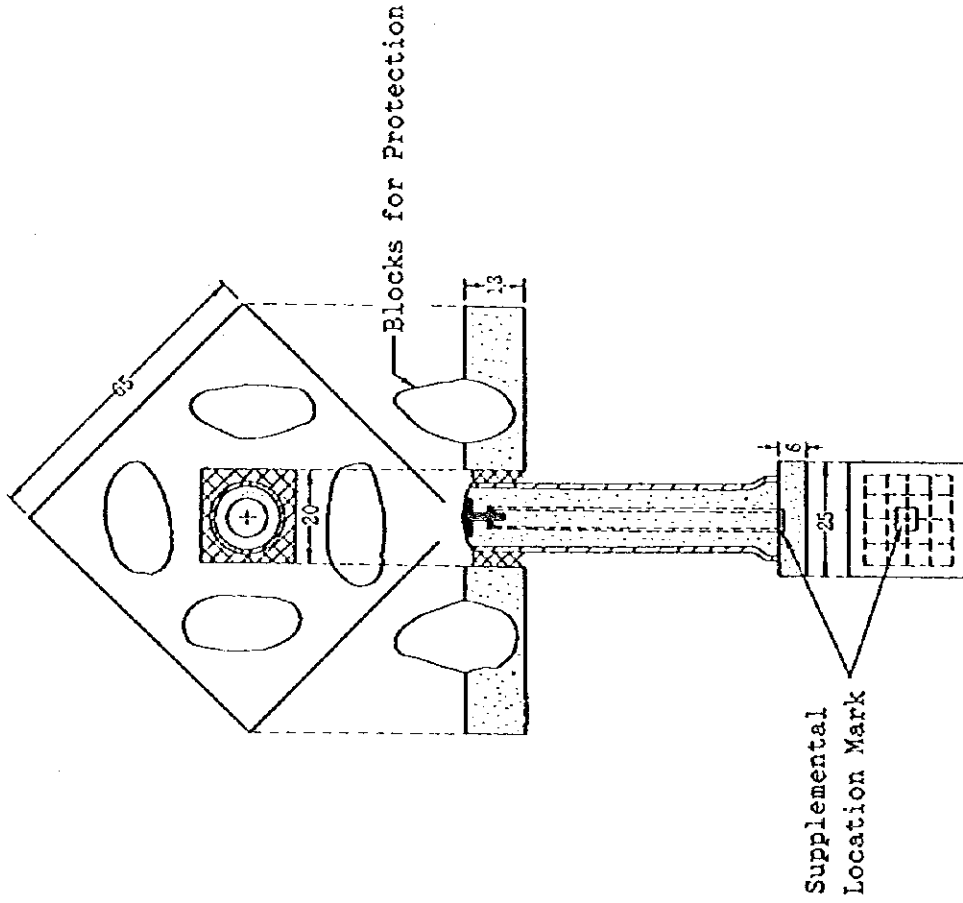
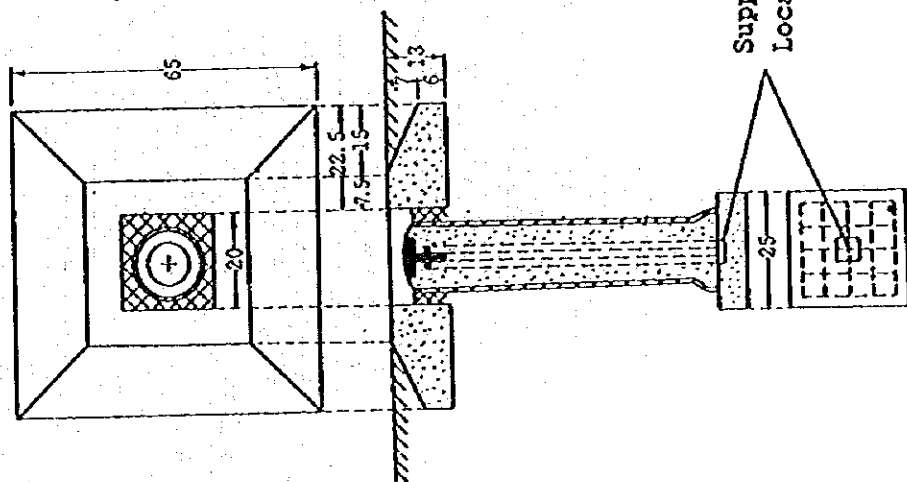
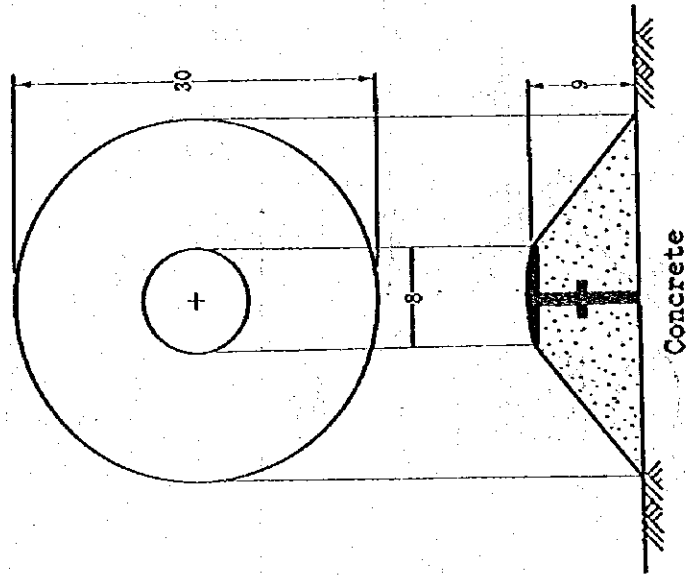


Fig. 2-2.2a STANDARD FORM OF CONTROL MONUMENT

Sample - 2 (Establishment in the Ground)



Sample - 3 (Establishment on the Top of Building)



Unit:cm

Fig. 2-2.2b STANDARD FORM OF CONTROL MONUMENT

## Adjustment of Traverse Net

### 1. General

A traverse consists of a series of straight line connecting successive established points along the route of a survey. The points defining the ends of the traverse lines are called traverse station or traverse points. Distances along the line between successive traverse points are determined either by direct measurement using a tape or electronic distance measuring (EDM) equipment. At each point where the traverse changes direction, an angular measurement is taken using a transit or theodolite.

Such traverse nets can include a large number of intersection points. If a strict analysis is wanted we can form all independent conditions according to "Method of variation (described the following page)".

### 2. Approximate Adjustment

For a number of applications a simplified adjustment of the traverse nets has been used. In several countries the angular condition adjustment is made in a primary step using the given misclosures in azimuths. After a computation of all corrected azimuths the misclosures of x and y are calculated. These misclosures are eliminated by independent adjustments for x and y. After completion of the adjustments all intersections will have their final coordinates as well as one azimuth defined. The adjustment inside the traverse can then be fulfilled using the method of simple traverse.

In this procedure we use the coordinates and one azimuth of each intersection point as unknown quantities. Also here a primary angular adjustment is made and then independent adjustments are made for x and y. From a theoretical point of view this procedure is inadequate because the coordinates and angles are correlated.

We obtain the following observation equations

$$\begin{array}{l} \psi_m - e_m = \hat{\psi}_m \quad (P_m = 1/m) \\ y_n - e_{yn} = \hat{y} \quad (P_n = 1/\Sigma d) \\ x_n - e_{xn} = \hat{x} \quad (P_n = 1/\Sigma d) \end{array}$$

where

- $\psi_m$  = azimuth of the intersection point (normally a direction to a neighbouring traverse point)
- $x_n, y_n$  = coordinates of the intersection point
- $\hat{x}, \hat{y}$  = adjusted coordinates of an intersection point
- $m$  = number of observed angles that are used to define the azimuth

In most cases the azimuths are given an a priori variance proportional to the number of observed angles. A coordinate determination can be

given an a priori variance that is proportional to the length of the traverse in question.

### 3. Example

We explain the method and procedure with the example.

Now, data for the adjustment are given in Table 2-2.1, and traverse nets are shown in Fig. 2-2.3.

Computer procedure are as follows:

- 1) Estimates the coordinates of intersection points from the simple traverse route.
- 2) Adjustment of azimuth from given azimuths and measured angles.
- 3) From the adjusted azimuth we take the coordinate errors ( x, y).

$$\Delta x = d \cos \psi \quad \Delta y = d \sin \psi$$

$W_x$  = misclosure in x-values

$W_y$  = misclosure in y-values

$$W_r = \sqrt{W_x^2 + W_y^2}$$

$W_x$  and  $W_y$  are compensated by corrections proportional to their lengths.

- 4) Error equation and its solutions

The differentiated observation equations are given in Table 2-2.3 and the solution follows from the Table.

$$\hat{\psi}_1 = 104^\circ 09' 12''$$

$$\hat{\psi}_2 = 3^\circ 15' 26''$$

- 5) Adjustment of coordinates

The approximate values are

$$x^{(85)} = 35,952.911 \quad x^{(84)} = 35,952.459$$

$$y^{(85)} = 67,047.810 \quad y^{(84)} = 66,880.969$$

The differentiated observation equations are given in Table 2-2.3.

$$\begin{array}{ll} y(85) = 67,047.856 \text{ m} & y(84) = 66,880.945 \text{ m} \\ x(85) = 35,952.927 \text{ m} & x(84) = 35,952.396 \text{ m} \end{array}$$



The numerical results for the individual traverses are given in Table 2-2.3.

The procedure by computer, and its results are shown in the following page.



Table 2-2.2 OBSERVED QUANTITIES OF TRAVESES I-V

Traverse	Point	Angle	Length (m)
I	14		
	19	125° 22' 15"	85.167
	1941	131° 18' 00"	91.903
	1942	155° 55' 59"	93.914
	1943	141° 36' 17"	80.012
	84	309° 39' 37"	
	5427		
II	20		
	54	143° 49' 41"	134.308
	5421	144° 25' 58"	154.871
	5422	194° 31' 19"	111.154
	5423	191° 01' 08"	127.229
	5424	236° 19' 12"	108.628
	5425	178° 47' 20"	127.906
	5426	182° 36' 55"	110.434
	5427	177° 20' 09"	123.196
III	1943		
	84	205° 42' 02"	166.934
	85	76° 02' 32"	
	5903		
IV	19		
	59	298° 31' 58"	81.960
	5901	191° 11' 29"	82.843
	5902	179° 39' 06"	83.380
	5903	151° 29' 32"	87.872
	85	107° 01' 23"	
	5502		
V	19		
	55	10° 37' 52"	93.584
	5501	157° 29' 40"	104.991
	5502	182° 02' 21"	154.276
	85		

Table 2-2.1 INPUT DATA FOR A TRAVERSE ADJUSTMENT

Known Quantities

Point	Bearing	Coordinates	
		(x)	(y)
19	A19-14=140° 17' 04"	66637.477	35825.923
54	A54-20= 95° 19' 08"	67140.219	36748.350
59	A59-19=155° 21' 51"	67392.556	36001.454
55	A55-19=193° 05' 13"	67075.237	35625.169

Unknown Quantities

Point	Bearing	Coordinates	
		(x)	(y)
84	A84-85= -	-	-
85	A85-5903= -	-	-

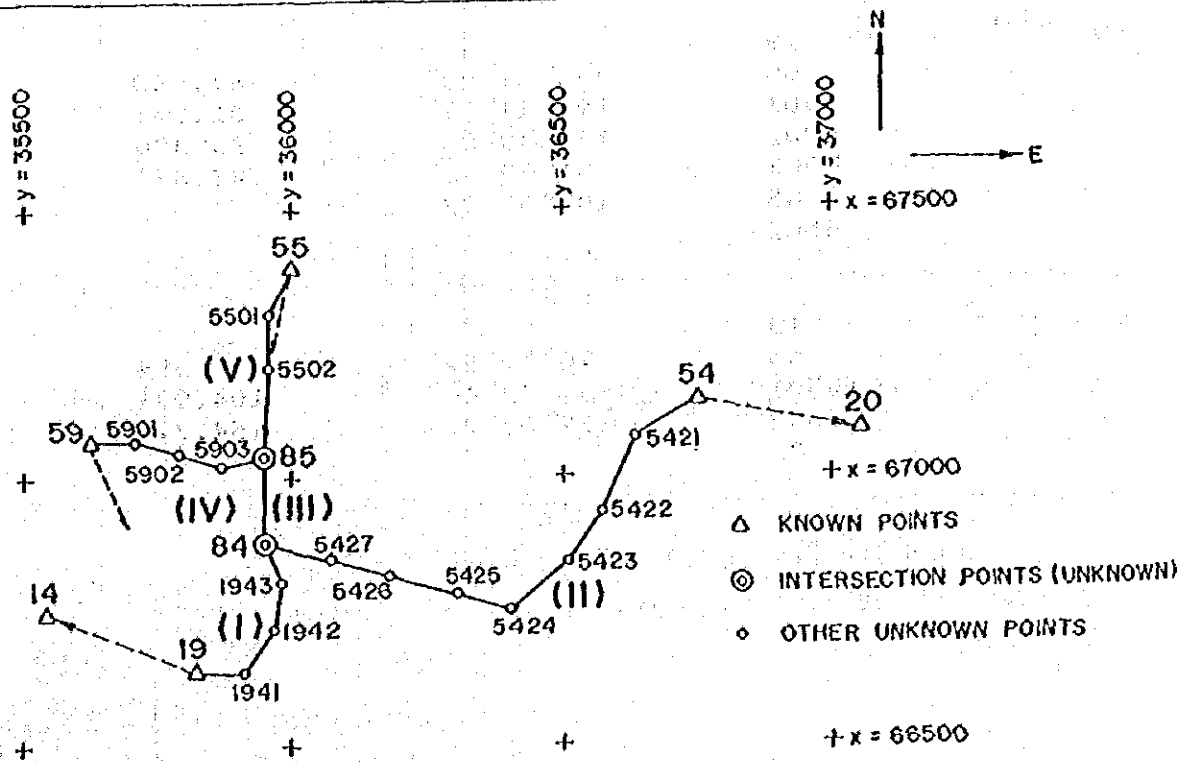


Fig. 2-2.3 TRAVERSE NET WITH FIVE POLYGONS

**ADJUSTMENT OF TRAVERSES**

**NAME OF AREA :**

**TYPE OF TRAVERSES :** II TYPE

**NAME OF ROUTE :** T001 T002 T004 T005 T003

**MEAN SQUAR ERROR:**

**ANGLE = 0 60**

**DISTANCE= 0.141 (M)**

**DN(X) = 0.064 (M)**

**DE(Y) = 0.125 (M)**

**DATE : / /**

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ADJUSTMENT OF TRAVERSES ( H TYPE )

ROUTE NO	T001	ANGLE	BEARING	DISTANCE	(N)	(E)
0 ( )			320 17 4			
0 ( 19)		125 22 15			66637.477	35825.923
1 (1941)		131 18 0	85 39 20	85.167	66643.924	35910.832
2 (1942)		155 55 59	36 57 21	91.703	66717.196	35965.949
3 (1943)		141 36 17	12 53 22	93.914	66808.735	35986.884
( 84)		309 39 37	334 29 40	80.012	66880.936	35952.407
( )			104 9 18			
OBSERVED	VALUES		104 9 12	350.796	66880.969	35952.459
CORRECTED	VALUES		0 6	0.062	-0.033	-0.052
ACCURACY				( 1 / 5676 )		
LIMIT			0 10	0.112		

ADJUSTMENT OF TRAVERSES ( H TYPE )

ROUTE NO	T002					
NO	NAME	ANGLE	BEARING	DISTANCE	(N)	(E)
	( )		95 19 8			
0	( 54 )	143 49 41	239 8 39	134.308	67140.219	36748.350
1	(5421)	144 26 58	203 34 27	154.871	67071.324	36633.045
2	(5422)	194 31 19	218 5 35	111.154	66929.375	36571.105
3	(5423)	191 1 8	229 6 33	127.229	66841.890	36502.526
4	(5424)	236 19 12	285 25 35	108.628	66758.595	36406.342
5	(5425)	178 47 20	284 12 45	127.906	66787.477	36301.612
6	(5426)	182 36 56	286 49 29	110.434	66818.865	36177.602
7	(5427)	177 20 9	284 9 28	123.196	66850.817	36071.879
	( 84 )	360 0 0	104 9 18		66880.936	35952.407
	( )					
OBSERVED VALUES			104 10 50	997.726	66880.917	35952.516
CORRECTED VALUES			- 1 32	0.110	0.018	-0.109
ACCURACY				( 1 / 9032 )		
LIMIT			0 26	0.153		

ADJUSTMENT OF TRAVERSES ( II TYPE )

ROUTE NO	T004				(N)	(R)
NO	NAME	ANGLR	BEARING	DISTANCE		
0	( 59 )	298 31 58	155 21 51		67075.237	35625.169
1	(5901)	191 11 29	93 53 50	81.960	67069.681	35706.944
2	(5902)	179 39 6	105 5 21	82.843	67048.131	35786.935
3	(5903)	151 29 32	104 44 28	83.380	67026.931	35867.576
	( 85 )	107 1 23	76 14 1	87.872	67047.856	35952.927
	( )		3 15 26			
OBSERVED VALUES			3 15 19	336.055	67047.810	35952.911
CORRECTED VALUES			0 7	0.049	0.046	0.016
ACCURACY				( 1 / 6863 )		
LIMIT			0 10	0.112		



ADJUSTMENT OF TRAVERSES ( H TYPE )

ROUTE NO		T005					
NO	NAME	ANGLE	BEARING	DISTANCE	(N)	(E)	
0	( 55 )	10 37 52	193 5 13		67392.556	36001.454	
1	(5501)	157 29 40	203 43 10	93.584	67306.868	35963.837	
2	(5502)	182 2 21	181 12 55	104.991	67201.894	35961.642	
	( 85 )	360 0 0	183 15 21	154.276	67047.856	35952.927	
	( )		3 15 26				
OBSERVED VALUES			3 15 6	352.851	67047.878	35952.853	
CORRECTED VALUES			0 20	0.078	-0.022	0.074	
ACCURACY				( 1 / 4545 )			
LIMIT				0 9	0.110		

**ADJUSTMENT OF TRAVERSES (H TYPE)**

ROUTE NO	T003	ANGLE	BEARING	DISTANCE	(N)	(E)
( )			154 29 40			
0 ( 84 )	205 42 2		0 11 36	166.934	66880.936	36952.407
( 85 )	76 2 32		256 14 1		67047.856	36952.927
( )						
<b>OBSERVED</b>	<b>VALURS</b>		256 14 9	166.934	67047.902	36953.022
<b>CORRRCTED</b>	<b>VALURS</b>		- 0 13	0.045	-0.013	-0.043
<b>ACCURACY</b>				(1 / 3720 )		
<b>LIMIT</b>			0 5	0.100		

### Method of Variation (or Variations of Parameters)

This method be used to determine the most probable value of unknowns that are functions of excessive observations and to estimate the error of the solution as well as that of the observations.

#### (1) The fundamental principle

Now we consider the following equation:

$$l_i - E_i = f_i(x_1, x_2, \dots, x_m) \quad (i = 1, 2, \dots, n) \quad (2-2.1)$$

where

$$\begin{aligned} l_i &= \text{observation (i-th value)} \\ E_i &= \text{error (i-th value)} \\ f_i(\quad) &= \text{function} \\ x_1, x_2, \dots, x_m &= \text{unknown quantities} \end{aligned}$$

We are obtain the error estimate after series expansion according to Tailor's as indicated in equation ( 2.2).

$$l_i - i = f((x_1), (x_2), \dots, (x_n)) + \frac{\partial f_i}{\partial x_1} \Delta x_1 + \frac{\partial f_i}{\partial x_2} \Delta x_2 + \dots + \frac{\partial f_i}{\partial x_m} \Delta x_m \quad (2-2.2)$$

where

$$\begin{aligned} (x_1), (x_2), \dots, (x_m) &= \text{approximate values of the unknown} \\ \Delta x_1, \Delta x_2, \dots, \Delta x_m &= \text{corrections to the approximate values} \end{aligned}$$

We introduce the following parameter

$$(l_i) = f_i((x_1), (x_2), \dots, (x_m)) \quad (2-2.3)$$

$$a_{i1} = \frac{\partial f_i}{\partial x_1}$$

$$a_{i2} = \frac{\partial f_i}{\partial x_2}$$

Now high order terms were neglected, then we can be transcribed the next equations.

$$l_i - (l_i) - E_i = a_{i1} \Delta x_1 + a_{i2} \Delta x_2 + \dots + a_{im} \Delta x_m \quad (2-2.4)$$

Further more we introduce an additional parameter.

$$l_i - (l_i) = \Delta l_i \quad (2-2.5)$$

This parameter means the difference between the observation and approximate values.

And, we have the final error equations.

$$-E_1 = a_{11}\Delta x_1 + a_{12}\Delta x_2 + \dots + a_{1m}\Delta x_m \quad (2-2.6)$$

If we can solve the above equations for  $\Delta x_1, \Delta x_2, \dots, \Delta x_m$ , new estimate values are as follows:

$$x_1 = x_1 + \Delta x_1, \quad x_2 = x_2 + \Delta x_2, \quad \dots, \quad x_m = x_m + \Delta x_m \quad (2-2.7)$$

And, if  $\Delta x_1, \Delta x_2, \dots, \Delta x_m$  make less than allowance, these estimate are suppose the most probable values of unknowns.

This is presently most useful adjustment method for triangulation, traverses, trilaterations, and above mixed surveys.

Error equations for angle observations and solution

We consider the plane coordinate system.

First, therefore, observed directions must be reduce to their projection on a plane by applying an "arc to chord" correction, i.e., (t-T) correction.

$$t_{AB} - T_{AB} = - (1/4 R_o^2 M_o^2) (X_A + X_B)(Y_B - Y_A) + (1/12 R_o^2 M_o^2) (Y_B - Y_A)(X_B - X_A) \quad (2-2.8)$$

where

- $R_o$  = Mean radices on the origin point
- $M_o$  = Scale factor on the central meridian

Fig. -1 Illustrated this conversion for a transverse Mercator Projection.

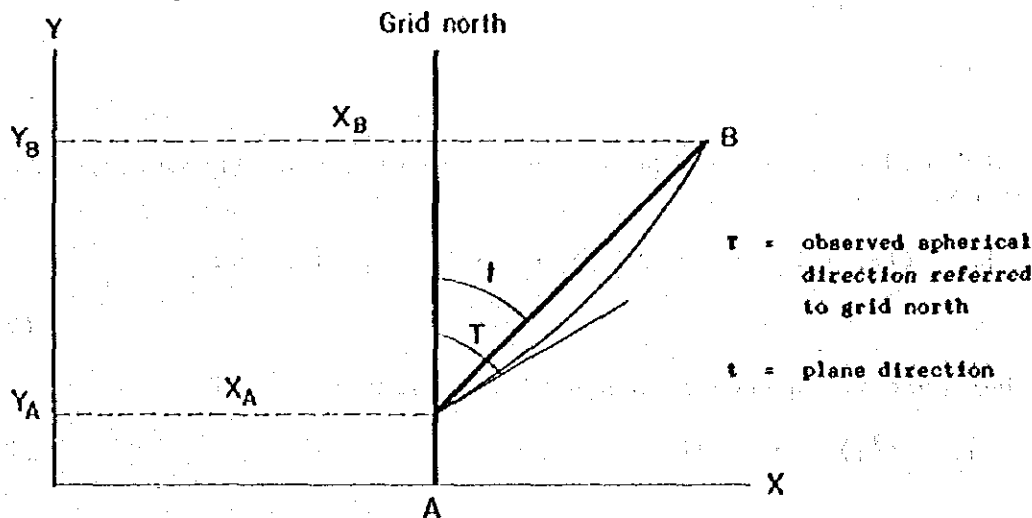


Fig. 2-2.4 "arc to chord" CORRECTION ON THE UTM GRID

In general case we consider a bearing (or direction) between two unknown point A and B.

Grid bearing may be written in the follows:

$$\tan t = \frac{X_B - X_A}{Y_B - Y_A} \quad (2-2.9)$$

Differentiation above equation,

$$\sec^2 t \, dt (Y_B - Y_A) + \tan t (dY_B - dY_A) = dX_B - dX_A \quad (2-2.10)$$

Rearrange the above equation,

$$dt = \cos^2 t \frac{dX_B - dX_A}{Y_B - Y_A} - \cos t \sin t \frac{dY_B - dY_A}{X_B - X_A} \quad (2-2.11)$$

Nothing that

$$\sin t = \frac{X_B - X_A}{S} \quad \text{and} \quad \cos t = \frac{Y_B - Y_A}{S} \quad (2-2.12)$$

where S is the distance between A and B, and we obtain the error equation as shown in the follows:

$$dt = -\frac{(Y_B - Y_A)}{S} dx_A + \frac{X_B - X_A}{S} dy_A + \frac{Y_B - Y_A}{S} dx_B - \frac{X_B - X_A}{S} dy_B \quad (2-2.13)$$

This equation represents the variation in a computed direction in radians as a function of coordinate change.

Above equation can make for one directed observation.

If point A or B are known position, then  $dx$ , and  $dy$  make to zero because the corrections are zero.

Making the normal equations from the error equations, then we can solve for  $dx_A$ ,  $dx_B$ ,  $dy_A$ , and  $dy_B$ .

We introduce one more parameter  $Z_i$ .

This parameter is meaning the station correction for station  $i$ , observing station.

Therefore, the Relation between observation angle and correction is represents the equation (2-2.14)

$$V_{ij} = L_{ij} - (\lambda_{ij} + Z_1) \quad (2-2.14)$$

$V_{ij}$  designates the correction to observations used to obtain consistency these corrections are independent as are the individual observation.

$Z_1$  is error for initial direction, and is affected for all observed directions.

Fig. -2 show the above relations.

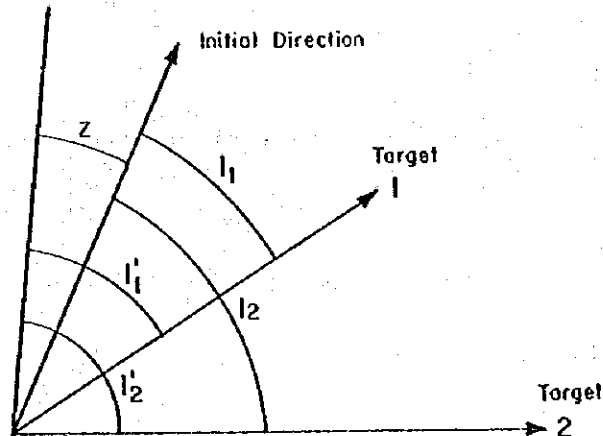


Fig. 2-2.5 OBSERVED ANGLES ( $\lambda_1$ ) AND CORRECTION

The random error portion of  $v$  must still be accommodated.

The correction  $dt$  to the computed observation must result in the same value as the correction to the actual observation, that is,

$$\lambda_{ij}^0 + dt = L_{ij} = \lambda_{ij} + V_{ij} + Z_{ij} \quad (2-2.15)$$

After insert this equation into equation (2-2.15), we obtain the general observation equation for one direction.

$$V_{ij} = -Z_1 - a_{ij} \Delta x_1 - b_{ij} \Delta y_1 + a_{ij} \Delta x_j + b_{ij} \Delta y_j - \lambda_{ij} \quad (2-2.16)$$

where

$\Delta x, \Delta y$  = correction to station coordinate

$Z_1$  = orientation error

and

$$a_{ij} = \frac{Y_j - Y_1}{S_{ij}^2} \rho'' \quad b_{ij} = - \frac{X_j - X_1}{S_{ij}^2} \rho''$$

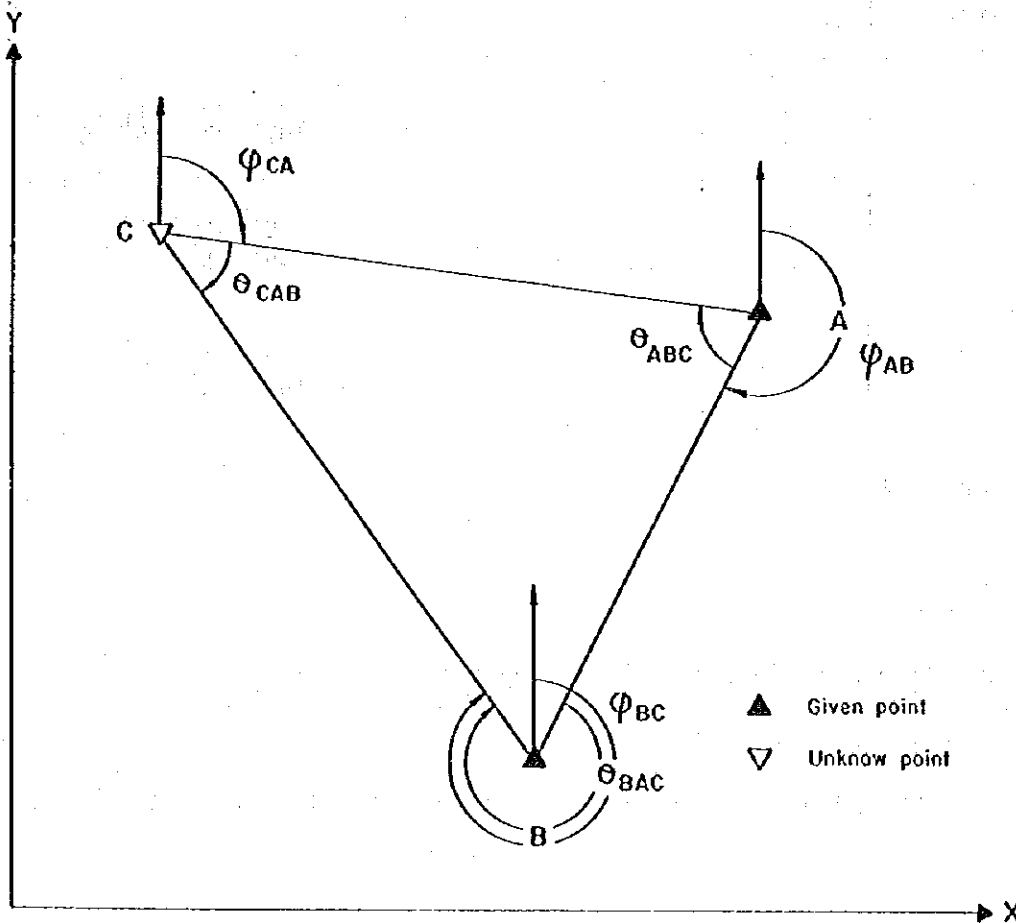
$\rho''$  = arc-second equivalent to one radian (206265")

We explain about the adjustment of directions in triangulation network with example below. (See Fig. 2-2.6)

1) Coordinates of triangulation station A, B and C

Station	y	x
A	67318.56	46608.05
B	63522.27	41961.53
C	(72144.70)	(40325.00)

Note : Coordinate of station C use the approximate values.



$\theta_{ijk}$  : observed angle from j to k at point i  
 $\phi_{ij}$  : Direction angle from i to j

Fig. 2-2.6 INTERSECTION OF A POINT C FROM TWO GIVEN POINTS A AND B

2) Computation of Lengths and Directions

Points	Difference of Coordinates		Distance S	Directions ( $\psi$ )
	$\Delta y$	$\Delta x$		
A - B	3796.23	4646.52	6000.125	230° 45' 04"
A - C	-4826.14	6283.05	7922.647	307° 31' 43"
B - C	-8622.43	1636.53	8776.362	349° 15' 11"

3) Observed Angles and Approximate Directions

Observed points	Object point	Observed angle	Approximate directions
A	B	0° 0' 0"	230° 45' 04"
	C	76° 46' 42"	307° 31' 46" *
B	A	0° 0' 0"	50° 45' 04"
	C	298° 30' 10"	349° 15' 14"
C	A	0° 0' 0"	127° 31' 43"
	B	41° 43' 10"	169° 14' 53" *

\* approximate direction from observation.

4) Error equation from above tables

Standard formula is given from equation (2-2.17).

$$V_{ij} = -Z_i - a_{ij} \Delta x - b_{ij} \Delta y + l_{ij} \quad (2-2.17)$$

. Coefficients  $a_{ij}$  and  $b_{ij}$  are obtain the following table.



Points	$a_{ij}$	$b_{ij}$	$l_{ij}$
A → B	-	-	= 0
A → C	$\frac{4826.14}{7922.6472} \rho''$ = 15.8593	$\frac{6283.05}{7922.6472} \rho''$ = -20.6469	307° 31' 46" 307° 31' 43" = 3
B → A	-	-	= 0
B → C	$\frac{8622.43}{8776.3622} \rho''$ = 23.0901	$\frac{163.53}{8776.3622} \rho''$ = -4.3825	= 3"
C → A	-15.8593	+20.6469	= 0
C → B	-23.0901	+44.3825	= -18"

Then, the error (observation) equations are given below according to equation (2-2.14).

$$\begin{aligned}
 \text{for point A} \quad v_{AB} &= -Z_A + \quad \quad \quad + \quad \quad \quad + 0.0 \\
 v_{AC} &= -Z_A + 15.8593 \Delta x - 20.6469 \Delta y + 3.0 \\
 \\
 \text{for point B} \quad v_{BA} &= -Z_B + \quad \quad \quad + 0.0 \\
 v_{BC} &= -Z_B + 23.0901 \Delta x - 4.3825 \Delta y + 3.0 \\
 \\
 \text{for point C} \quad v_{CA} &= -Z_C - 15.8593 \Delta x + 20.6469 \Delta y + 0.0 \\
 v_{CB} &= -Z_C - 23.0901 \Delta x + 4.3825 \Delta y - 18.0
 \end{aligned}$$

Note : -  $Z_A$ ,  $Z_B$ , and  $Z_C$  : orientation error at each point

(2-2.18)

If we use the matrix notation for above equations, the observation equations are given below:

$$\begin{vmatrix} -1 & 0 & 0 & 0.0 & 0.0 \\ -1 & 0 & 0 & 15.8593 & -20.6469 \\ 0 & -1 & 0 & 0.0 & 0.0 \\ 0 & -1 & 0 & 23.0901 & -4.3825 \\ 0 & 0 & -1 & -15.8593 & 20.6469 \\ 0 & 0 & -1 & -23.0901 & 4.3845 \end{vmatrix} = \begin{vmatrix} -Z \\ -Z \\ -Z \\ \Delta x \\ \Delta y \end{vmatrix} = \begin{vmatrix} 0.0 \\ 3.0 \\ 0.0 \\ 3.0 \\ 0.0 \\ -18.0 \end{vmatrix}$$

(2-2.19)

We can obtained the corrections for approximate values. After solving the error equation by mean of the method of least squares, and corrections for x and y are as follows:

$$\Delta x = 0.59, \quad \Delta y = -0.35$$

and the estimated values are as follows:

$$\begin{aligned} x &= (72144.70) + 0.59 = 72145.29 \\ y &= (43025.00) - 0.35 = 40324.66 \end{aligned}$$

Error equations for distances measurements and solution.

Recently the distance measurement using electro-optical instruments has received practical application. Distances of up to 2 and more kilometers can be measured directly, and its accuracy is higher in proportional to the distance than angle observation.

In modern survey system, therefore, the trilateration is often utilized in stead of Angle observation.

Now we consider the plane coordinate system. First, measured distances should be reduce to their projection on a plans by applying an spherical distance (S) to plane distance (s) correction, i.e., (s/S) correction.

Its formula is indicated as follows:

$$(s/S)_{AB} = M_0 (1 + (1/6 R_0^2 M_0^2) (X_A^2 + X_A X_B + X_B^2)) \tag{2-2.20}$$

where

- $M_0$  = Scale factor on the central meridian
  - in case of UTM-grid = 0.9996
  - in case of state coordinate and X-ordinates less than 100 km = 0.9999
  - in case of local coordinate = 1.0000

$R_0$  = Mean radius on the origin point

Fig. -2 Illustrates this conversion.

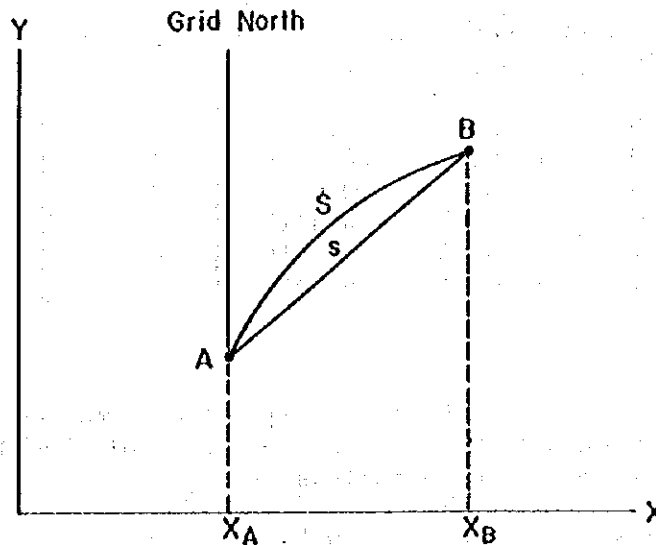


Fig. 2-2.7 SPHERICAL DISTANCE (S) AND PLANE DISTANCE (S)

Therefore, we have plane distance (s) from equation (2-2.21).

$$S = m_0 S + m_0 S (1/6 R_0^2 m_0) (X_A^2 + X_A X_B + X_B^2) \quad (2-2.21)$$

Hereafter S is treated as plane distance.

Fig. 2-2.8 illustrate the trilateration from a given point A to an unknown point B.

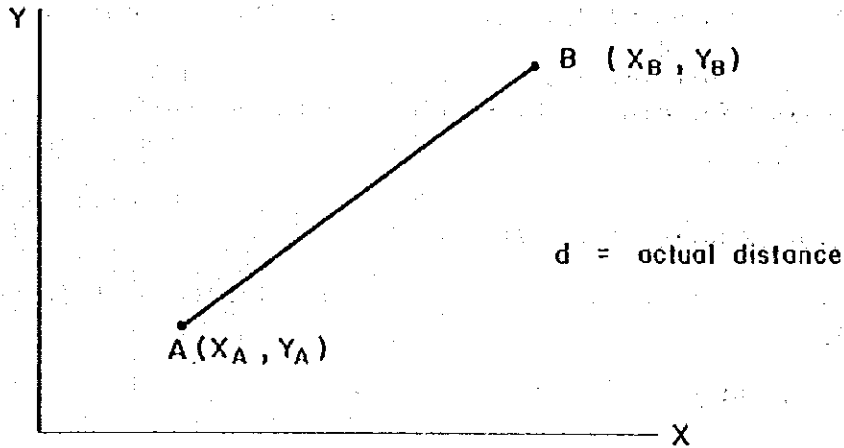


Fig. 2-2.8 TRILATERATION BETWEEN A AND B

In a distance measurement between point A and point B we use the following relation.

$$S_{AB}^2 = (X_B - X_A)^2 + (Y_B - Y_A)^2 \quad (2-2.22)$$

differentiation above equation by  $X_A$ ,  $Y_A$ ,  $X_B$  and  $Y_B$  we obtain the next equation.

$$dS_{AB} = S_{AB} - (S_{AB}) = \frac{\partial S_{AB}}{\partial X_A} dX_A + \frac{\partial S_{AB}}{\partial Y_A} dY_A + \frac{\partial S_{AB}}{\partial X_B} dX_B + \frac{\partial S_{AB}}{\partial Y_B} dY_B \quad (2-2.23)$$

where  $(S_{AB})$  = approximate computed distance Derivatives:

$$\frac{\partial S_{AB}}{\partial X_A} = -\frac{X_B - X_A}{S_{AB}}, \quad \frac{\partial S_{AB}}{\partial Y_A} = -\frac{Y_B - Y_A}{S_{AB}}$$

$$\frac{\partial S_{AB}}{\partial X_B} = \frac{X_B - X_A}{S_{AB}}, \quad \frac{\partial S_{AB}}{\partial Y_B} = \frac{Y_B - Y_A}{S_{AB}}$$

Now we introduce the parameters

$$a = \frac{X_B - X_A}{S_{AB}} \quad b = \frac{Y_B - Y_A}{S_{AB}}$$

and equation (2-2.23) can be transcribed.

$$dS_{AB} = S_{AB} - (S_{AB}) = -a \Delta x_A - b \Delta y_A + a \Delta x_B + b \Delta y_B \quad (2-2.25)$$

where

- $S_{AB}$  = measured distance value
- $(S_{AB})$  = Computed distance value from the approximate coordinate
- $a, b$  = Coefficients for the distance A and B
- $\Delta X_A, \Delta Y_A, \Delta X_B, \Delta Y_B$  = Corrections to the approximate values

This is error equation for trilateration.

Since point A is given point, the correction for point A is zero, i.e.,  $\Delta X_A = \Delta Y_A = 0$

Then above equations change to the following form.

$$dS_{AB} = + a \Delta X_B + b \Delta Y_B \quad (2-2.25)$$

We explain with the example below.

[I] Given data

a) Coordinated of the given points and approximate point

Point	X	Y	note
A	5311.736	2589.811	given
B	5964.874	3988.647	given
C	4915.689	4082.736	given
P	(4915.689)	(3076.500)	unknown

b) Measured distances

Distance	Measured (d)	A priori standard deviations ( $\sigma$ )
A P	515.538	1
B P	1032.220	2
C P	1154.379	2.5

Note :  $\sigma = (1.0 \text{ cm} + d \cdot 10^{-6})$  : temporary used

[II] Making the Coefficient table

a) Computation of approximate distance

Point	Difference between coordinates			dS <sub>i</sub>
	dx <sub>i</sub>	dy <sub>i</sub>	S <sub>i</sub>	
(P) - A	169.864 <sup>m</sup>	486.689 <sup>m</sup>	515.4803 <sup>m</sup>	5.77 <sup>cm</sup>
(P) - B	-483.273	-912.147	1032.2625	-4.25
(P) - C	565.911	-1006.236	1154.4549	-7.59

b) Computation of Coefficients

Point	a $\left( = \frac{(X_P) - X_i}{S_i} \right)$	b $\left( = \frac{(Y_P) - Y_i}{S_i} \right)$
(P) - A	0.3295	0.9441
(P) - B	-0.4682	-0.8836
(P) - C	0.4902	-0.8716

[III] Error equations (observation equations)

No. Points

1. (P) - A  $V_1 = 0.3295 \Delta x_P + 0.9441 \Delta y_P - 5.77$
2. (P) - B  $V_2 = -0.4682 \Delta x_P - 0.8836 \Delta y_P + 4.25$
3. (P) - C  $V_3 = 0.4902 \Delta x_P - 0.8716 \Delta y_P + 7.59$

Above equations can be written in the matrix form

$$\begin{pmatrix} 0.3295 & 0.9441 \\ -0.2341 & -0.4418 \\ 0.1961 & -0.3486 \end{pmatrix} \begin{pmatrix} \Delta x_p \\ \Delta y_p \end{pmatrix} = \begin{pmatrix} 5.77 \\ -2.12 \\ -3.04 \end{pmatrix}$$

(A) (X) (B)

**{IV} Normal equations**

From above error equations we can obtain the normal matrix

$$\begin{pmatrix} 0.2018 & 0.3461 \\ 0.3461 & 1.2080 \end{pmatrix} \begin{pmatrix} \Delta x_p \\ \Delta y_p \end{pmatrix} = \begin{pmatrix} 1.8014 \\ 7.4438 \end{pmatrix}$$

(A \* A) (X) (A \* B)

Note : A\* = transposed matrix

and, unknown values are obtained after solution of matrix

$$\begin{pmatrix} \Delta x_p \\ \Delta y_p \end{pmatrix} = (A * A)^{-1} A * L \begin{pmatrix} -3.23 \\ 7.09 \end{pmatrix}$$

Then, the estimate values of coordinates are as follows:

$$P_x = (x) + \Delta x_p = 5481.600 - 0.0323 = 5481.568$$

$$P_y = (y) + \Delta y_p = 3076.500 + 0.071 = 3076.571$$

### Combined triangulation and trilateration

Under circumstance that prohibit achieving the necessary strength of figure by trilateration or triangulation alone, it is feasible to perform a network.

Then, new survey system combined triangulation and trilateration are developed.

These operation can be called "combined control survey" or "traverse-net adjustment survey", and the subsequent adjustment is designated as a combined adjustment.

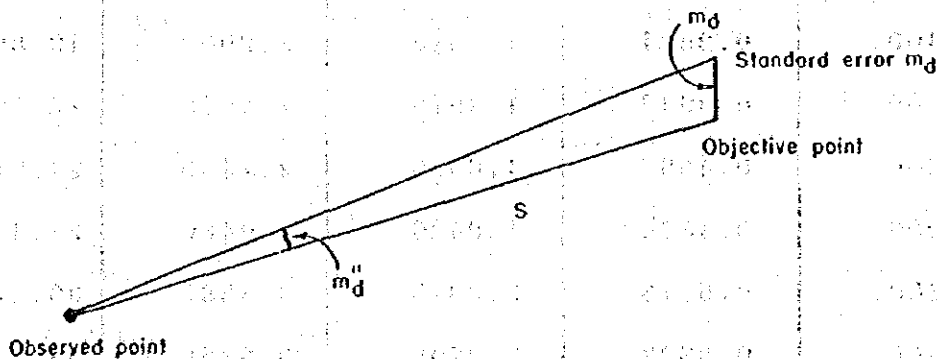
"Method of variation" can be used for the adjustment of this combined system only change the weight for the observation equations of trilateration. We explain the relation between angle observation and distance measurement.

First, we carry out the angle observation and if the standard error of a direction is known from past experience and is signified with  $m$ , that of the angle resulting from such a direction ( $m_1$ ) is

$$m_1 = (2)^{1/2} \cdot m \quad (2-2.26)$$

For the distance measurements, the standard error ( $m_d$ ) in linear units is converted to arc seconds with multiplication by  $\rho''/s$ , it indicate as the below.

$$m_d'' = \frac{\rho''}{s} m_d \quad (2-2.27)$$



The standard error of observed values  $L_1$  and  $L_3$  are popularly written as:

$$M_1 = \frac{m}{(P_1)^{1/2}} \quad \text{and} \quad M_d = \frac{m}{(P_d)^{1/2}} \quad (2-2.28)$$

where

- $m$  = same standard error for one measurement
- $P_1$  = weights
- $M_1$  = standard error of the weighted mean

Table 2-2.3 DISTANCE WEIGHT FOR ANGLE WEIGHT (=1)

DISTANCE (m.)	1 ST CLASS	2 ND CLASS	3 RD CLASS	4 TH CLASS
400.	0.0049	0.0184	0.0305	0.2742
600.	0.0110	0.0415	0.0685	0.6168
800.	0.0195	0.0737	0.1218	<u>1.0966</u>
1000.	0.0305	0.1152	0.1904	1.7135
1200.	0.0439	0.1658	0.2742	2.4674
1400.	0.0597	0.2257	0.3732	3.3584
1600.	0.0780	0.2948	0.4874	4.3864
1800.	0.0987	0.3732	0.6168	5.5516
2000.	0.1218	0.4607	0.7615	6.8538
2200.	0.1474	0.5574	0.9214	8.2930
2400.	0.1755	0.6634	<u>1.0966</u>	9.8694
2600.	0.2059	0.7785	1.2870	11.5828
2800.	0.2388	0.9029	1.4926	13.4332
3000.	0.2741	<u>1.0365</u>	1.7134	15.4207
3200.	0.3119	1.1793	1.9495	17.5452
3400.	0.3521	1.3313	2.2008	19.8068
3600.	0.3948	1.4925	2.4673	22.2054
3800.	0.4398	1.6630	2.7490	24.7411
4000.	0.4874	1.8426	3.0460	27.4138
4200.	0.5373	2.0315	3.3582	30.2235
4400.	0.5897	2.2295	3.6856	33.1702
4600.	0.6445	2.4368	4.0282	36.2540
4800.	0.7018	2.6533	4.3861	39.4747
5000.	0.7615	2.8790	4.7592	42.8325
5200.	0.8236	3.1139	5.1475	46.3273
5400.	0.8882	3.3580	5.5510	49.9590
5600.	0.9552	3.6113	5.9697	53.7277
5800.	<u>1.0246</u>	3.8739	6.4037	57.6334
6000.	1.0965	4.1456	6.8529	61.6761



Squaring and equating above expression yield.

$$\frac{P_d}{P_t} = \frac{M_t^2}{M_d^2} \quad (2-2.29)$$

The relative weight between the angle (t) and distance (d) equations is determined as:

$$\frac{P_d}{P_t} = \frac{m_t^2}{m_d^2} = \frac{m_t^2 \cdot S^2}{(\rho'' m_d)^2} \quad (2-2.30)$$

Assuming the  $m_t > m_d$ , it is convenient to be  $P_t = 1$ , and we obtain the weight of distance.

$$P_d = \frac{m_t^2 \cdot S^2}{(\rho'' m_d)^2} \quad (2-2.31)$$

For the geodetic distance with Electro-magnetic distantmeter (EDM) we can use the following a priori standard deviation from the manufacture.

$$m_d = (0.01 \text{ cm} + r \cdot 10^{-6} \cdot S) \quad (2-2.32)$$

Therefore we obtain the following popular formula for the weight of distance ( $P_d$ ).

$$P_d = \frac{m_t^2 \cdot S^2}{(M_s + r \cdot 10^{-6} \cdot S)^2 \rho''} \quad (2-2.33)$$

where

- $M_s$  = constant value and 10.0 mm.
- $r$  = constant value and 5.00 (from the manufacture)
- $m_t$  = angle mean error for the survey classes and its values are given below

1st class	2nd class	3rd class	4th class
1.8"	3.5"	4.5"	13.5"

The relations between the distance and the weight for distance are given in above Table.

This table shows the relation between the Angle observations ( $p = 1$ ) and standard measuring distance for each class.

Now we can be written the error (observation) equation for angle below.

$$V = -Z + a \Delta x + b \Delta y - l \quad (2-2.34)$$

The weight (P) of above equation is 1.

Next, the error equation for the distance is multiplied by  $(P_d)^{1/2}$ , the dimension less weight is given below.

$$(P_d)^{1/2} \cdot v'' = (P_d)^{1/2} a'' \Delta x + b'' \Delta y - l'' \quad (2-2.35)$$

After modified above distance equation, we can obtain the corrections for approximate values by solving the mixed error equations of combined triangulation and trilateration.

### (3) Observation

#### (Efficiency of Instrument)

Instruments to be used for survey shall have the following efficiency:

Classes	Efficiency	Remarks
1st Theodolite	Minimum reading scale of 1 second of arc.	1st and 2nd control survey
2nd Theodolite	Minimum reading scale of 10 second of arc.	2nd and 3rd control survey
3rd Theodolite	Minimum reading scale of 20 second of arc.	4th control survey
Electro-magnetic distance measurement equipment (EDM)	( $\pm 5$ mm $\pm 5$ ppm D)	1st 4th control survey D : Measured distance
Steel Tape		3rd and 4th control survey
3rd Level Unit		

#### (Calibration of Theodolite)

Allowance of check by horizontal angle observation are as follows:

Classes	Double angle difference	Difference of observation	Allowance of (T1 - T2)
1st theodolite	15"	8"	6"
2nd theodolite	30"	20"	12"
3rd theodolite	50"	40"	20"

Note : Values or (T1-T2) are the difference between two mean values of sets at the position  $0^\circ$ ,  $60^\circ$ ,  $120^\circ$  and at the position  $30^\circ$ ,  $90^\circ$ ,  $150^\circ$

The double angle is the summation of the observed angle in the direct and the reverse and the double angle difference is the difference between the maximum and minimum value of the double angle obtained by the observations in two or more sets.

The difference of observation is the difference of the observed angle in the direct and reverse when the observation is carried out in two or more sets, the difference of maximum of minimum values of "difference of observation" is also called same word.

Allowance of check by vertical angle observation are as follows:

Classes	Allowance of difference of elevation constant
1st theodolite	10"
2nd theodolite	30"
3rd theodolite	50"

Note : Observations shall be done a pair for three different target

(Angle Observation and Measurement of Distance)

Angles shall be observed according to the following rules:

(1) Angle observation

Item\Classes	1st	2nd	3rd	4th
Set numbers	2	2	2	2
Position	0°, 90°	0°, 90°	0°, 90°	0°, 90°
Allowance of double angle	15"	20"	30"	60"
Allowance of difference angle	8"	10"	20"	40"
Difference of elevations constant	10"	15"	30"	60"

(2) Measurement of Distance

Item\Classes	1st	2nd	3rd	4th
Set numbers	2	2	1	1

4) Calculation

(Calculation)

Allowance of closure and closure ratio of the obtained results are shown in the following table.

In case of exceeding the allowance, re-observation shall be conducted after re-checking the results.

(1) Traverses

Item \ Classes	1st	2nd	3rd	4th
Closure of bearing	$8''\sqrt{n}$	$10''\sqrt{n}$	$20''\sqrt{n}$	$50''\sqrt{n}$
Closure rate of coordinate	$1 \text{ cm}\sqrt{n} \text{ ES}$	$1.5 \text{ cm}\sqrt{N} \text{ ES}$	$2.5 \text{ cm}\sqrt{N} \text{ ES}$	$5 \text{ cm}\sqrt{N} \text{ ES}$
Closure rate	$5 \text{ cm} \frac{\text{ES}}{\sqrt{N}}$	$10 \text{ cm} \frac{\text{ES}}{\sqrt{N}}$	$15 \text{ cm} \frac{\text{ES}}{\sqrt{N}}$	$30 \text{ cm} \frac{\text{ES}}{\sqrt{N}}$

n : Number of angles  
 N : Numbers of distances  
 S : Total length

(2) Triangulation

Item \ Classes	1st	2nd	3rd	4th
Closure of sum of interior angle	10"	20"	-	-
Closure of side length	30 cm	30 cm	-	-
Closure of elevation difference	30 cm	30 cm	-	-

(Adjustment)

Residual error of net adjustment for traverses and triangulation shall not exceed the value of the following table.

(1) Strict adjustment

Item \ Classes	1st	2nd	3rd	4th
Standard Deviation for a direction	12"	15"	-	-
Residual of length	8 cm	10 cm	-	-
S.D of angle	10"	12"	15"	20"
S.D of establish point	10 cm	10 cm	10 cm	10 cm

Note : In case of adopting the method of strict adjustment, weight for distance ( $P_s$ ) shall be used the following value.

$$P_s = \frac{m_f^2 \cdot S_{ij}}{(m_s^2 + r^2 \cdot S_{ij}^2) \cdot 0.002}$$

Item \ Classes	1st	2nd	3rd	4th
Coefficient ( $m_f$ )	1.8"	3.5"	4.5"	13.5"

$$m_s = 10 \text{ mm}$$

$$r = 5 \times 10^{-6}$$

(2) Simplified adjustment

Item \ Classes	1st	2nd	3rd	4th
S. D of a direction			50"	120"
S. D of coordinate			30 cm	30 cm

## 5) Results

### (Results)

Results of surveying and record are as follows:

- (1) Results of surveying
- (2) Index map of control point
- (3) Field notes
- (4) Calculation notes
- (5) Description about points
- (6) Check list concerning the accuracy

## 2.3 Levelling

### 2.3.1 General

#### (Purpose)

Levelling in this specification means to determine the height of bench mark with level equipment and staff. Trigonometric levelling are included in this specification.

#### (Category of Levelling)

Levelling is classified into 1st, 2nd, 3rd, 4th and simplified class respectively according to kinds of known height points, accuracy of observation.

1st class levelling: The 1st class levelling shall start from national 1st class bench mark and return there.

2nd class levelling: The 2nd class levelling is required to be connected to existing 1st or 2nd class bench mark.

The levelling of each class shall be according to the following Table.

Classification	Kinds of known point	Length of Route Between know points
1st Class Levelling	1st Bench mark	less than 150 km
2nd Class Levelling	1st Bench mark 1st Level point	less than 150 km
3rd Class Levelling	1st 2nd Bench mark 1st 2nd Level point	less than 50 km
4th Class Levelling	1st 3rd Bench mark 1st 3rd Level point	less than 50 km
Simplified Levelling	1st 3rd Bench mark 1st 4th Level points	

(Classification of Works and Procedure)

Classification of works and procedure are as follows:

(1) Planning

Before field surveying, levelling route and locations of bench mark shall be selected from a topographic map, photo map or road map.

(2) Reconnaissance

(3) Establishment of permanent monuments

As a rule, permanent monuments shall be established at selected position according to the specification.

4) Observation

(Instrument)

Main instruments to be used shall be equal or better than those shown in the following Table.

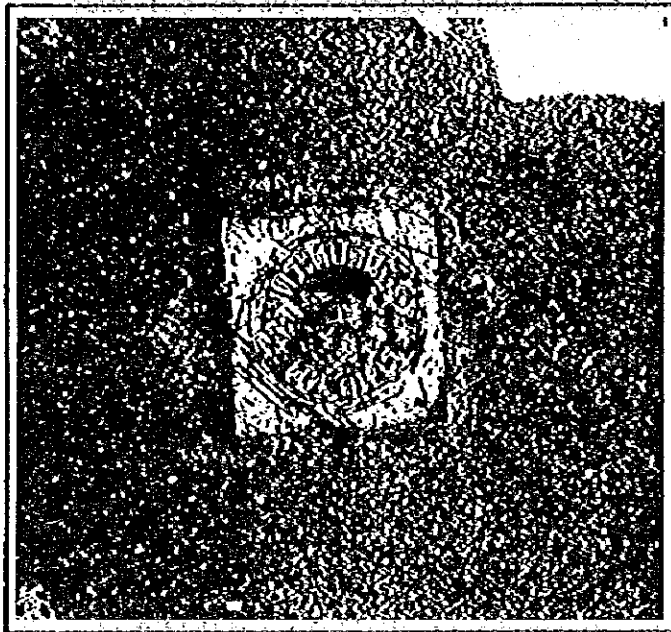
Selection shall be clearly designated in the stage of operation plan.

Class		Remarks
1st Level	sensitivity of spirit level shall be better than 10"/2mm 20"/2mm 40"/2mm	1st class levelling
2nd Level		2nd class levelling
3rd Level		3rd, 4th class levelling
1st Staff	Staff made with invar tape	1st, 2nd class levelling
2nd Staff	Staff made with wooden	3rd, 4th class levelling

(Inspection)

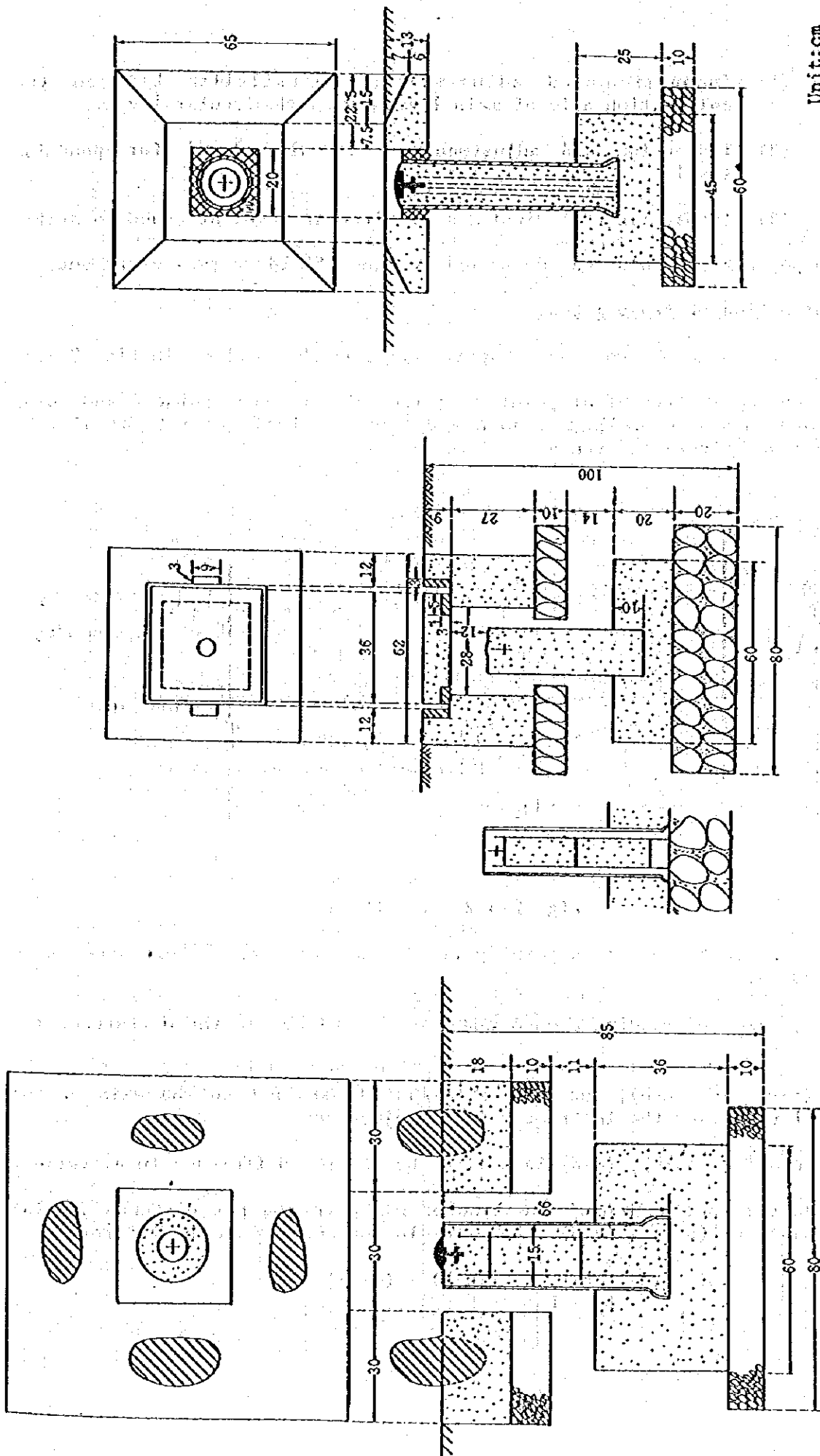
The level and staff to be used shall be inspected and adjusted for the items shown in the following paragraphs before beginning observation.

**SOME EXAMPLES OF CONTROL MONUMENT**



**ROYAL THAI SURVEY DEPARTMENT**





Unit:cm

Fig. 2-3.1 STANDARD FORM OF BENCH MARK

- (1) Inspection and adjustment of parallelism between the collimation axis of main level tube and circular level.
- (2) Inspection and adjustment of circular level for pendulum level.
- (3) Inspection and adjustment of circular level attached to staff.

Two-peg test should be done before the field survey carry out.

Explanation of two-peg test

Set two pegs 80 to 100 m. apart on ground as shown in Fig. 2-3.2.

Set up the instrument at point M equally distant from point A and point B, and take rod readings a on A and b on B. Difference (a-b) will be the true height difference.

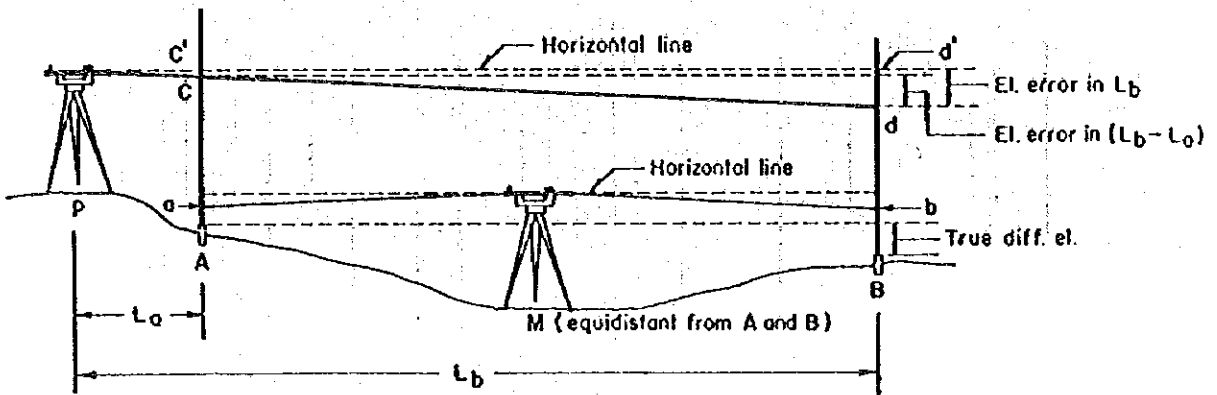


Fig. 2-3.2 TWO-PEG TEST

Move the instrument to a point p near A, and measure the distance a to A and b.

Next, take rod reading c on A and d on B, and obtain the difference (c-d).

If (c-d) = (a-b), the line of sight is parallel to the axis of the level tube, and the instrument is in adjustment.

If (c-d) ≠ (a-b), (c-d) is called the "false" difference in elevation.

Since the inclination of the line of sight in the net distance (Lb-La) is equal to (a-b) - (c-d), the error in the reading on the far rod is

$$e_{fr} = \frac{L_b}{L_b - L_0} \{ (a-b) - (c-d) \}$$

Subtract the amount of this error from the reading  $d$  on the far rod to obtain the correct reading  $d'$  at B.

Note:

Main mistakes in leveling are as follows:

1. Confusion of numbers in reading the rod, as, for example, reading and recording 492 when it should be 3.92. The mistake is not likely to occur if the observer notices the numbers on both sides of the observed reading.
2. Recording backsights in the foresight column and vice versa.
3. Faulty additions and subtractions; adding foresights and subtracting backsights. As a check, the difference between the sum of the backsights and the sum of the foresights should be computed for each page or between bench marks.
4. Rod not held on same point for both foresight and backsight. This is not likely to occur if the turning points are marked or otherwise clearly defined.
5. Not having the Philadelphia rod fully extended when reading the long rod. Before a reading on a turning point is taken, the clamp should be inspected to see that it has not slipped.
6. Wrong reading of vernier when the target rod is used.
7. When the target is used on the extended rod, not having the vernier on the target set to read exactly the same as the vernier on the back of the rod when the rod is not extended.

(Observation)

Observation shall be done under the following rules:

- (1) Observation shall comprise the double running except simplified levelling.
- (2) Distance between the position of back-sight and fore-sight for the position of level unit shall be equal, and those relation
- (3) Sight length and units of reading shall satisfy the requirement as shown in the following Table.

Classes	1st	2nd	3rd	4th	Simplified
Sight length	max 50 m	max 60 m	max 70 m	max 70m	max 80 m
Units of Reading	0.1 mm	1 mm	1 mm	1 mm	10 mm

(Re-observation)

Allowance of different value at double running shall be the following Table:

Classes	1st	2nd	3rd	4th	.
Different values of running	$2.5 \text{ mm}\sqrt{S}$	$5 \text{ mm}\sqrt{S}$	$10 \text{ mm}\sqrt{S}$	$20 \text{ mm}\sqrt{S}$	.

Note : S (km) one-way

(Calculation and Checking)

Hight of bench marks shall be calculated from observed values after performing the staff correction and orthometric correction.

Allowance of observed mean values are as follows:

Classes	1st	2nd	3rd	4th	Simplified
Closure error of ling	$2 \text{ mm}\sqrt{S}$	$5 \text{ mm}\sqrt{S}$	$10 \text{ mm}\sqrt{S}$	$20 \text{ mm}\sqrt{S}$	$40 \text{ mm}\sqrt{S}$
Closed error of known points	$3 \text{ mm}\sqrt{S}$	$6 \text{ mm}\sqrt{S}$	$12 \text{ mm}\sqrt{S}$	$25 \text{ mm}\sqrt{S}$	$50 \text{ mm}\sqrt{S}$

(Adjustment)

Adjustment of levelling shall be calculated from solving the observation equation or condition equation.

In this equation, weight of each equation must be in propotional to length.

Allowance of mean square error by adjustment are as follows:

Classes	1st	2nd	3rd	4th	Simplified
Mean square error (S. D)	2 mm	5 mm	10 mm	20 mm	40 mm

## 5) Results

Final results of levelling are as follows:

- (1) Results of observation and final results
- (2) Field maps
- (3) Lists of calculation
- (4) Schack map of established points
- (5) Route map of levelling

### 2.3.2 Adjustment by the Method of Least SQUARES

#### 1) Principle

The method of least squares is closely linked to the normal distribution.

It will be shown that for a normal distribution we obtain the solution with the minimum likelihood when minimizing the variance.

This is the estimate according to the method of least squares.

The normal distribution is shown in the following form:

$$\frac{dF(\epsilon)}{d\epsilon} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\epsilon\epsilon/2\sigma^2} = f(\epsilon) \quad (2-3.1)$$

where

- $\epsilon$  = error
- $\sigma^2$  = theoretical variance
- $F(\epsilon)$  = (cumulative) normal distribution function

The incremental probability  $dP$  for an error inside the interval  $(\epsilon, \epsilon + d\epsilon)$  is as follows:

$$dP = f(\epsilon)d\epsilon = \frac{1}{\sigma\sqrt{2\pi}} e^{-\epsilon\epsilon/2\sigma^2} d\epsilon \quad (2-3.2)$$

For  $n$  observations we get its corresponding simultaneous probability  $P$

$$\begin{aligned} P &= dP \times dP \times \dots \times dP = \frac{1}{\sigma_1\sqrt{2\pi}} e^{-\epsilon_1\epsilon_1/2\sigma_1^2} d\epsilon_1 \dots \frac{1}{\sigma_n\sqrt{2\pi}} e^{-\epsilon_n\epsilon_n/2\sigma_n^2} d\epsilon_n \\ &= \left(\frac{C}{\sigma\sqrt{2\pi}}\right)^n e^{-\sum \epsilon_i^2/2\sigma_i^2} d\epsilon_1 \times d\epsilon_2 \times \dots \times d\epsilon_n = e^L \times d\epsilon_1 \times d\epsilon_2 \times \dots \times d\epsilon_n \end{aligned} \quad (2-3.3)$$

where  $e^L$  is the likelihood function

$$L = -n \ln \sigma - n \ln \sqrt{2\pi} - \frac{\sum \epsilon_i^2}{2\sigma^2} + n \ln C \quad (2-3.4)$$

and

$$\begin{aligned} P_1 \sigma_1^2 &= P_2 \sigma_2^2 = \dots = P_n \sigma_n^2 = \sigma^2 \\ C^n &= (P_1 \times P_2 \times \dots \times P_n)^{1/2} \end{aligned}$$

The maximum likelihood is obtained for  $\Sigma P \epsilon \epsilon = \min.$

Furthermore

$$\frac{dL}{d\sigma} = \frac{d}{d\sigma} \left( -n \ln \sigma - n \ln \sqrt{2\pi} - \frac{\Sigma P \epsilon \epsilon}{2\sigma^2} \right) \quad (2-3.5)$$

from which follows the estimate of  $\sigma^2$

$$\sigma^2 = \frac{\Sigma P \epsilon \epsilon}{n} \quad (2-3.6)$$

where P is the "weight".

According to equation (2-3.5) we obtain the desired solution for  $\Sigma P \epsilon \epsilon =$  minimum.

## 2) Fundamental Least Squares Applications

For a normal distribution we can find the solution of maximum likelihood (x) according to the method of least squares.

We consider the following relations

$$\hat{x} = l_i + v_i \text{ or } \hat{x} - l_i = v_i = \text{Correction} \quad (2-3.7)$$

where

$$\begin{aligned} \hat{x} &= \text{solution of maximum likelihood} \\ l_i &= \text{observed data} \\ v_i &= \text{correction} \end{aligned}$$

According to the method of least squares we have to minimize the estimated variance of above problem.

$$\begin{aligned} V_1^2 &= l_1^2 + \hat{x}^2 - 2 \hat{x} l_1 & \text{Weight } P_1 &= 1/\sigma_1^2 \\ V_2^2 &= l_2^2 + \hat{x}^2 - 2 \hat{x} l_2 & P_2 &= 1/\sigma_2^2 \\ \dots & \dots & \dots & \dots \\ V_n^2 &= l_n^2 + \hat{x}^2 - 2 \hat{x} l_n & P_n &= 1/\sigma_n^2 \\ \hline \Sigma P v v &= \Sigma P l l + \hat{x} \Sigma P - 2 \hat{x} \Sigma P l \end{aligned} \quad (2-3.8)$$

The minimum is obtained for the first derivative equal to zero:

$$\frac{d \Sigma P v v}{dx} = 2 \hat{x} \Sigma P - 2 \Sigma P l = 0 \quad (2-3.9)$$

Therefore,

$$\hat{x} = \frac{\sum P_i l_i}{\sum P_i} \quad (\text{Weighted mean}) \quad (2-3.10)$$

if  $P_1 = P_2 = \dots = P_n$ , then

$$\hat{x} = \frac{\sum l_i}{n} \quad (\text{Simple mean}) \quad (2-3.11)$$

### 3. Indirect Observation of Its Unknown

The method of least squares can easily be extended to any finite number of unknowns which can be indirectly determined by the observation equations.

Now, we consider the three unknowns  $x$ ,  $y$  and  $z$ .

#### (1) Observation equations

$$\begin{aligned} a_1 x + b_1 y + c_1 z &= l_1 + V_1 \\ a_2 x + b_2 y + c_2 z &= l_2 + V_2 \\ &\dots \\ &\dots \\ a_n x + b_n y + c_n z &= l_n + V_n \end{aligned} \quad (2-3.12)$$

where

$l_1, l_2, \dots, l_n$  = observed data

$a_1, b_1, c_1, \dots$  = known coefficients

$V_1, V_2, \dots, V_n$  = corrections

We want to find the solution that satisfies the condition,

$$\sum P V^2 = \text{Minimum} \quad (2-3.13)$$

After squaring its above equation we obtain the equation,

$$\begin{aligned} \sum P V^2 &= \sum P \{l^2 + EPaa + yEPbb + zEPcc + 2yzEPbc + 2xyEPab \\ &+ 2xzEPac - 2xEPa\} - 2yEPb\} - 2zEPC\} \end{aligned} \quad (2-3.14)$$

In order to obtain the minimum we put the partial differentials of equal to zero.

$$\begin{aligned}\frac{\partial \Sigma PVV}{\partial x} &= 2(x \Sigma P_{aa} + y \Sigma P_{ob} + z \Sigma P_{oC} - \Sigma P_{oI}) = 0 \\ \frac{\partial \Sigma PVV}{\partial y} &= 2(x \Sigma P_{ob} + y \Sigma P_{bb} + z \Sigma P_{bC} - \Sigma P_{bI}) = 0 \\ \frac{\partial \Sigma PVV}{\partial z} &= 2(x \Sigma P_{oC} + y \Sigma P_{bC} + z \Sigma P_{CC} - \Sigma P_{CI}) = 0\end{aligned}\quad (2-3.15)$$

Equation ( 3.15) can be modified the following equations:

$$\begin{aligned}(\Sigma P_{aa})x + (\Sigma P_{ob})y + (\Sigma P_{oC})z &= \Sigma P_{oI} \\ (\Sigma P_{ob})x + (\Sigma P_{bb})y + (\Sigma P_{bC})z &= \Sigma P_{bI} \\ (\Sigma P_{oC})x + (\Sigma P_{bC})y + (\Sigma P_{CC})z &= \Sigma P_{CI}\end{aligned}\quad (2-3.16)$$

This equations give the so-called normal equation. This is the linear equations and we can solve the x, y, z value from above equations.

#### 4) Matrix Approach of Method of Least Squares

Above equation can be shown in the matrix forms.

Observation equations are now written

$$AX = L + V \quad |A^* A| \neq 0 \quad (2-3.17)$$

where

$$\begin{aligned}L &= \text{observed data} \\ A &= \text{given parameter} \\ X &= \text{unknown parameter} \\ V &= \text{corrections making the system of equations consistent}\end{aligned}$$

After squaring the above equations, we obtain the following:

$$V^* V = L^* L + X^* A^* A X - X^* A^* L - L^* A X \quad (2-3.18)$$

Differentiation gives

$$d(V^* V) = X^* A^* A dX + dX^* A^* A X - dX^* A^* L - L^* A dX \quad (2-3.19)$$

Minimum or maximum is obtained for

$$(X^* A^* A - L^* A) dX = 0 \quad (2-3.20)$$

Therefore we obtain

$$(A^* A) X = A^* L \quad \text{AND} \quad X = (A^* A)^{-1} A^* L \quad (2-3.21)$$

This is popular formular for the method of least squares.

A matrix with the elements  $A_{ij}$  gives the transposed matrix  $A^*$  with the elements.



$$(A^*)_{ij} = A_{ji}$$

and,  $A^{-1}$  is called the inverse matrix of the matrix A.

And we also can be obtained the least squares estimate of its corrections by means of the following procedure

(1) from equations (2-3.17) and (2-3.21)

$$\hat{\Delta} = A(A^*A)^{-1}A^*L-L \quad (2-3.22)$$

and

$$\hat{V}^* \hat{\Delta} = L^*L - L^*A(A^*A)^{-1}A^*L \quad (2-3.23)$$

$$s^2 = V^*V / (n-m) \quad (2-3.24)$$

This type of solution is only valid when all observations are uncorrelated and of equal quality.

But in many cases the observations are given with different weight such as different measured distances, utility of instrument with different accuracy.

Therefore minimize  $V^*pV$  in stead of  $V^*V$  is used for the method of least squares where  $p$  is the weight matrix. Then we obtain the solution

$$\begin{aligned} \hat{\Delta} &= (A^*PA)^{-1}A^*PL \\ s^2 &= V^*PV / (n-m) \end{aligned} \quad (2-3.25)$$

Fig. 2-3.3 show the general flow chart for the solution of levelling net.

### 5) Adjustment of a Junction Point for Levelling

We consider the following net with two unknown points.

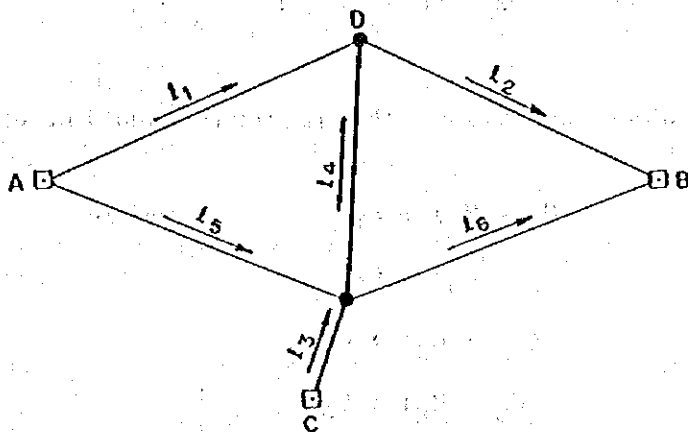


Fig. 2-3.3 LEVELLING NET WITH POINT D AND E UNKNOWN

Known heights and measured quantities are given below:

A = 1.75 m	$l_1 = 4.00$ m
B = 2.25 m	$l_2 = -3.20$ m
C = 11.70 m	$l_3 = -4.70$ m
D = unknown	$l_4 = -1.25$ m
E = unknown	$l_5 = 5.20$ m
	$l_6 = -4.65$ m

- (a) Firstly, we must assign the approximate values of the height for unknown points, D and E.

We adopted the approximate values from known points and measured height difference as shown in the below.

$$D \text{ approximate height} = H_A + h_{AD} = 1.75 + 4.00 = 5.75 \text{ m}$$

$$E \text{ approximate height} = H_C + h_{CE} = 11.70 - 4.70 = 7.00 \text{ m}$$

- (b) Product its residual equations

As we obtained the approximate values for unknown points, D and E, the corrections for approximate values are shown in the following form.

$$\left. \begin{array}{l} H_D, \text{ approximate} + dH_D \\ H_E, \text{ approximate} + dH_E \end{array} \right\} \Rightarrow \text{Estimate values}$$

The residual equations are produced for the each route.

$$\text{for } l_1 \quad H_D + d_{HD} = H_A + l_1 \quad \text{distance } (S_1)$$

$$\text{for } l_2 \quad H_D + d_{HD} = H_B + l_2 \quad \text{distance } (S_2)$$

$$\text{for } l_3 \quad H_E + d_{HE} = H_C + l_3 \quad \text{distance } (S_3)$$

$$\text{for } l_4 \quad H_D + d_{HD} = H_E + d_{HE} + l_4 \quad \text{distance } (S_4)$$

$$\text{for } l_5 \quad H_E + d_{HE} = H_A + l_5 \quad \text{distance } (S_5)$$

$$\text{for } l_6 \quad H_E + d_{HE} = H_B + l_6 \quad \text{distance } (S_6)$$

(2-3.26)

After rearranged, above equations, the residual equation are shown in the following forms.

$$dH_D = (H_A - H_D) + l_1 \quad \text{weight} = (1/S_1)$$

$$dH_D = (H_B - H_D) - l_2 \quad (1/S_2)$$

$$dH_D - dH_E = (H_D - H_E) + l_3 \quad (1/S_3)$$

$$dH_E = (H_C - H_E) + l_4 \quad (1/S_4)$$

$$dH_E = (H_A - H_E) + l_5 \quad (1/S_5)$$

$$\Delta H_E = (H_B - H_E) - I_6 \quad (1/S6)$$

$$(2-3.27)$$

or it can be shown in the matrix form below:

$$\begin{array}{c} \text{(ai)} \\ \left| \begin{array}{c} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{array} \right| \end{array} \quad \begin{array}{c} \text{(bi)} \\ \left| \begin{array}{c} 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \right| \end{array} \quad \begin{array}{c} \left| \begin{array}{c} dH_D \\ \\ dH_E \end{array} \right| \end{array} = \begin{array}{c} \text{(li)} \\ \left| \begin{array}{c} (H_A - H_D) + I_1 \\ (H_B - H_D) - I_2 \\ (H_C - H_E) + I_3 \\ -(H_D - H_E) + I_4 \\ (H_A - H_E) + I_5 \\ (H_B - H_E) - I_6 \end{array} \right| \end{array}$$

$$(2-3.28)$$

where

$H_D, H_E$  = approximate values

(c) Each terms of normal equations make form above, residual equations

$$\begin{aligned} \text{[aa]} &= (1 \times 1) + (1 \times 1) + (1 \times 1) + (0 \times 0) + (0 \times 0) = 3 \\ \text{[ab]} &= (1 \times 0) + (1 \times 0) + (1 \times -1) + (0 \times 1) + \dots = -1 \\ \text{[bb]} &= (0 \times 0) + (0 \times 0) + (-1 \times -1) + (1 \times 1) + \dots = 4 \end{aligned}$$

as a same rule, normal equation are as follows:

$$\begin{aligned} \text{[paa]} dH_D + \text{[pab]} dH_E &= \text{[paf]} \\ \text{[pab]} dH_D + \text{[pbb]} dH_E &= \text{[pbf]} \end{aligned} \quad (2-3.29)$$

Each terms are computed as follows:

$$\begin{aligned} \text{[paa]} &= (1 \times 1) + (1 \times 1) + \dots + (0 \times 0) = 3 \\ \text{[pab]} &= (1 \times 0) + (1 \times 0) + \dots + (1 \times -1) = -1 \\ \text{[pbb]} &= (0 \times 0) + (0 \times 0) + \dots + (1 \times 1) = 4 \end{aligned}$$

and, normal equation are shown in the following form

$$\begin{aligned} 3.0 dH_D - 1.0 dH_E &= -0.35 \\ -1.0 dH_D + 4.0 dH_E &= 0.05 \end{aligned} \quad (2-3.30)$$

Above are linear equations and we can obtain the unknowns dHD and dHE by matrix approach.

$$\begin{array}{c} \left| \begin{array}{c} dHD \\ dHE \end{array} \right| = \left| \begin{array}{cc} 3.0 & -1.0 \\ -1.0 & 4.0 \end{array} \right|^{-1} \left| \begin{array}{c} -0.35 \\ 0.05 \end{array} \right| \end{array} \quad (2-3.31)$$

Solving above matrix, we obtained

$$dH_D = -0.0182 \qquad dH_E = -0.1220$$

And adopted values are

$$H_D = H_D \text{ . approximate} + dH_D = 5.75 - 0.0182 = 5.628 \text{ m}$$

$$H_E = H_E \text{ . approximate} + dH_E = 7.00 - 0.1220 = 6.932 \text{ m}$$

(2-3.32)

This computation from (b) to (c) iterative until  $dH_D$ ,  $dH_E$  are less than allowance.

The iteration are less than 3 times in general.

(d) Residual ( $v_i$ ) are computed using equation (3-3.27).

Now we obtained its following values

$$v_1 = -122.7 \qquad v_4 = -54.5$$

$$v_2 = -177.3 \qquad v_5 = -18.2$$

$$v_3 = -68.2 \qquad v_6 = -31.8$$

(e) Computation of mean squares error

For mean squares error we can obtained from the following formula

$$m_0 = \sqrt{\frac{\sum v_i^2}{n-m}} = \sqrt{\frac{0.5545}{6-2}} = 117.74 \text{ mm/km}$$

where

$(\sum v^2)$  = total sum of squares

$n$  = number of observations

$m$  = number of unknowns

Computation Procedure for Leveling Net

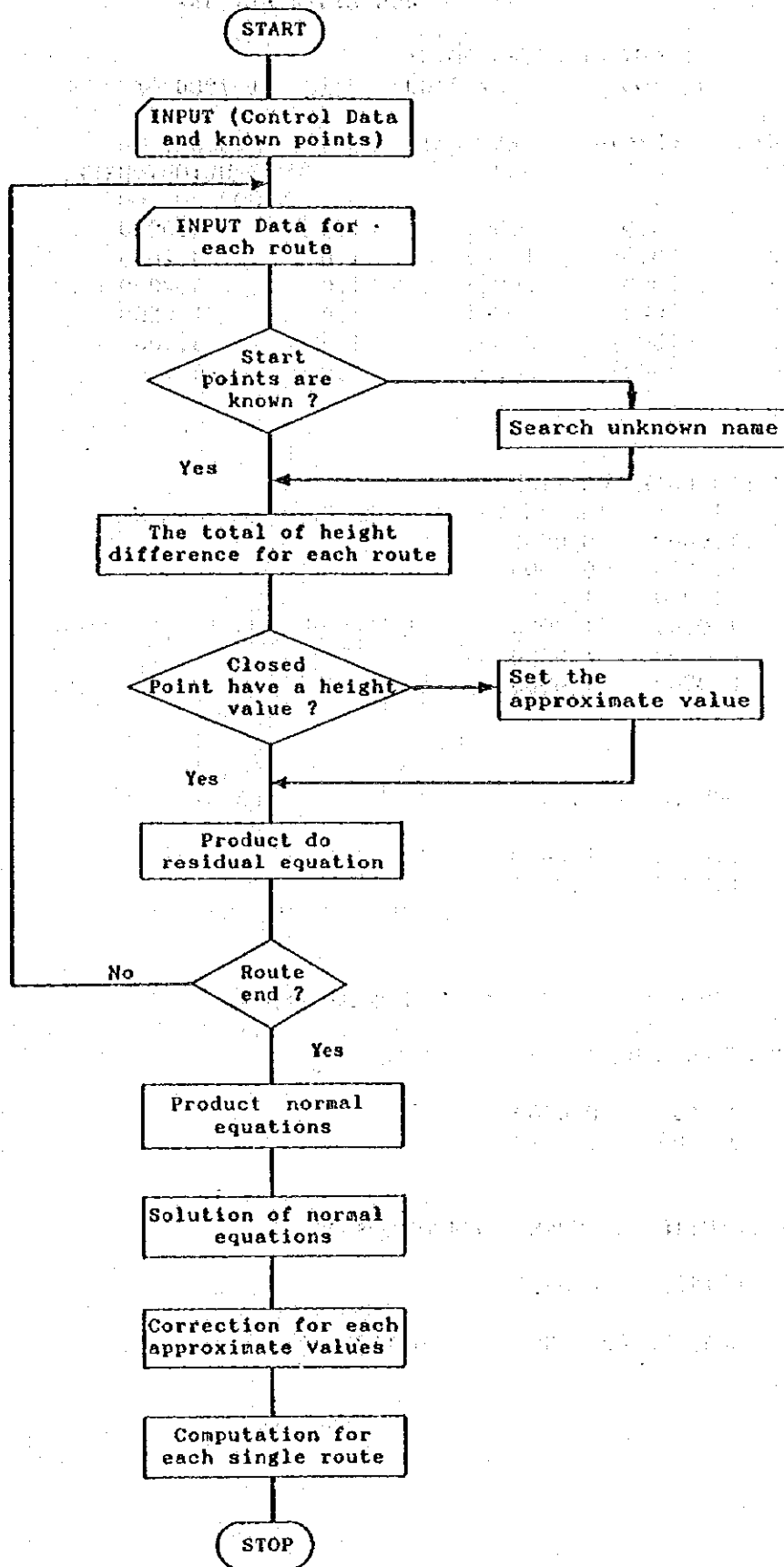


Fig. 2-3.4 GENERAL FLOW CHART FOR SOLUTION OF LEVELING NET

NET ADJUSTMENT OF LEVELLING FOR (CASE STUDY CASE- )

\*\*\*\* CHECK LIST (1) \*\*\*\*

HEIGHT OF GIVEN POINT  
1.7500                      2.2500                      11.7000

ROUTE NO.	ORIGIN NAME	DESTINATE NAME	DIST.	CORR. HEIGHT DIFF.	RESIDUAL
1	10001	20001	1.0	4.0000	0.0000
2	20001	10002	1.0	-3.2000	-0.3000
3	10003	20002	1.0	-4.7000	0.0000
4	20002	20001	1.0	-1.2500	0.0000
5	10001	20002	1.0	5.2000	0.0500
6	20002	10002	1.0	-4.6500	-0.1000

\*\* RESIDUAL EQUATIONS \*\*

	1	2	CONST.
1	1.0000	0.0000	0.0000
2	-1.0000	0.0000	-0.3000
3	0.0000	1.0000	0.0000
4	1.0000	-1.0000	0.0000
5	0.0000	1.0000	0.0500
6	0.0000	-1.0000	-0.1000

\*\* NORMAL EQUATIONS \*\*

1	3.0000	-1.0000	0.3000
2	-1.0000	4.0000	0.1500

-SOLUTION OF MATRIX IS NORMAL END.-

\*\* INVERSE MATRIX \*\*

1	0.3636	0.0909	0.1227
2	0.0909	0.2727	0.0682

\*\* SOLUTION OF NORMAL EQUATIONS \*\*

0.1227      0.0482

MEAN ERROR    NO    -    0.11774395

**NET ADJUSTMENT OF LEVELLING**

**BY METHOD OF INDIRECT OBSERVATION**

**SURVEYED AREA : CASE STUDY CASE-3**

**NO. OF FIXED**            3  
**NO. OF UNKNOWN**       2  
**NO. OF ROUTE**           6

**MEAN ERROR**            117.744 (MM/KM)

**SURVEYED AT : 20/07/1988**  
**COMPUTED AT : 27/07/1988**

**( DEPARTMENT OF TOWN AND COUNTRY PLANNING )**

**HEIGHT OF FIXED BENCH MARK**

NO.	NAME	GIVEN HEIGHT
1	10001	1.7500
2	10002	2.2500
3	10003	11.7000

**INPUT AND ORTHOMETRIC CORR.**

**ROUTE NO.    1**

B.M.	ASS.H M	DIST. KM	OBS.DIFF M	LAT. D M	CORR. MM	ORTH. MM
10001		1.0	4.0000		0.0	
20001						
<b>TOTAL</b>		1.0	4.0000		0.0	

INPUT AND ORTHOMETRIC CORR.

ROUTE NO. ____ 2						
B.M.	ASS.H M	DIST. KM	OBS.DIFF M	LAT. D M	CORR. MM	ORTH. MM
20001		1.0	-3.2000		0.0	
10001						
TOTAL		1.0	-3.2000		0.0	

INPUT AND ORTHOMETRIC CORR.

ROUTE NO. ____ 3						
B.M.	ASS.H M	DIST. KM	OBS.DIFF M	LAT. D M	CORR. MM	ORTH. MM
10003		1.0	-4.7000		0.0	
20002						
TOTAL		1.0	-4.7000		0.0	

INPUT AND ORTHOMETRIC CORR.

ROUTE NO. ____ 4						
B.M.	ASS.H M	DIST. KM	OBS.DIFF M	LAT. D M	CORR. MM	ORTH. MM
20002		1.0	-1.2500		0.0	
20001						
TOTAL		1.0	-1.2500		0.0	

INPUT AND ORTHOMETRIC CORR.

ROUTE NO. ____ 5						
B.M.	ASS.H M	DIST. KM	OBS.DIFF M	LAT. D M	CORR. MM	ORTH. MM
10001		1.0	5.2000		0.0	
20002						
TOTAL		1.0	5.2000		0.0	



INPUT AND ORTHOMETRIC CORR.

ROUTE NO. 6						
B.M.	ASS. H	DIST.	OBS. DIFF	LAT.	CORR.	ORTH.
	M	KM	M	D M	MM	MM
20002		1.0	-4.6500			
					0.0	
10002						
TOTAL		1.0	-4.6500			0.0

ADJUSTED HEIGHT OF JUNCTION BENCH MARK

B.M.	ASS. H	CORR.	ADJ. H	M.E.
	M	M	M	M
20001	5.7500	-0.1227	5.6273	0.0710
20002	7.0000	-0.0682	6.9318	0.0615

ADJUSTED DIFFERENCE OF HEIGHT

ROUTE NO.	B.M.	DIST. KM	OBS. DIFF. M	ADJ. DIFF. M	V MM
1	10001	1.0	4.0000	3.8773	-122.7
2	20001	1.0	-3.2000	-3.3773	-177.3
3	10002	1.0	-4.7000	-4.7682	-68.2
4	20002	1.0	-1.2500	-1.3045	-54.5
5	20001	1.0	5.2000	5.1818	-18.2
6	20002	1.0	-4.6500	-4.6818	-32.8
	10002				
TOTAL		6.0			

ADJUSTED HEIGHT				ROUTE NO. 1
( 10001) --- ( 20001)				
B.M.	DIST. KM	OBS. DIFF. M	V M	ADJ. H M
10001	1.0	4.0000	-0.1227	1.7500
20001				5.6273
TOTAL	1.0	4.0000	-0.1227	

ADJUSTED HEIGHT				ROUTE NO. 2
( 20001) --- ( 10002)				
B.M.	DIST. KM	OBS. DIFF. M	V M	ADJ. H M
20001	1.0	-3.2000	-0.1773	5.6273
10002				2.2500
TOTAL	1.0	-3.2000	-0.1773	

ADJUSTED HEIGHT				ROUTE NO. 3
( 10003) --- ( 20002)				
B.M.	DIST. KM	OBS. DIFF. M	V M	ADJ. H M
10003	1.0	-4.7000	-0.0682	11.7000
20002				6.9318
TOTAL	1.0	-4.7000	-0.0682	

ADJUSTED HEIGHT				ROUTE NO. 4
( 20002) --- ( 20001)				
B.M.	DIST. KM	OBS. DIFF. M	V M	ADJ. H M
20002	1.0	-1.2500	-0.0545	6.9318
20001				5.6273
TOTAL	1.0	-1.2500	-0.0545	

B.M.	ADJUSTED HEIGHT		ROUTE NO. 5	
	DIST.	( 10001) --- (		20002)
	KM	CORRECTED.	V	ADJ.H
		M	M	M
10001				1.7500
20002	1.0	5.2000	-0.0182	6.9316
TOTAL	1.0	5.2000	-0.0182	

B.M.	ADJUSTED HEIGHT		ROUTE NO. 6	
	DIST.	( 20002) --- (		10002)
	KM	CORRECTED.	V	ADJ.H
		M	M	M
20002				6.9318
20001	1.0	-4.6500	-0.0318	2.2500
TOTAL	1.0	-4.6500	-0.0318	

END

## 2.4 Topographic Mapping

### 2.4.1 General

A topographic map shows by the use of suitable symbols, (1) the spatial configuration of the earth's surface, which includes such features as hills and valleys; (2) Other natural features such as forests, lakes, and streams; (3) the physical changes brought upon the earth's surface by the works of man, such as buildings, roads, canals, and cultivation.

The distinguishing characteristic of a topographic map, as compared with other maps, is the representation of the terrestrial relief.

Topographic maps are used in many ways for planning. They are a necessary aid in the design of any engineering project that requires a consideration of land forms, elevations, or gradients, and they are used to supply the general information necessary to the studies of geologists, economists, and others interested in the broader aspects of the development of natural resources.

The term "Topographic map" is used for the general map. Photo map and compilation are included in this category.

#### (Classification of Topographic Survey)

Topographic survey are classified into control survey, plane table survey or aerial photogrammetry, photo map with contours, map compilation.

#### (Accuracy of Topographic Map)

Accuracy of topographic map are as following table as a principle rule:

		Map Scale		Remarks
		More than 1/500	Less than 1/1,000	
Horizontal position		less than $\pm 0.5$ mm	less than $\pm 0.7$ mm	on the map
Elevation	Dot point	less than $\pm \Delta h/4$	less than $\pm \Delta h/3$	h are interval of
	Control	less than $\pm \Delta h/2$		Same as above

**(Projection of Map)**

The projection is required to be based upon the specification. In case of no specifications, plain rectangular coordinate system is used.

Origin of coordinate, scale factor of map projection are decided in the specification.

**(Interval of Contours)**

Interval of contours are standardized the following table:

Scale of Map \ Classification	Main contours	Index contours	Supplementary contours	Remarks
1/8,000	10 m	50 m	5.0 m	
1/4,000	5	25	2.5	
1/2,900	2	10	1.0	
1/1,000	1	5	0.5	
1/ 500	1	5	0.5	

**2.4.2 Map Design**

The sheet and format sizes indicated are as follows. But it can be altered to fit the specific requirement of a community.

**(Map Content)**

The following data shall be included in the margin of map.

- (1) Items common to all maps:

- Title block
- Project name
- Project location (Contracting authority)
- Sheet name
- Map scale
- Map Type
- North Arrow
- Bar Scale
- Accuracy note

- (2) Items that must be specifically tailored to map:

- Position in map location diagram
- Adjoining sheet designations
- Geographic coordinates

Preparation data and photo data

Land classification

Housing, land symbol and other special symbols

Datum

## 2.5 Photogrammetric Mapping

### 2.5.1 General

#### (Definition)

"Aerial photogrammetry" in this specification are mapping works of obtaining reliable measurements or information of land features from aerial photographs.

#### (Mapping Scale)

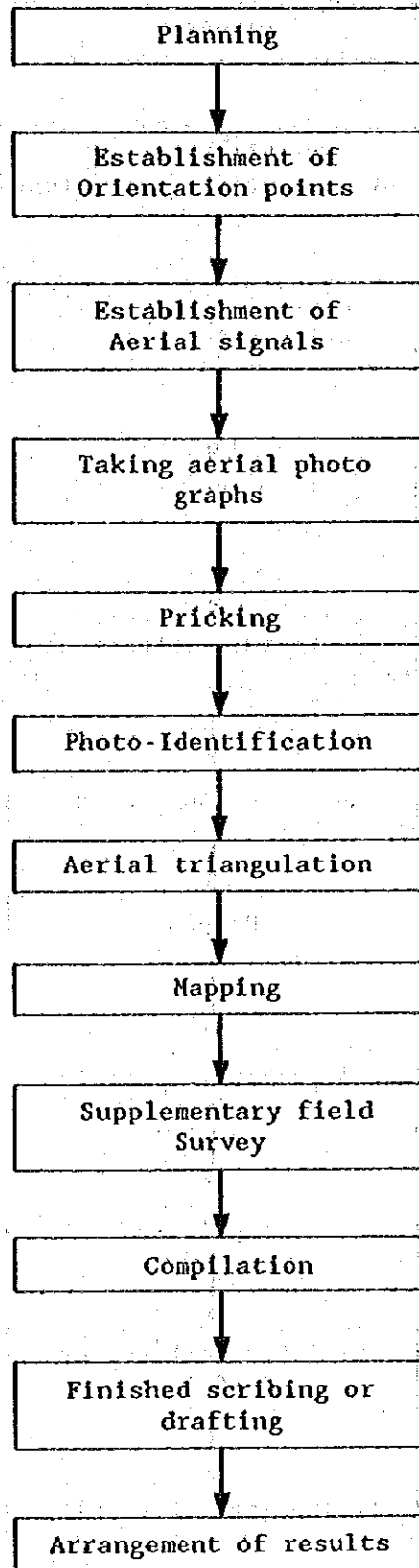
Mapping Scale by method of aerial photogrammetry are less than 1/1,000 in principle.

Map scale are standardize 1/1,000, 1/2,000, 1/4,000 and 1/10,000.

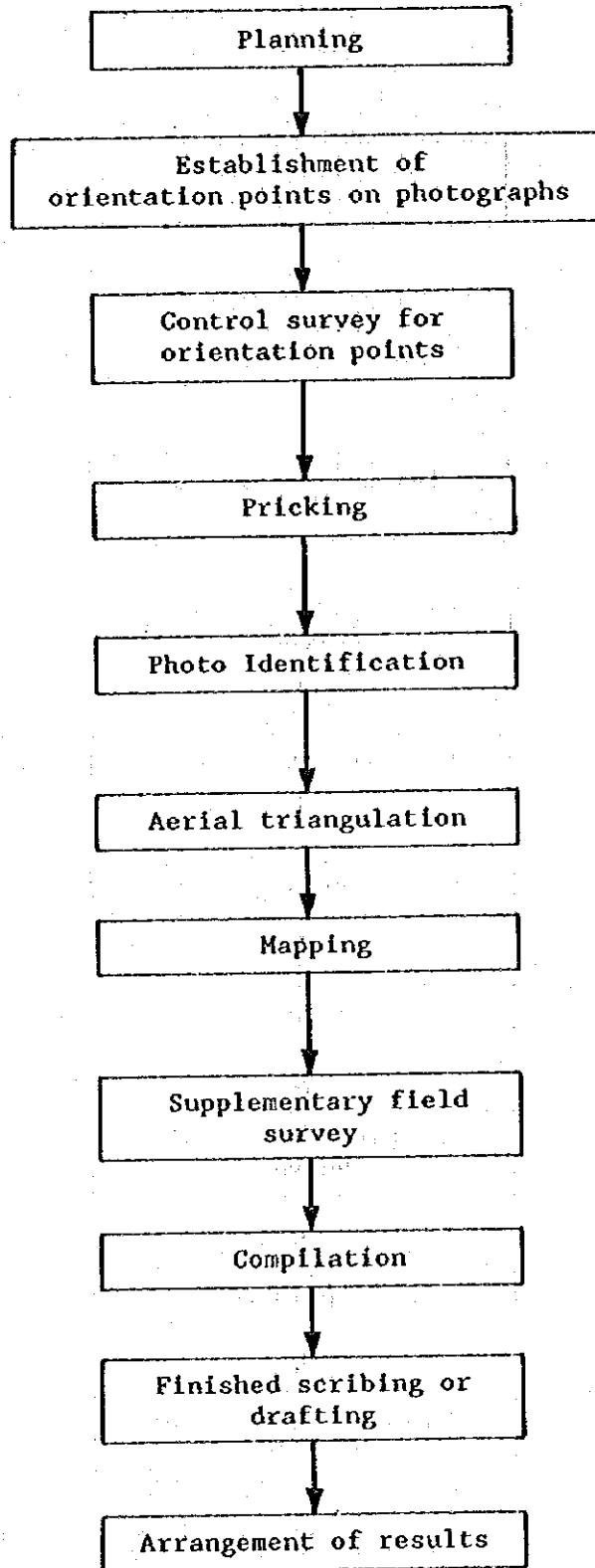
#### (Classification of Works and Procedure)

Classification of works are as follows:

Start from taking aerial photographs:

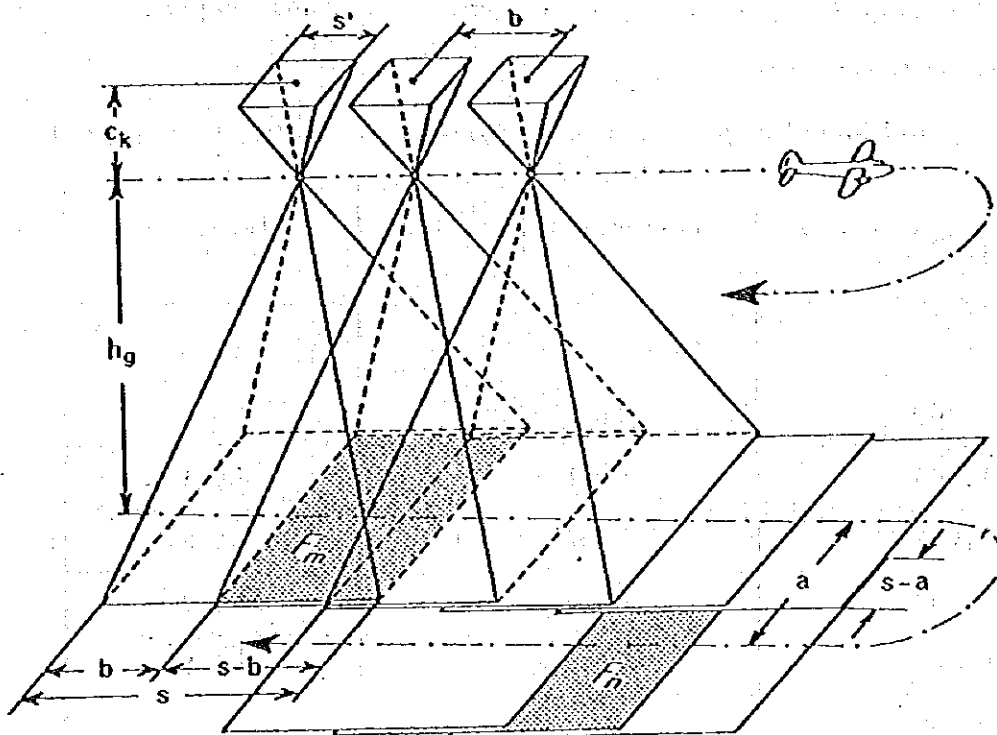


Start from existing aerial photographs





Note: Elements to be required for the flight plan of taking air-photo



End lap	$p(\%) = \frac{s-b}{s} \cdot 100$	Length of base with p % end lap	$b = s \left(1 - \frac{p}{100}\right)$
side lap	$q(\%) = \frac{s-a}{s} \cdot 100$	Distance between flight lines with q % side lap	$a = s \left(1 - \frac{q}{100}\right)$
Photo scale	$M_b = 1 : m_b = \frac{c_k}{h_g} = \frac{s'}{s}$	Base-height ratio	$= \frac{b}{h_g}$
Photo scale figure	$m_b = \frac{h_g}{c_k} = \frac{s}{s'}$	Number of photographs per strip	$n_p = \frac{l_p}{b} + 1$
flying height above ground	$h_g = c_k \cdot m_b = \frac{s}{s'} \cdot c_k$	Number of strips	$n_q = \frac{l_q - s}{a} + 1$
Ground distance	$s = s' \cdot m_b = \frac{h_g}{c_k} \cdot s'$	Model area	$F_m = (s-b) \cdot s$
Area covered by a single photograph	$f_g = s^2 = s'^2 \cdot m_b^2$		

where

Negative size	$s'$	Ground area to be covered	$F_g$
calibrated focal length	$c_k$	Length of flight strip	$l_p$
flying speed	$v_g$	Width of area to be covered	$l_q$

## 2.5.2 Ground survey for photogrammetry

(General)

Orientation (or control) points survey is done to establish orientation points necessary for performing aerotriangulation and mapping.

(Accuracy of Orientation Points)

Accuracy of orientation points are as follows according to map-scale.

Map-scale \ Accuracy	Horizontal position (S. D)	Elevation (S. D)
	less than	less than
1/ 500	$\pm 0.1$ m	$\pm 0.1$ m
1/ 1,000	$\pm 0.1$ m	$\pm 0.1$ m
1/ 2,000	$\pm 0.2$ m	$\pm 0.2$ m
1/ 4,000	$\pm 0.2$ m	$\pm 0.2$ m
1/ 8,000	$\pm 0.5$ m	$\pm 0.5$ m
1/10,000		

(Method)

Establishment of control points is carry out by means of traverses or triangulation.

Class of surveying are as follows in principle:

Map-scale	Horizontal points	Elevation
more than 1/1,000	3rd class control survey	Simplified levelling
less than 1/1,000	4th class control survey	Same as above

**(Selection of Orientation Points)**

Orientation points shall be selected the right place necessary for performing aerotriangulation and mapping based on the flight plan or flight index map.

Orientation points shall be selected the right place for the establishment of air photo signal or pricking.

**(Final Results)**

Final results are as follows:

- (1)
- (2)
- (3)
- (4)

**2.5.3 Establishment of Air Photo Signal**

**(General)**

"Establishment of airphoto signal" means the setting work of airphoto signal on the control points to aid in its identification on a photograph.

**(Standard of Signal)**

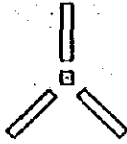
The airphoto signal shall have shape, size and color allowing it to be identified on the enlarged aerial photograph.

The signal shall be strongly established with well materials.

The size of signal-board are as follows:

Photo-Scale	Type of A or C	Type of C
1 : 4,000	20 cm x 10 cm	20 cm x 20 cm
1 : 6,000	30 cm x 10 cm	30 cm x 30 cm
1 : 10,000	45 cm x 15 cm	45 cm x 45 cm
1 : 20,000	90 cm x 30 cm	30 cm x 90 cm

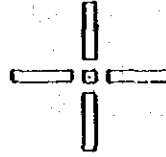
A-type



B-type



G-type



(Standard types of airphoto signal)

#### 2.5.4 Aerial photograph

(General)

The operator shall take vertical aerial photographs, free of clouds, clouds shadow and atmospheric haze, during the specified season.

When urban areas are photographed, the sun angle must not be less than 30 .

(Aerial Camera)

Only precision aerial cameras and magazines which have been calibrated by the Authority.

The Calibration of the camera shall include the magazine matched to it and only that combination of camera cone and magazine shall be used to take the photographs.

(Film)

The film must not have passed the suggested expiration date, and must have been stored in accordance with the manufacture's instructions.

(Flight Plan)

With any proposal, the operator shall submit a plan showing proposed flight lines designed to acquire the photographic coverage specified herein.

(Spacing of Photographs)

Overlapping photographs in each flight line shall provide full stereoscopic coverage of the area to be mapped.

Endlap (in the line of flight) shall not be less than 55 percent, and shall average approximately 60 percent. Sidelap shall not be less than 20 percent, no more than 40 percent, and shall average approximately 30 percent.

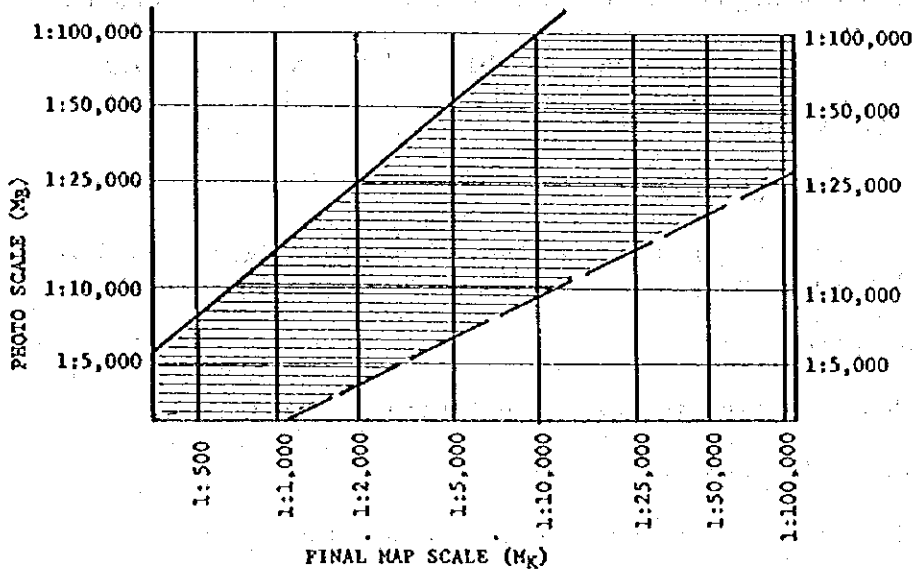
(Tilt)

Tilt of the camera from verticality at the instant of exposure shall not exceed 3 degrees nor shall it exceed 5° grades between successive exposure stations.

(Negative Scale)

Negative scale of Aerial photographs is related to the mapping scale by the following table:

Mapping scale	Negative scale of photographs
1 : 500	1 : 3,000 ~ 1 : 4,000
1 : 1,000	1 : 6,000 ~ 1 : 8,000
1 : 2,000	1 : 8,000 ~ 1 : 10,000
1 : 4,000	1 : 16,000 ~ 1 : 20,000
1 : 8,000	1 : 24,000
1 : 10,000	1 : 30,000



(Reflights)

Unacceptable coverage resulting from deviation from the flight plan shall be corrected with reflight coverage overlapping accepted coverage by two stereomodels.

(Image Quality)

Maximum shutter speed, considering aperture and film speed shall be used to minimize image motion.

All negative and fiducial mark images shall be clear and sharp.

(Photoindex)

The photoindex shall be prepared. The index shall include title information identifying the area, name of contracting, photographic scale, focal length of the aerial camera, flight height, date of photography.

(Accuracy Control)

Accuracy control shall be carried out according to specification.

2.5.5 Pricking

(General)

"Pricking" means the works to mark the position of control points on the aerial photograph at field.

(Execution)

Pricking is required to be executed in the following case.

- (1) Airphoto signal cannot distinguishable on the photographs.
- (2) Aerial photographs not established the airphoto signal.

Field Identification

(Use of Aerial Photographs)

Aerial photographs enlarged on same scale as plotting are used for field classification in principle.

(Stage of Execution)

The field identification is carried out before mapping.

(Execution of field identification)

The field identification is carried out the following items:

- (1) Complicated land features to be interpreted on the aerial photographs.
- (2) Change which appeared in area after photography.

(Editing)

The gathering information at survey field shall be edited on the enlarged aerial photographs.

- (1) The geographic features shall be laid down precisely on the positions.
- (2) The significant names data shall show on the photographs.

#### 2.5.6 Aero triangulation (phototriangulation)

##### 1) General

Aerial triangulation (or phototriangulation) is defined as the process of the extension of horizontal and/or vertical control whereby the measurements of angles and/or distances on overlapping photographs are related into a spatial solution using the perspective principles of the photographs. Aerial triangulation is classified into many categories criteria, data acquisition and data processing.

1. With respect to the unit analyzed
  - a. Single model used.
  - b. Strip triangulation.
  - c. Block triangulation.
2. With respect to the data acquisition and data processing devices and methods.
  - a. Analog triangulation, using analogical stereo-instruments.
  - b. Computational (analytical) triangulation comparators for photo-coordinate and electronic computers for numerical solutions.
  - c. Semi-analytical (or semi-analog) triangulation, which is a combination of analog and computational triangulation.
3. Others

##### Instruments

Various instruments may be required in performing an aerial triangulation.

Some of the instrumentation which may be used is listed as follows:

1. For analog, computational (analytical) or Semi-analog methods
  - a. First-order type stereo instruments
  - b. Second-order stereo-plotters
  - c. Analytical plotters
  - d. Other

2. For purely computational methods
  - a. Stereo-comparator or mono-comparator
  - b. Analytical plotters
3. For computation and adjustment work
  - a. Electronic computers
  - b. Analytical plotters
4. For aid in preparation, photo inspection
  - a. Point-transfer devices

#### Operation Procedure

All aerial triangulation worked can be divided into three distinct phases.

1. Preparation with respect to material, equipment, control data, and others.
2. Data acquisition involving instrument work, observation, data arrangements, and others.
3. Data processing involving computation and adjustment of the data as well as final evaluation and analysis.

The each details of above phases are different from the method of triangulation .

General procedure is shown in Fig. 2-5.1.

Recently new instruments such as points marker, points transfer devices, comparators, and analytical plotters were developed in U.S.A., Germany, and other.

Here we introduce the several principle and standard instruments below.



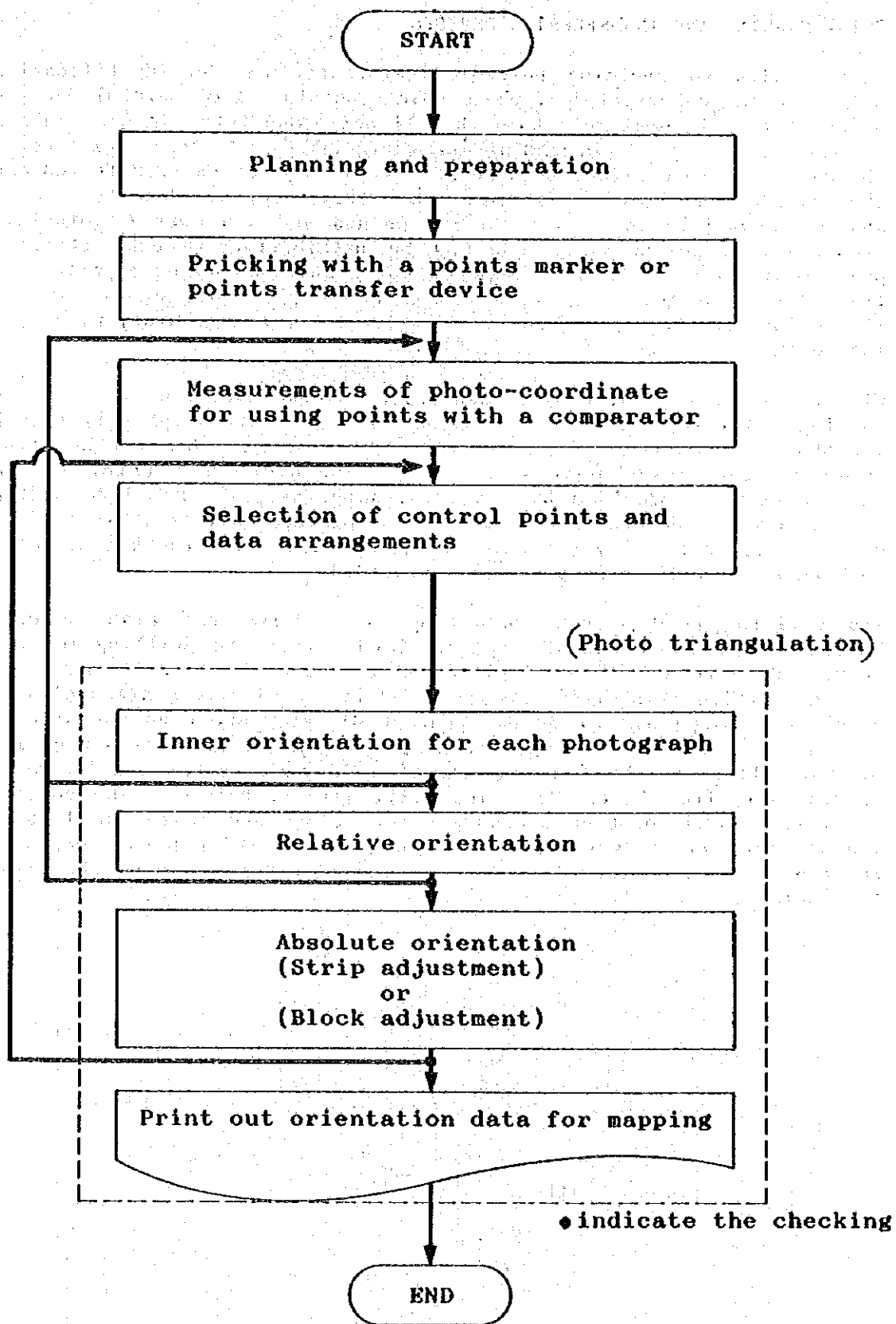


Fig. 2-5.1 GENERAL FLOW CHART OF THE PROCEDURE

## POINT MARKING AND TRANSFERRING DEVICES

With a view to avoiding possible ambiguity in the identification, selection and description of photo points obtained from natural details, often it is felt desirable to mark well defined points in the emulsion of the photograph. Depending on whether or not mono- or stereo-observations are desired, such points may or may not be transferred onto the adjacent photographs. In some areas (e.g., forest, desert, etc. it may be impossible to locate natural points and the cost of providing presignalized points on ground may be prohibitive. Such artificial points are desirable in these cases. Several point marking and transferring devices are available. Each device has, of course, its limitations in the type of point it can mark and the accuracy with which such points are marked and transferred.

WILD PUG is most popular type (see below) This instrument is meant for marking and transferring of points on film by stereoscopically viewing the photographs and drilling holes in the emulsion. A metal box frame supports two glass plates which act as picture carriers. The illumination is provided by two fluorescent tubes. Two fine setting screws permit a common translation of the picture carriers, with a range of 10 mm, in x and y directions. Independent differential movements in both x and y directions are also provided.

Two drilling heads, each consisting of a drive mechanism, a drill holder, a drill and a marker are provided. When the drilling lever is depressed, a rotation and simultaneous lowering of the drill holder is caused. PUG drills of 60, 100, 220 and 180  $\mu\text{m}$  diameters are available. Black measuring marks of 30  $\mu\text{m}$  apparent diameter are used for accurate setting under stereo viewing. Each optical system includes a zoom lens of range 1:4, whereby the magnification in each system can be varied continuously from 6x to 24x. The drills are motorized. The constant drill speed and constant pressure ensure clean and sharp round holes. The viewing system is equipped with Dove prisms which permit necessary rotations of the observed pictures. The pictures are actually viewed from below through the carrier plates.

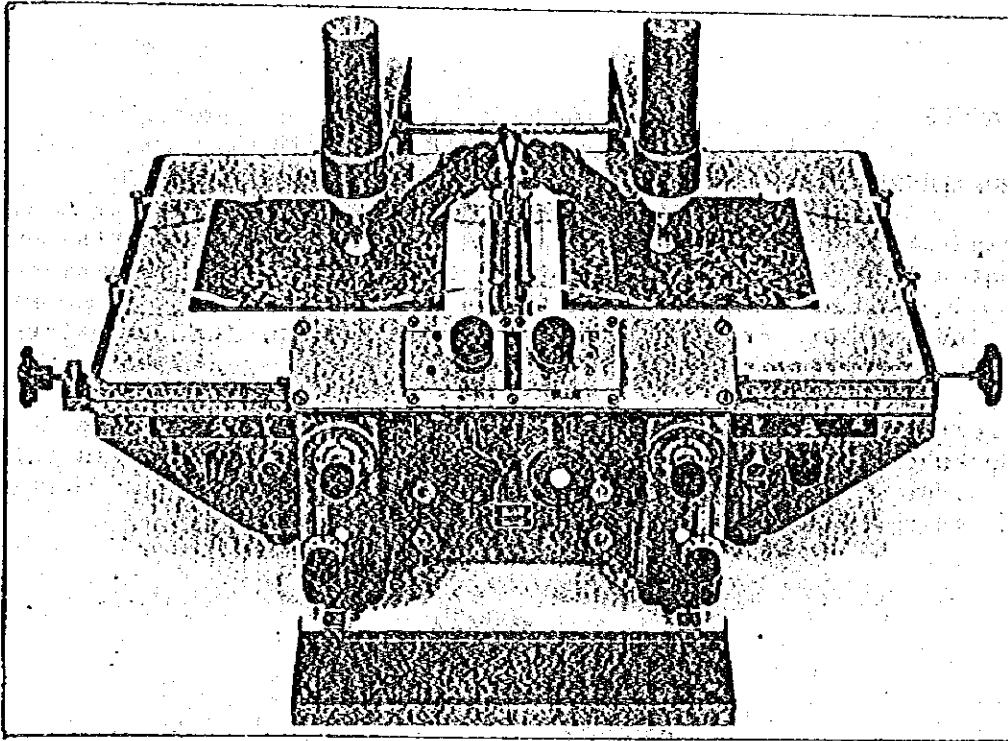


Fig. 2-5.2 PUG-4 POINT TRANSFER INSTRUMENT  
(COURTESY, WILD HEERBRUGG INSTRUMENTS, INC.)

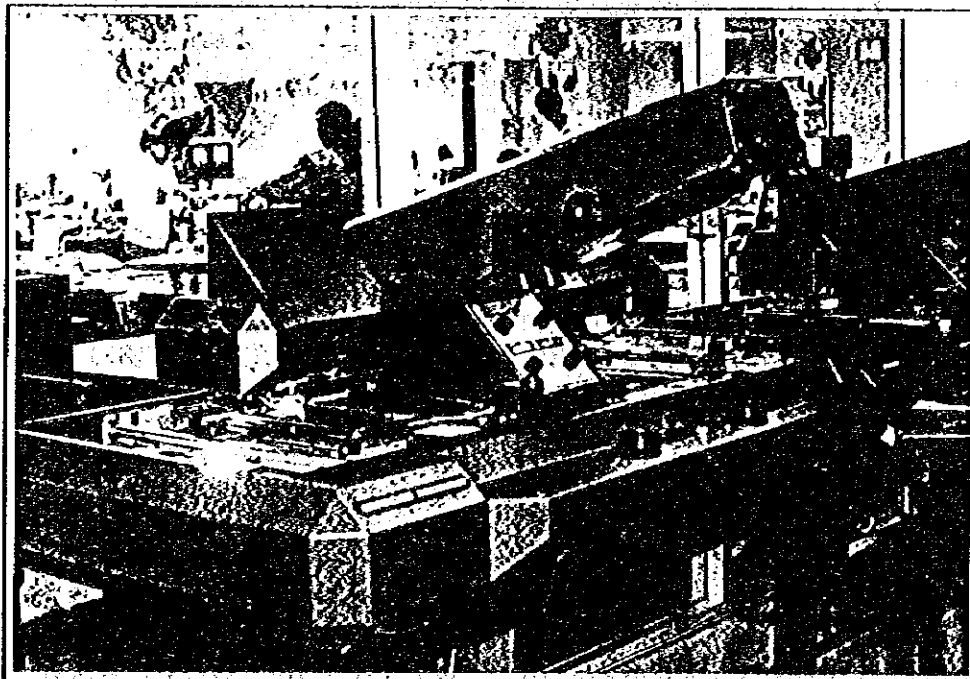


Fig. 2-5.3 STEREO POINT TRANSFER INSTRUMENT  
(at Department of Lands)  
(COURTESY, WILD HEERBRUGG INSTRUMENTS, INC.)

## COMPARATORS

### Monocomparators

Monocomparators make measurements (x,y coordinates) of points on one photo at a time. In such an approach, the point images to be measured have to be marked permanently on the photo as natural features on the ground (object) or, artificially, by a point marking (or, transferring) device in appropriate locations.

A representative example is the PK-1 Monocomparator (see below) introduced in 1976 by Carl Zeiss, Oberkochen, West Germany. In this, a photo-holder carries the photograph on its upper side and the index grating on the under side moves along a plane surface.

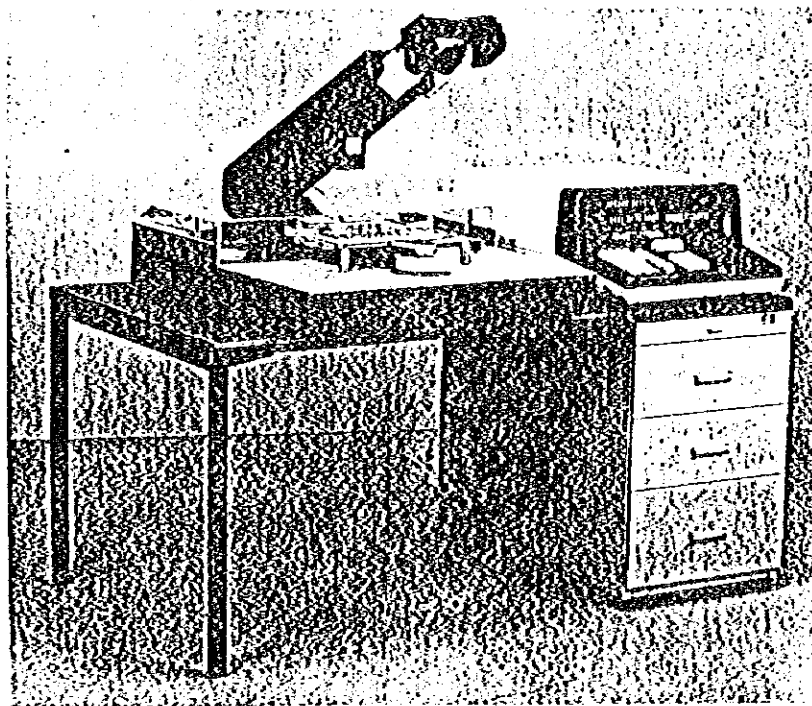
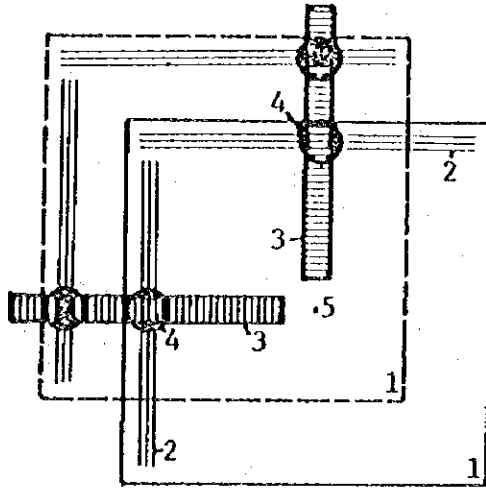


Fig. 2-5.4 MONOCOMPARATOR PK-1 WITH ECOMAT 12  
(COURTESY, CARL ZEISS, OBERKochen, WEST GERMANY)

There is a guide ensuring parallel motions of the photo-holder. Test points are set by first shifting the photo-holder by free hand and tilting a scale is mounted on a U-beam which at the same time serves to guide the carriage with the scanning head. An image of the photograph is formed in a nearby intermediate image plane in which the measuring marks are located. It offers the operator a choice of using one of three types of luminous floating marks or a black floating mark. Binocular viewing is possible with 5x, 12x, 20x or 30x magnification.

The two-dimensional linear measuring system of this monocomparator is illustrated in Fig. 2-5.5. This uses a photoelectric linear measuring system. The index grating is contained on the photo-holder. This grating, in conjunction with the scale-grating, produces Moire fringes, which are required for counting. The length of scale lines is equivalent to the test length in the opposite coordinate. The design combines the principles of linear pulse measurements with that of guide-error compensation.



- |                      |                  |
|----------------------|------------------|
| 1 Photo-holder plate | 4 Scanning head  |
| 2 Index grating      | 5 Measuring mark |
| 3 Scale              |                  |

Note : Broken line indicates center position, solid line indicates off-center position

Fig. 2-5.5 TWO-DIMENSIONAL LINEAR MEASURING SYSTEM WITH INDEX GRATING AND FIXED SCALES OF PK-1 MONOCOMPATOR

### Stereocomparators

A stereocomparator is used for measuring simultaneously the coordinates of corresponding points on a stereopair of photographs, particularly when such readings are made without resorting to some form of point transfer. Usually, the measuring, viewing and read out systems are of the same type as those employed in the monocomparator.

It is advantageous for the computational photogrammetric techniques to have completely separate measuring systems for the two photographs of a stereo pair. It is, however necessary to translate both stage plates simultaneously under the viewing optics in order to continuously scan the stereo pair.

In the classical construction of stereocomparators, two stages for measuring are utilized: (i) The lower stage with its own measuring

system, carries the left side photograph and also the upper stage on which the right side photograph is mounted. It records the coordinates  $(x_1, y_1)$  of a point on the left plate. (ii) The upper stage which can be translated by the amounts necessary to remove and record  $x$  and  $y$  parallaxes ( $px$  and  $py$ ). Thus, the observed values are:  $x_1, y_1, px$  and  $py$ . Coordinates  $x_2, y_2$  for the right photo (see below) are Zeiss - Jena Stecometer.

$$x_2 = x_1 + px \quad \text{and} \quad y_2 = y_1 + py$$

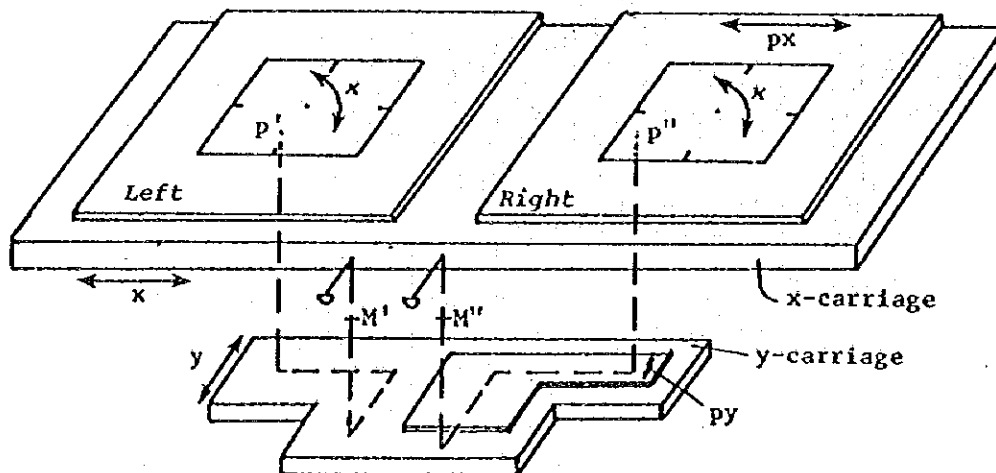


Fig. 2-5.6 SCHEMATIC DIAGRAM OF A STEREOCOMPARATOR  
[THE MEASURING MARKS ARE M' AND M'';  
THE OPTICAL PATHS SHOWN IN BROKEN LINES]

In this instrument the common  $x$  motion is given to the lower stage and common  $y$  motion is given to the optics (microscope). The  $px$  motion is given to the right photograph on one upper stage and the  $py$  motion to the left photograph on a second upper stage. It can use photographs up to 23x23 cm size. The photographs are illuminated through a condenser from above and are observed from below with a magnification, 6x, 12x or 18x. It is possible to choose one of six luminous measuring marks, each differing in size and shape. Three different colors of measuring marks can be selected also. One can optically transform  $px$  into  $py$  with built-in Dove prisms.

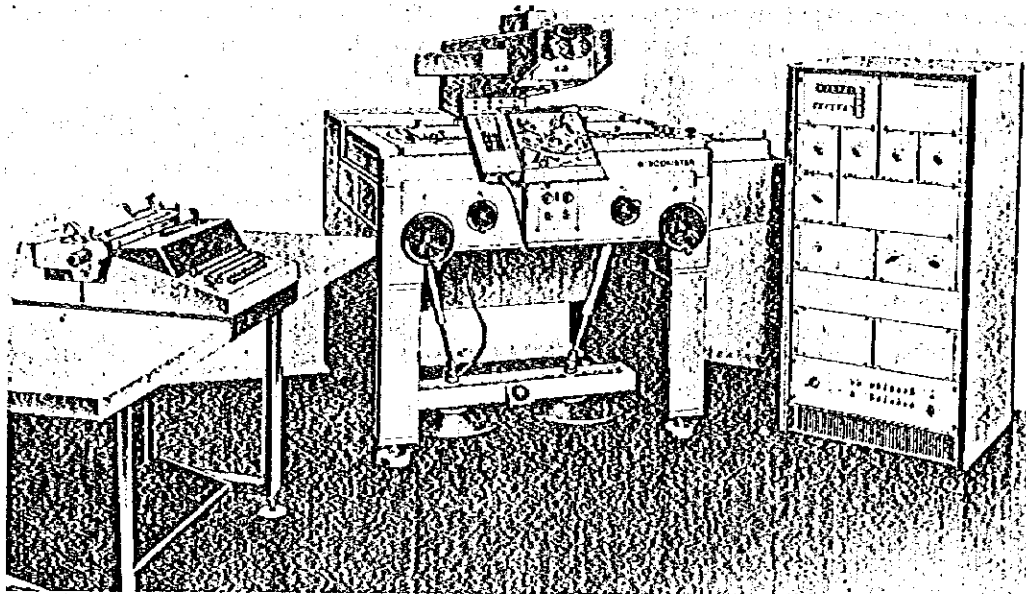


Fig. 2-5.7 ZEISS-JENA STECOMETER  
(COURTESY, JENOPTIK JENA G.m.b.H, EAST GERMANY)

#### ANALYTICAL PLOTTERS

An analytical plotter consists of a precision stereocomparator and coordinatograph interfaced with an electronic digital computer (e.g., AP/C4 and AS11A1 of the OMI Corporation, C-100 Planicomp of Zeiss, US-1 of Bendix Corporation, APPS-IV of Autometric, Inc., etc.). These instruments are capable of solving a wide variety of photogrammetric problems, conventional and non-conventional.

More recent ones incorporate electronic image correlators and other components, which make them operate almost completely automatically. Their capabilities are limited by the dimensions of the photo-carriers in the comparator and the limitation of the computer. Many analytical solutions like Relative Orientation, Scaling, Absolute Orientation, Orthophoto printing, etc. are possible in such an instrument apart from it being used as a stereocomparator only.

The instrument operates by solving analytical equations based on the observation data (the points locations), the stored information and the mathematical model relating the problem. Output from the comparator-

computer combination activates servo motors to drive the coordinatograph plotting device (pencil or scribe), which presents the graphical, plotted information.

For their use with conventional (frame) photography, the operations follow a pattern similar to any conventional optical-mechanical stereoplotter, e.g., with the AP/C4 or C-100 planicomp,

1. Interior Orientation of each photo. The inputs at this stage are,
  - a) Diapositives (or negatives) places on the plate carriers of comparator.
  - b) Constants (for the computer):
    - F - Camera focal length corrected for uniform film shrinkage;
    - $C_f$  - Differential film-shrinkage constant, which is used to calculate such correction as is applicable to one set of photo coordinates (say, y) compared to the other (say, x);
    - R - Radius of the Earth Sphere in the model scale;
    - X, Y - The coordinates of the points in the model at which the Earth Sphere is assumed to be tangent to the X-Y plane of the model;
    - $C_e, C_h$  - Correction constants which are determined by fitting correction equations to appropriate distortions due to lens and atmospheric distortions;
    - $X_n, Y_n$  - Coordinates of the camera Nadir point in the model system;
    - $K_{im}$  - The image motion constant which corrects for the Image Motion Compensation (IMC) of the camera(AP/C4).
  - c) Photo centers ( $x_o, y_o$ ), established from the observed fiducial marks constitute the only observation data for Interior Orientation.
2. Relative Orientation of the stereo pair.
  - a) Y-parallaxes observed at five or more locations (in the stereo mode);
  - b) The elements of relative orientation, generally 'Dependent' case (i.e.,  $b_x, b_y, w, \phi$  and K) are computed and stored.

Note : With appropriate software, the relative orientation can be



performed by also applying the Collinearity or the Coplanarity condition.

3. Absolute Orientation of the stereo model.

- a) Three control points (at least) are identified and used for absolute orientation (scaling and leveling). The elements of transformation between model coordinates and ground (survey) coordinates are computed and stored.
- b) Aero-triangulation are computed with the mini-computer.

4. Mapping (compilation) of necessary details and contour lines in the model.

- a) DX and DY are computed from model coordinates and applied to the photo-carriages on a real time basis.
- b) Map coordinates are computed from model coordinates using the stored parameters of transformation from Step 3 above.

Note : DX, DY indicate the coordinate differences between the existing point and next point.

- c) The compilation proceeds on a real time basis. Alternately, model coordinates ( $X_m, Y_m, E_m$ ) or the corresponding map or ground coordinates ( $X_p, Y_p, E$ ) for discrete points can be recorded or displayed.
- d) Data gathering for DTM can be recorded easily.

The fundamental system diagram of AP/C4 shows the Fig. 2-5.8 with the photograph, and C-100 planicomp system also illustrate in Fig. 2-5.10.

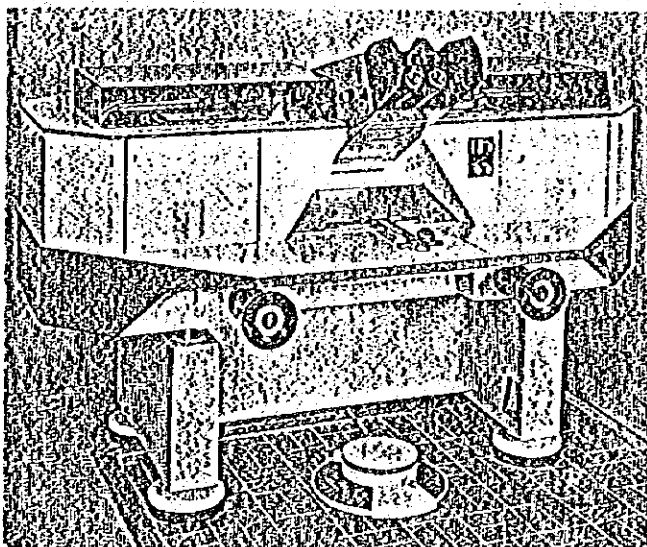


Fig. 2-5.8 ANALYTICAL PLOTTER AP/C4  
(COURTESY, OMI CORPORATION OF AMERICA, N.Y.)

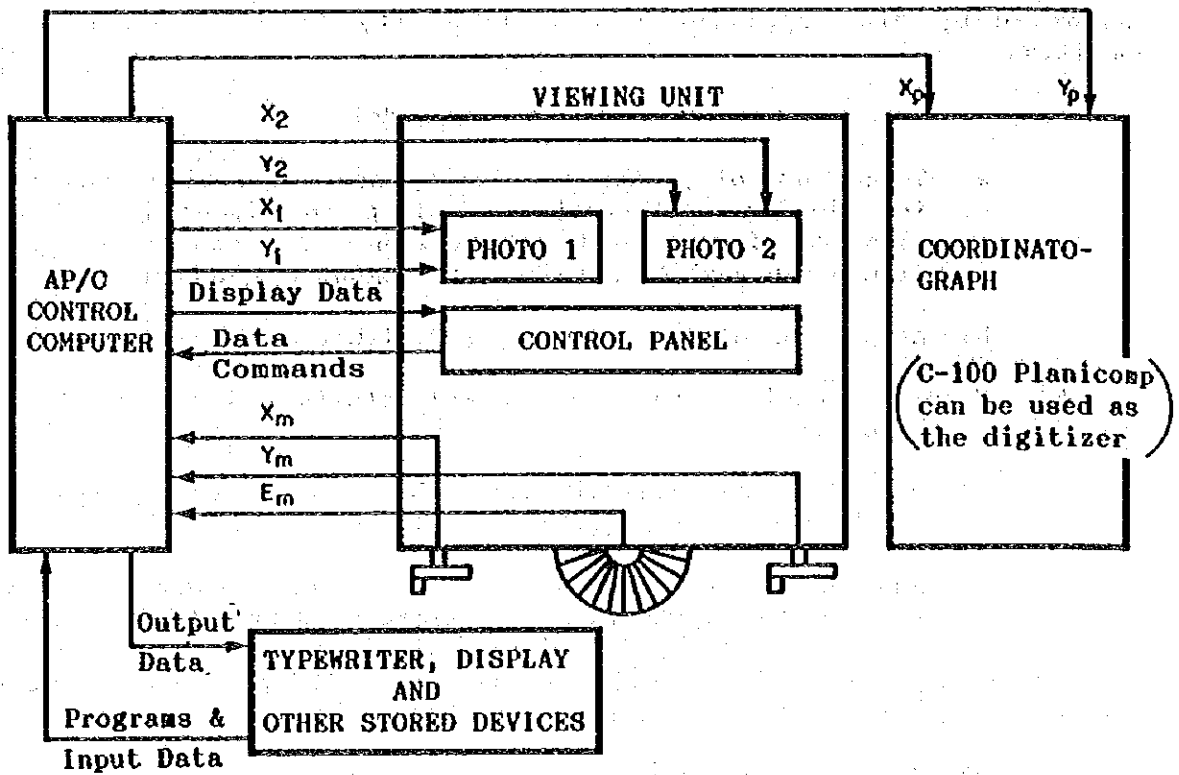


Fig. 2-5.9 SYSTEM DIAGRAM FOR ANALYTICAL PLOTTER AP/C4 (OR C-100 PLANICOMP)

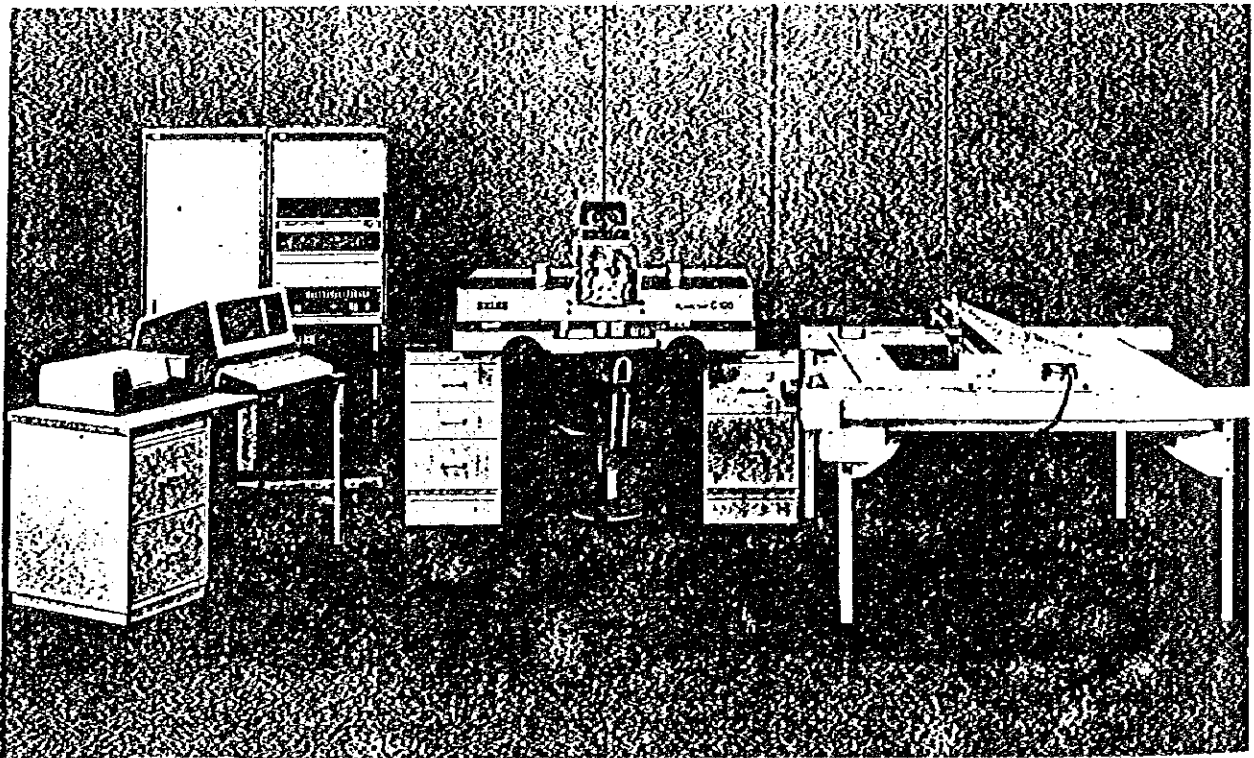


Fig. 2-5.10 PLANICOMP C100 (COURTESY, CARL ZEISS, OBERKOCHEN, WEST GERMANY)

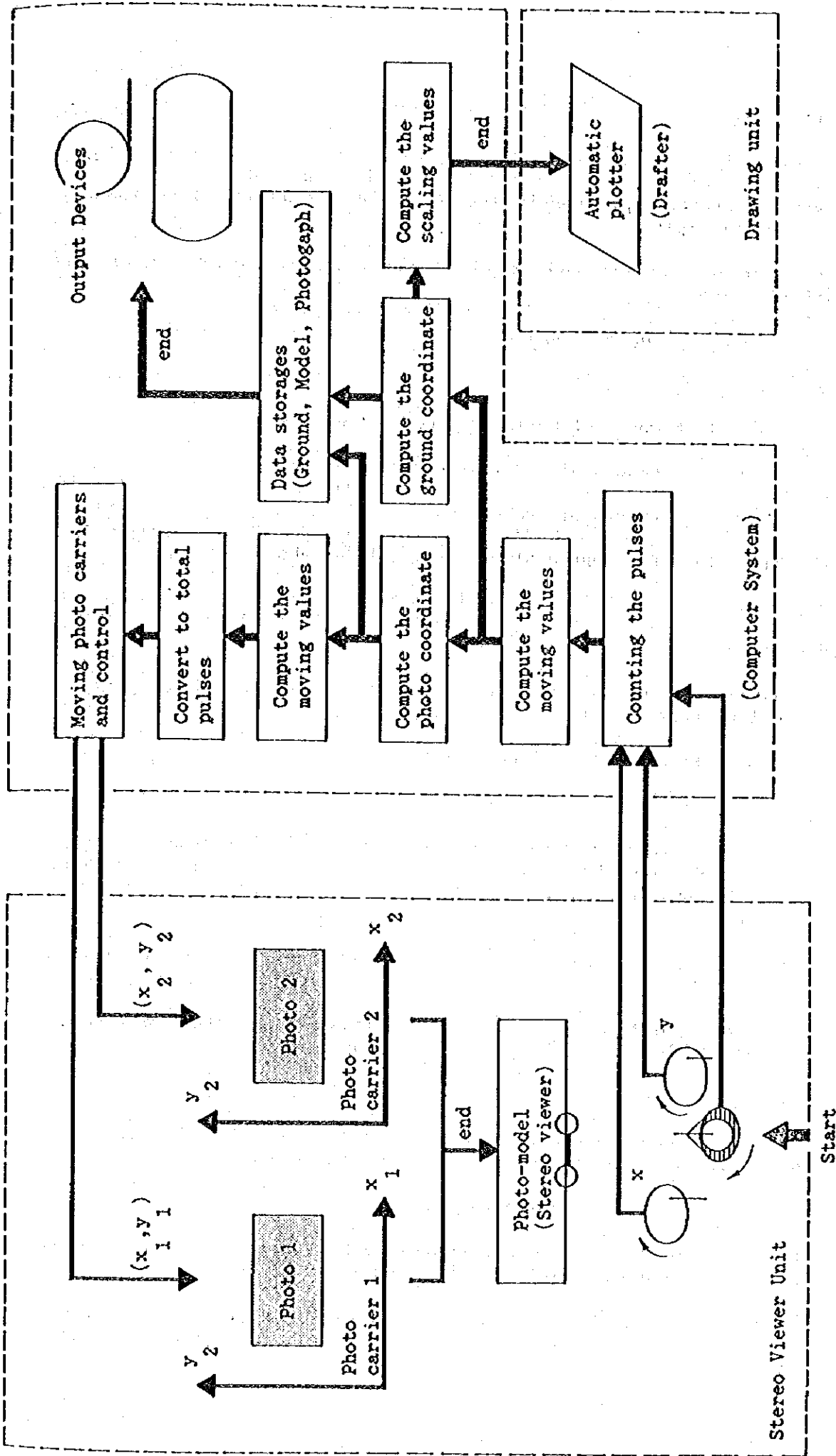


FIG. 2-5.11 DATA FLOW CHART OF ANALYTICAL PLOTTER (PLANICOMP C-100)

## 2. Planning

### (Standards)

Numbers of stereoscopic models for a single strips are less than 15 models in principle.

Numbers of horizontal and vertical control points are under the following rules.

$$N_h = N_v = n/2 + 2$$

where

$N_h$  : Numbers of horizontal control points

$N_v$  : Numbers of vertical control points

$n$  : Numbers of stereoscopic models

Block adjustment are adopted almost same rule as above mentioned.

## 3. Execution

### (Instruments)

Main instruments to be used for aerotriangulation are comparater, analytical plotter or 1st class stereo plotter.

### (Selection of Pass Points and Tie Points)

#### (1) Pass Points

- a) Pass point is fixed near the center of photograph and wing points on the line drawn through or near the center of photograph and perpendicular to the base line, except on water part.
- b) Wing points are placed at a distance of 7-10 cm from pass point.
- c) Pass point and wing points are fixed on as flat an area as possible.

#### (2) Tie Points

- a) More than one tie point shall be selected in a model.
- b) They shall be identifiable on the photographs of both strips.

### (Marking of Pass Points and Tie Points)

Pass-points, tie-point and other control point shall be marked on the positive films with the point transfer device.

### (Measurements of Photo Coordinates)

Coordinated of control points, pass points, tie point and fiducial mark on the photographs shall be measured twice with mono (or stereo) comparater instrument when the difference is less than 0.02 mm the mean is adopted.

### (Inner Orientation)

Distortion and shrinkage included each diapositive is required to be corrected in principle.

Measured error of distance between fiducial marks shall be less than 0.03 mm.

### (Relative Orientation)

All points of a model shall be used for relative orientation.

Residual error of relative orientation shall be less than 0.03 mm on the diapositive.

Discrepancies of model coordinates at a common point between successive models shall be less than 0.05% of flight height for planimetry and height.

### (Adjustment)

Adjustment shall be used all control points included the concerned strip in principle.

Effect of earth curvature shall be corrected.

Discrepancies of coordinates at a common point between adjacent models shall be less than 0.08% of flight height for the planimetry and the height.

### (Results)

Final results are as follows:

- (1) Final results of Aerotriangulation
- (2) Existing map of Aerotriangulation
- (3) Residual list of used control points
- (4) Measured list of photo coordinates of used points
- (5) Final list of adjustment
- (6) List of accurate checking

## 2.5.7 Mapping

### 1. General

#### (General)

"Mapping" in this chapter means the products of original drawing map with stereoscopic plotter in relation to the field identification survey and aerotriangulation.

(Stereoscopic Plotter)

Plotting equipment to be used shall have satisfied the ability of drawing or measurements.

(Scale of Original Map)

The original drawing map are usually same scale as the final map.

(Standard of Sheet Thickness)

The material and thickness of the sheet will be specified.

The sheet are usually used the stable polyester with a minimum thickness of 0.1 mm.

(Sheet Size)

The sheet size specified. Typical size are:

Map Scale	Typical size
1 : 500	x
1 : 1,000	x
1 : 2,000	x
1 : 4,000	x
1 : 8,000	x
1 : 10,000	x

(Plot of Control Points)

Plotting of control points to be used the orientation of photo-models and drawing of neat line and coordinate grid are done with the Coordinate-plotting Instrument or the Automatic drafter.

The maximum error of plotting are less than 0.2 mm.

(Control points are included the pass points, tie points and supplemental points)

(Orientation)

The relative orientation is carried out by using more than 6 points on a photo-model.

The absolute orientation is carried out by using pass points and control points.

Residuals of orientation are as follows:

- (1) Residual parallax of relative orientation shall not be exceed 0.02 mm.
- (2) Height error of absolute orientation shall not exceed the following value for each map scale.

Map Scale	Maximum Error
	less than
1 : 500	0.2 m
1 : 1,000	0.3
1 : 2,000	0.5
1 : 4,000	1.0
1 : 8,000	1.4
1 : 10,000	1.5

#### (Plotting)

Plotting of geographic details is restricted to inside a limit obtained by connecting pass points.

#### Planimetric Drawing

The maps shall shown planimetric features identifiable on or interpretable from the aerial photographs, including such features as buildings; canals, ditches, and reservoirs; trails, roads, highways, sidewalks, and alleys; rail-roads; ferry slips; fords; quarries and borrow pits; cemeteries; orchards and wooded areas; large lone trees; visible traces of utility lines and their poles and towers; underground cables; pipelines and sewers; billboards; and fences and walls. Such structures as bridges, trestles, tunnels, piers, retaining walls, dams, power plants, transformers, transportation terminals, airfields, and tanks shall also be shown. Such drainage features as rivers, streams, lakes, ponds and swamps shall be shown as well as such recreational facilities as parks, golf courses, and athletic fields.

Buildings and similar dimensionable objects shall be accurately outlined on the maps to actual scale.

Monumented horizontal control stations and bench marks used in making the maps shall be shown. In addition, other permanent control marks recovered during the course of the project shall also be shown, the

objective being to present an even distribution of control on the published maps.

All mapped information shall be shown in accordance with the symbols, style, and lineweights.



Expression of Map Contents  
Administrative Boundary

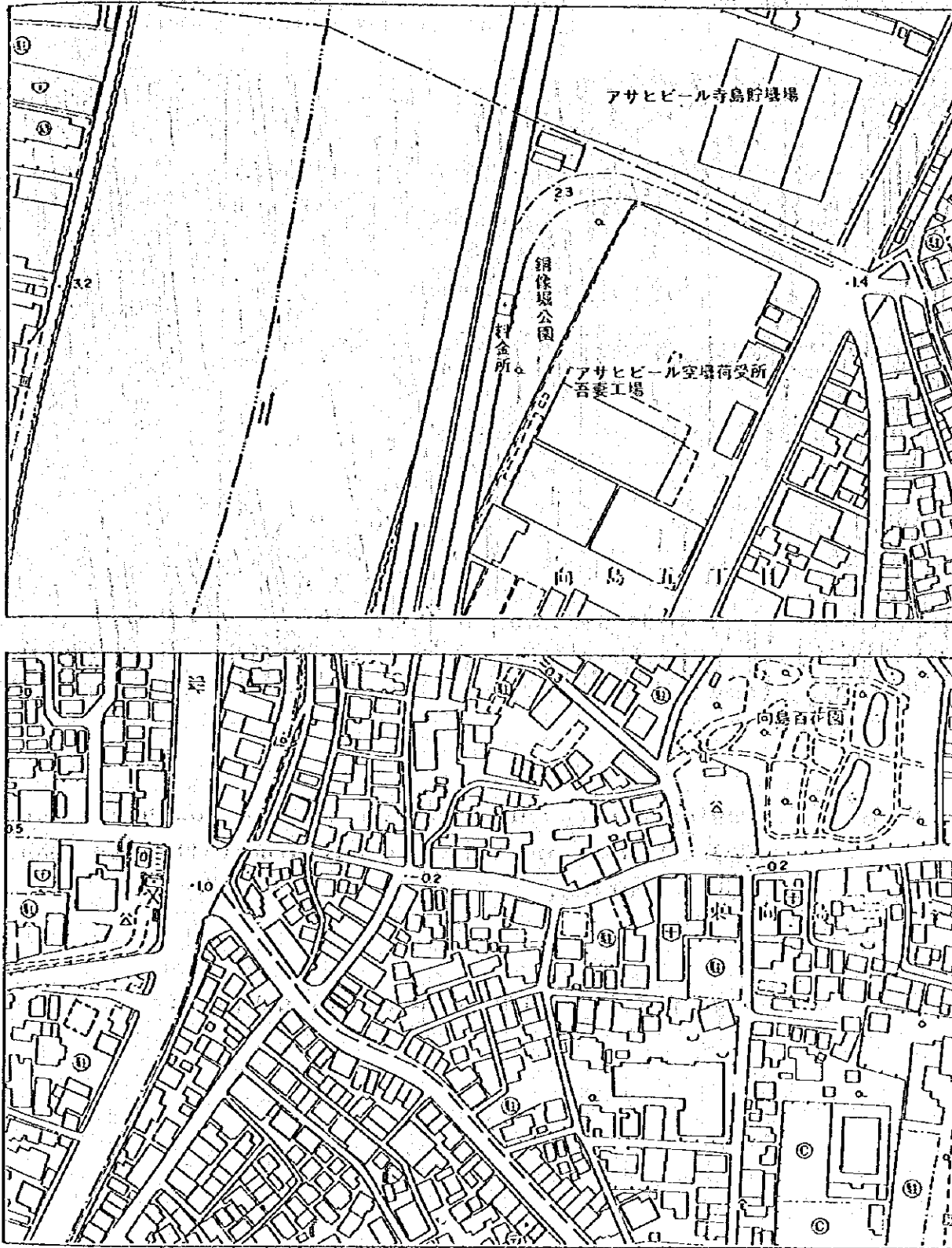
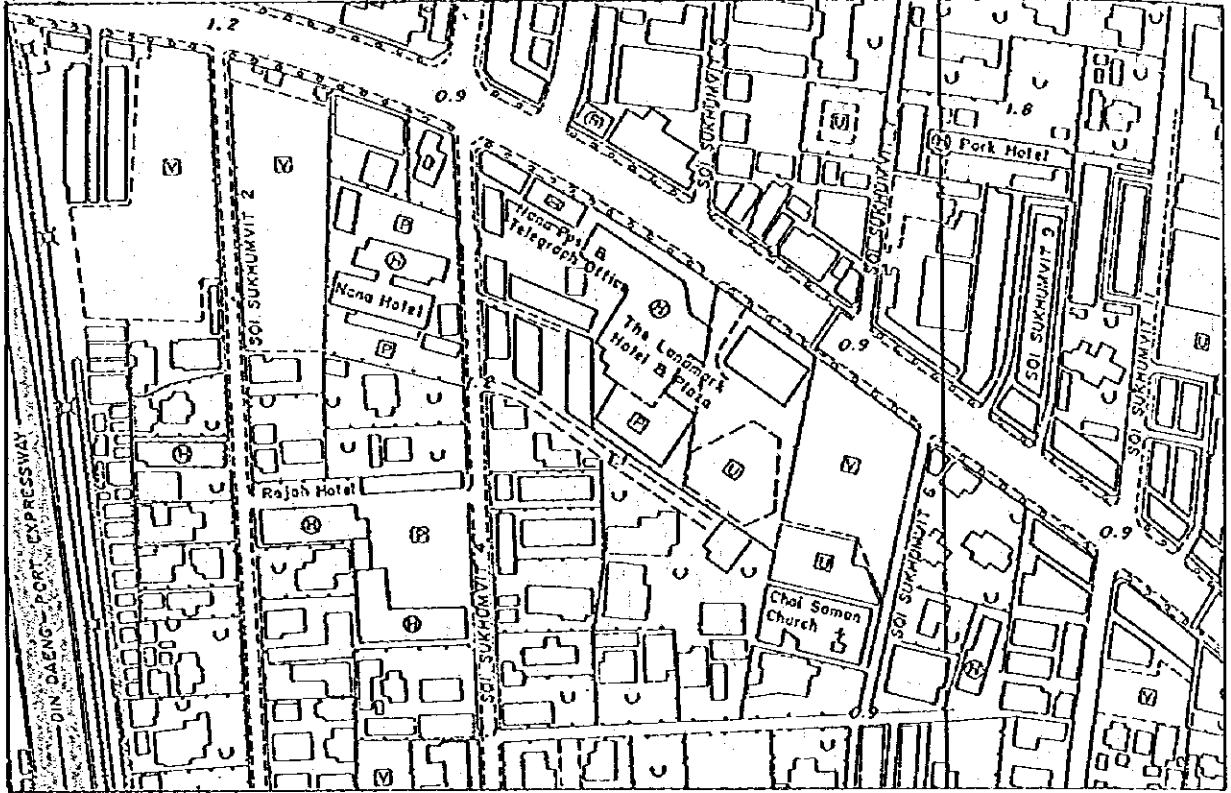


Fig. 2-5.12 EXPRESSION OF MAP CONTENTS

Expression of Map Contents  
Housing, Symbols and Names



Note : drawing a clear distinction between low house (or wooden house) and concrete building with more than 2 or 3 stories.

Fig. 2-5.13 EXPRESSION OF MAP CONTENTS

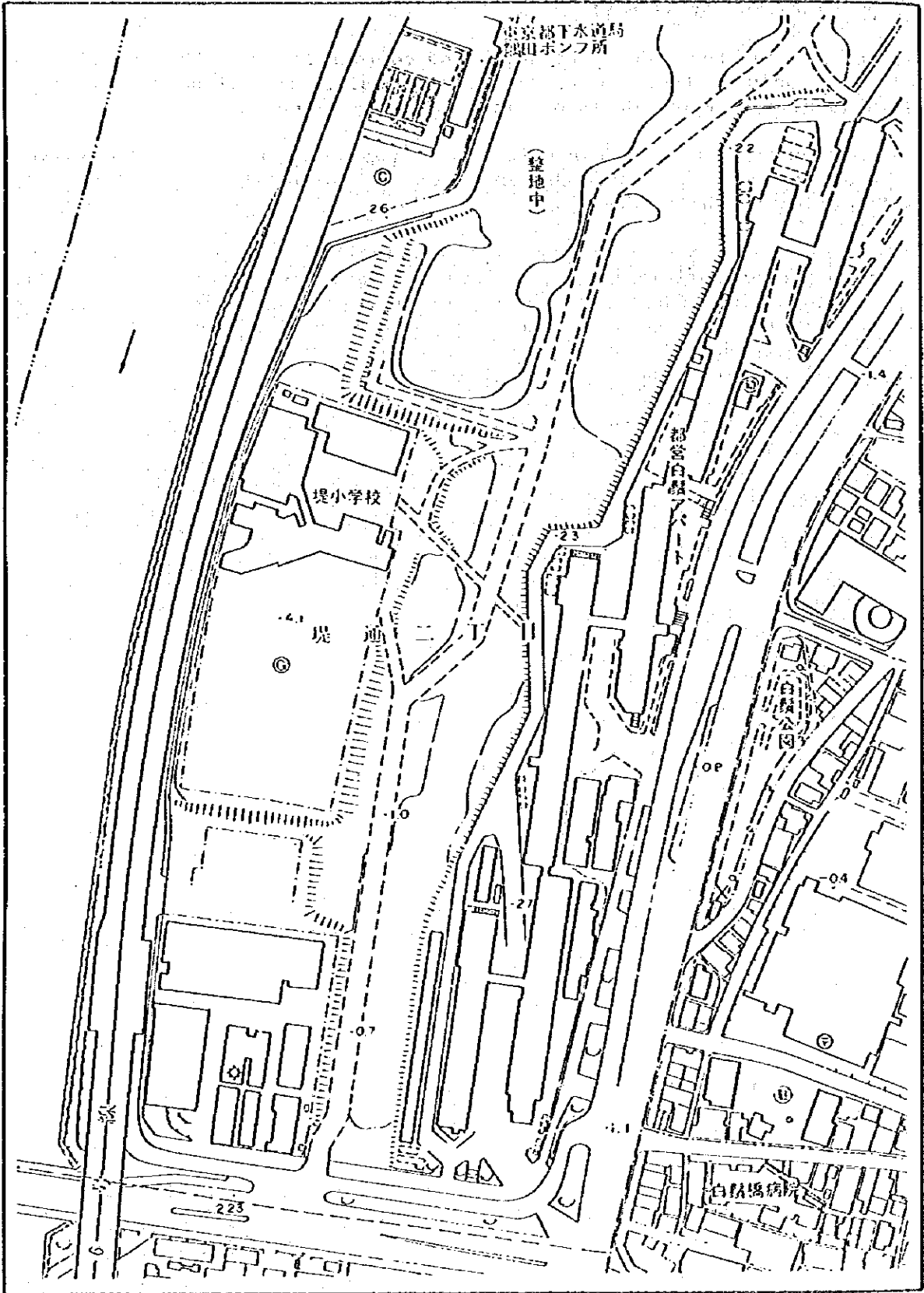
(Representation of Relief)

Relief may be presented by hachures and contour line of the symbols used on maps, only contours indicate elevation directly and quantitatively. It also give a important information for city planners.

The contours and spot elevations are popularly used as the relief information of maps, and the hachures are also given the important information for plain area.

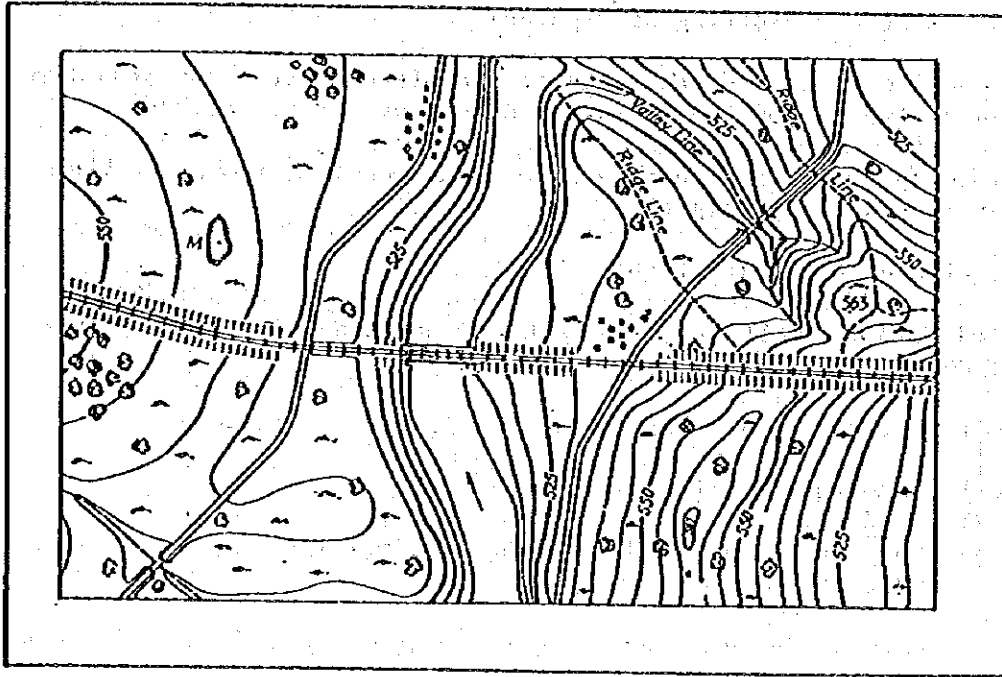
Spot elevations determined photogrammetrically shall be shown on the maps in proper position at water level on lakes, reservoirs, and ponds; on hilltops; in saddleles; at bottoms of depressions; at intersections of principal streets; and at ends of bridges. If a direction of the water-flow is drawn on the stream position, it is more effective to know the ground slope for alluvial topographic area.

Artificial structures (bank or cut places) are usually expressed with the hachures as shown in Fig. 2-5.14 .



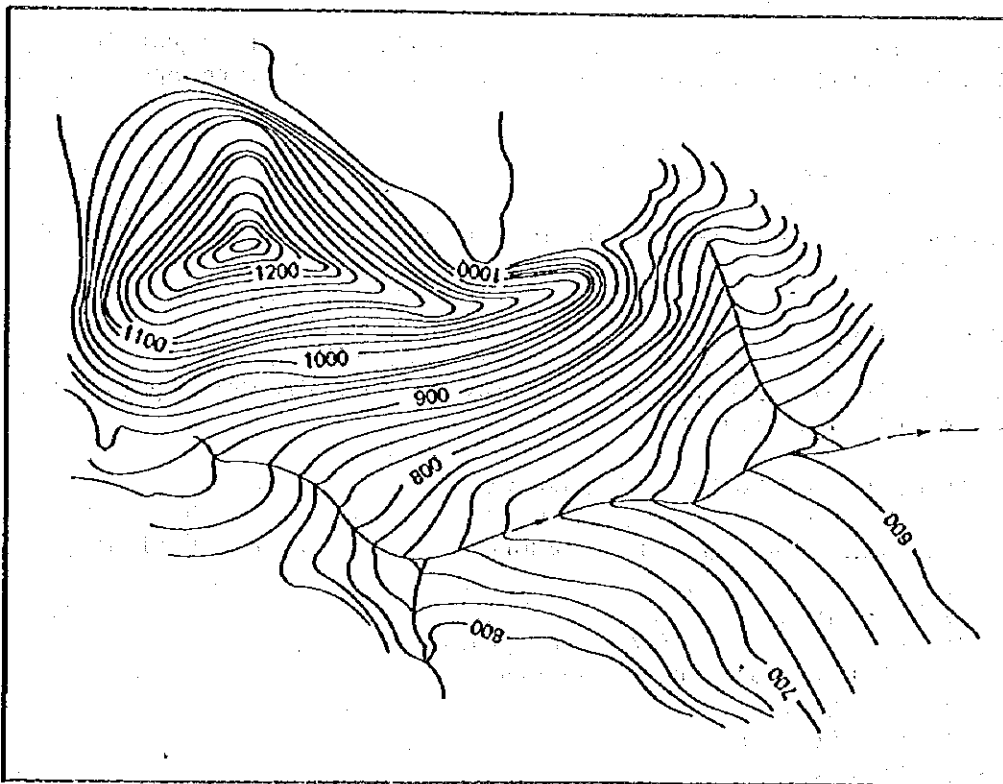
Artificial ground surface are expression with the hachures  
 (Town map 1:2500, JAPAN)

Fig. 2-5.14 AN EXAMPLE OF THE HACHURES



(A) Contour line and hachures

Note : Direction of water stream in plain area is given the important information for map interpreters (planners)



(B) Mountainous terrain showing a river and ridge  
(by Introduction to Surveying)

Fig. 2-5.15 CONTOUR LINE

(Measurement and Plotting of Figures)

Horizontal positions of the figures are plotted after the measurement of distance directly by the radiation method or off-set method.

Relief of the land surface are constructed contour line after measurement of spot height.

(Selection of Spot Elevation)

Spot elevation shall be plotted to be same density considering to be interpreted the topography.

Spot elevations shall be measured the significant points.

- 1) Summits and depressions
- 2) Road intersection
- 3) Mouth of valley and junction of river
- 4) Bridges and water surface

Density of spot height standardize one point per 4 cm square on the sheet.

2.6 Field Editing

2.6.1 General

The plane Table survey are works for product the topographic map will plane Table Unit to measure the topography and feature and plot on the Table.

(Control Point to be Used)

The plane Table survey are based on the 4th control points or more accurate points.

(Mapping Scale)

Mapping Scale are 1/1,000 or more large scale in principle. Standard Scale are 1/500 or 1/1,000.

(Classification of Works and Procedure)

Classification of works and procedure are shown in the following items:

- (1) Planning
- (2) Establishment of control points
- (3) Plotting of control points
- (4) Detailed survey
- (5) Compilation
- (6) Drafting
- (7) Arrangement of results

### 2.6.2 Establishment of Control Points

(General)

"Control points" in this chapter means establishment of new control points required for the orientation of plane table at detailed survey.

(Number of Control Points)

Density of control points per unit area standardize as the following table:

Distribution density per 100 x 100 m			
Map Scale \ Area	Dense area of city	Sub-urban area	Others
1/ 250	7 points	6 points	7 points
1/ 500	6	5	6
1/1,000	5	4	4

(Method)

Control points survey is done by the rules of 4th control survey.

#### Plotting of Control Points

(General)

Established control points shall be plotted on the sheet accurately according the coordinate system.

(Method)

Plotting dare done by the coordinate-plotting instruments or same accurate method.

Coordinate grid lines and horizontal control points shall be plotted within 0.2 mm.

### 2.6.3 Detailed Survey

(General)

(Instruments to be Used)

Main instruments to be used the detailed survey are as follows:

- (1) Plain table unit
- (2)
- (3) Alidade
- (4) Distant measurement instrument of tape (more than 1/1,000 accurately)
- (5) Wooden pole

## 2.7 Map Compilation

### 2.7.1 General

"Compilation" means the description of the results of the detailed survey according to the map specification.

### 2.7.2 Map content

#### a. Marginal data

The items constitute the minimum information to be shown in the margins, and the following items shall be included in margin of each map:

#### Items common to all maps:

Title block  
Project name  
Project location  
Contracting authority  
Sheet name  
Map scale  
Credit notes  
North arrow  
Bar scale  
Accuracy note  
Map location diagram

#### Items that must be specifically tailored to each map:

Position in map location diagram  
Adjoining sheet designations  
Geographic coordinates  
Preparation date and photo date  
Road classification  
Route symbols and other special symbols (poles, manholes, culverts, underground utilities).  
Datum  
Required signatures  
Contracting official  
Professional engineer/surveyor  
Revision block



### 2.7.3 Names and Labels

The maps shall show the significant names data for map interpreters (planners).

Selection between names and symbols is decided by the informational quality.

The following are examples of categories of named features in an urban area.

Corporate, locality, and boundary names

Parks, public squares, monuments, and cemeteries

Linear and hydrographic features

Universities, colleges, public schools, and large private schools

Historic, landmark, and unusually important churches

Shopping centers

Main and secondary streets, rail-roads, transit lines

Streets and roads are named and spelled in accordance with official designation.

The generic part of the name is spelled out in full, if space permits. Names are positioned within the casings of the streets where space permits.

Important and prominent buildings are named or identified. Normally, individual buildings within a complex are not named or identified.

### 2.7.4 Symbols

Objects are represented on a map by symbols, many of which are conventional now.

Topographic map published by Royal Thai Survey Department is represented the symbols on map scale 1 to 25,000 and 1 to 50,000. But it seems that the informative symbols for urban facility, public house, and physical feature of earth's ground conditions are a few.

Therefore, symbols in current use shall be as shown the symbol chart.

Symbols are useful informations for the urban analysis and urban planning.

COMPARISON OF CURRENT SYMBOLS FOR TOWN MAPS

Name	Line Width	Symbol (Thai)	Note Symbol (Japan, U.S.A.)
National Boundary			
Country Boundary			
Provincial Boundary			
Ampoe Boundary			
Taambon Boundary			
Military and Other Area			
For Boundaries			
Comprehensive Planning Zoning			
Specific Planning Zoning			
Municipal Area			
Sanitary Area			
Taambon Zoning			
Village Zoning			
For Planning Boundaries			
Low House Building			
Foundation, Ruin or Temporal House			
For Structures			
Government Building			
Changvat Office			
Ampoe Office			
King Ampoe Office			
School			
			(Primary)
			(Lower Secondary)
			(Higher Secondary)
Monastery without Temple			
Monastery			
Church			
Mosque			
Hindu			
Kinds of Structures			

Fig. 2-7.1a MAP SYMBOLS (1)

COMPARISON OF CURRENT SYMBOLS FOR TOWN MAPS (CONT.)

Name	Line Width	Symbol (Thai)	Note Symbol (Japan, U.S.A.)
Public Health Center			
Kindergarden			
Sewage Disposal			
Filtration			
Tax Office			
Count House			
Hospital			
Police Office			
Police Station			
Fire Station			
Post Office			
Tele-communication Office			
Factory			
Shrine			
Tomb			
Kinds of Structures			
Artificial Surrounding Material			
Fence With Surrounding Wall with Soils			
Land Marks			37.21 37.21 14.83 25.17 25.2     

Fig. 2-7.1b MAP SYMBOLS (2)

COMPARISON OF CURRENT SYMBOLS FOR TOWN MAPS (CONT.)

Name	Line Width	Symbol (Thai)	Note Symbol (Japan, U.S.A.)
Footpath			10 20 05 15 
Road with Side Walk			10 15 
Tunnel			10 15 
Concrete Bridge			
Wooden Bridge			
Road Side Tree			10 15 
Cutted Road			10 15 
Embanked Road			10 15 
Crossing and Underpass			10 15 
Rail Road			
Crossing and Underpass			
Bridge			
Specific Rail Road			
For Roads			
For Railroads			
For Land Classifications			
Paddy Field			" " " " V V V V Y Y Y Y
Plantation			o o o o o o o o
Woods (Broad Leaf)			o o o o o o o o
Wood (Needle Leaf)			o o o o o o o o
Uncultivated Land			o o o o o o o o
Laundation area Swamp			o o o o o o o o
Note Symbol (Japan, U.S.A.)			

FIG. 2-7.1c MAP SYMBOLS (3)

COMPARISON OF CURRENT SYMBOLS FOR TOWN MAPS (CONT.)

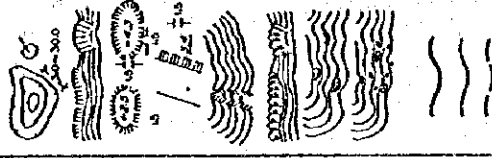
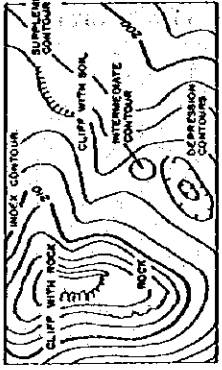

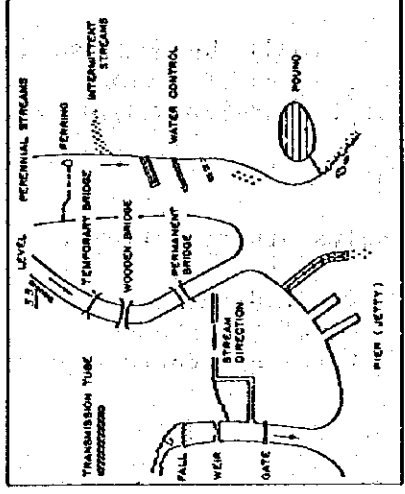
	Name	Line Width	Symbols (Thai Standard)	Line Width	Symbols (Japan and International Standard)
For Physical Land Features	Index Contour Intermediate c. Supplementary c.				
For Hydrography	Permanent River Pound Sea Side Swamp Area Intermittent Stream				 <p>Note:</p>

Fig. 2-7.1d MAP SYMBOLS (4)

## 2.8 Drafting

Final maps shall be scribed or drafted on stable polyester with a minimum thickness of mm.

Symbol, style and lineweights shall be shown on the chart.

## 2.9 Final Results

Final results are as follows:

- (1) Drafted sheets
- (2) Completed sheets
- (3) Lists of Accurate checking

**Chapter 3**  
**Map Projection**





Chapter 3  
Map Projection

3.1 Transverse Mercator Projection

3.1.1 History and Features

Thailand is adapted the Universal Transverse Mercator (UTM) projection for designating rectangular coordinates on state survey system.

The UTM is the ellipsoidal transverse mercator and its spherical form was invented by cartographer Johan Heimrich Lambert (1728-77).

While Lambert only discussed about its ellipsoidal form, Carl Friedrich Gauss led up this problem in 1822. Kruger published the providing formulas suitable for calculation relative to the ellipsoid in 1919. This is most famous formulas for computation. It is, therefore often called the "Gauss conformal projection" or "Gauss-Kruger projection".

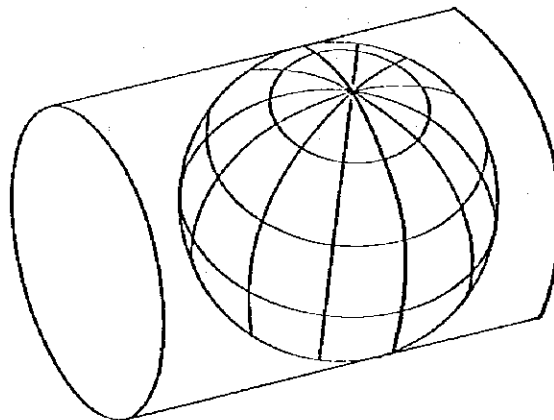


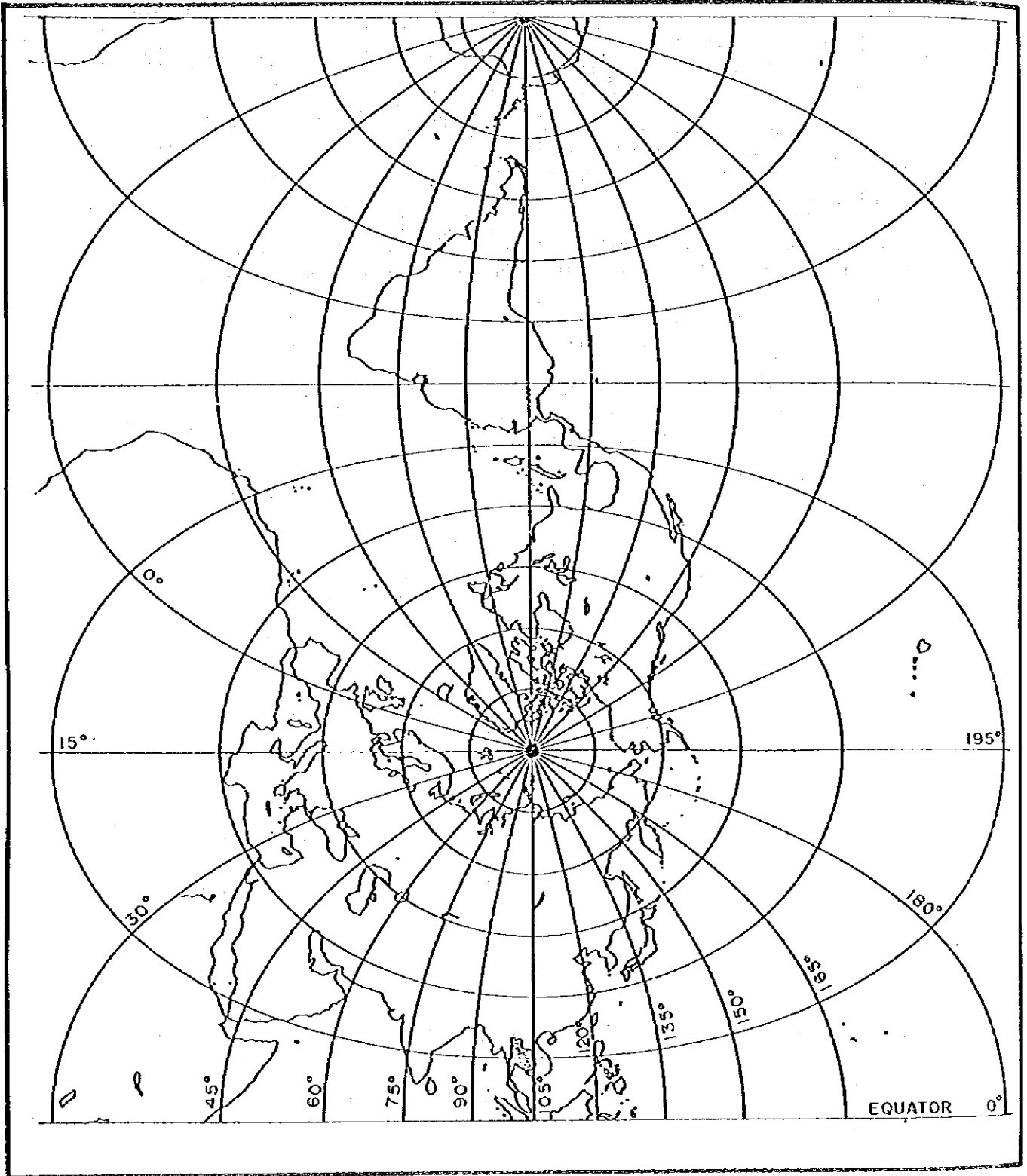
Fig. 3-1.1 TRANSVERSE CYLINDRICAL PROJECTION

This projection is shown in Fig. -1. The meridian and parallels are no longer the straight line except for the equators, control meridian, and each meridian 90 away from central meridian.

Other meridian and parallels are complex curve as shown in figure.

The ellipsoid form is also exactly conformal, but its scale error is slightly affected by factor other than the distance alone from the central meridian and were limited to series for relatively narrow bands of about 4 longitude.

Therefore, many countries' maps were drawn with reference to their own central meridians of the UTM zones with the 0.9996 central scale factor.



**Fig. 3-1.2 TRANSVERSE MERCATOR PROJECTION (UTM)**

This projection has constant scale along any chosen central meridian

In Japan the Gauss-Kruger projection was adopted as national coordinate system with the 0.9999 central scale factor.

Its coordinate system are divided into 17 zones, each zone is limited in width about 90 km. from the central meridian.

This system are applied to making middle scale or large scale maps, cadastral survey, and many types of engineering works.

In case of local coordinate system, the central scale factor should be 1.0000.

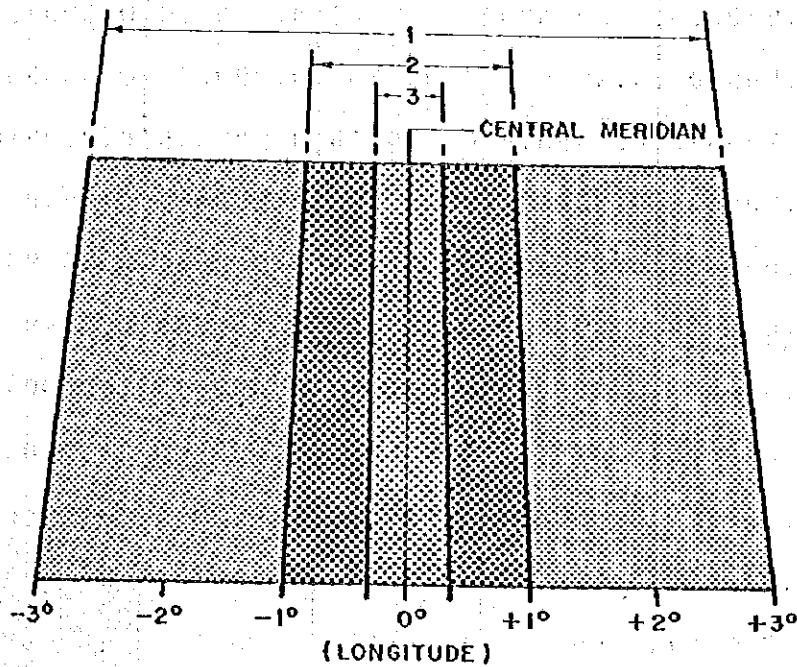
The coordinate systems in Japan are given in the following Table.

Table 3-1.1 COORDINATES SYSTEM AND SCALE FACTORS AT ORIGIN (JAPAN)

Map Projection	Scale Factor	Survey and Maps	Limits of One Zone
Oblique poly-conic projection		National map	
Transverse Mercator projection	0.9996 Universal transverse mercator P.	National base map (1:25,000, 1:50,000)	-3°~+3° for longitude (-300 km ~ +300 km)
	0.9999  State coordinate (17 zones)	City or Town map (1:2,500)  Prefecture map (1:5,000 1:10,000) Cadastral survey Engineering works Public works	-1°~+1° (-100 km ~ +100 km)
	1.0000 Local coordinates system	Local engineering project Local map	

Note : Prefecture's map scale are conventional,  
Figures for limits are approximate values.

The limits of mapping area for each scale at the central meridian is according to the Table 3-1.3.



LEGEND:

- |   |  |                  |            |
|---|--|------------------|------------|
| 1 |  | UTM 1 ZONE       | ( 0.9996 ) |
| 2 |  | STATE COORDINATE | ( 0.9999 ) |
| 3 |  | LOCAL COORDINATE | ( 1.0000 ) |

Fig. 3-1.3. COMPARISON OF EACH 1 MAP ZONE

Table 3-1.2 SCALE ON CENTRAL MERIDIAN AND SCALE FACTORS AT THE LONGITUDINAL DIFFERENCE

(LATITUDE IS FIXED AT 15°)

LONGITUDE DIFFERENCE	SCALE FACTORS AT ORIGIN					
	0.9995	0.9996	0.9997	0.9998	0.9999	1.0000
0° 0'	0.999500	0.999600	0.999700	0.999800	0.999900	1.000000
0° 10'	0.999504	0.999604	0.999704	0.999804	0.999904	1.000004
0° 20'	0.999516	0.999616	0.999716	0.999816	0.999916	1.000016
0° 30'	0.999536	0.999636	0.999736	0.999836	0.999936	1.000036
0° 40'	0.999564	0.999664	0.999764	0.999864	0.999964	1.000064
0° 50'	0.999599	0.999699	0.999799	0.999899	0.999999	1.000099
1° 0'	0.999643	0.999743	0.999843	0.999943	1.000043	1.000143
1° 10'	0.999695	0.999795	0.999895	0.999995	1.000095	1.000195
1° 20'	0.999754	0.999854	0.999954	1.000054	1.000154	1.000254
1° 30'	0.999822	0.999922	1.000022	1.000122	1.000222	1.000322
1° 40'	0.999897	0.999997	1.000097	1.000197	1.000297	1.000397
1° 50'	0.999981	1.000081	1.000181	1.000281	1.000381	1.000481
2° 0'	1.000072	1.000172	1.000272	1.000372	1.000472	1.000572
2° 10'	1.000172	1.000272	1.000372	1.000472	1.000572	1.000672
2° 20'	1.000279	1.000379	1.000479	1.000579	1.000679	1.000779
2° 30'	1.000395	1.000495	1.000595	1.000695	1.000795	1.000895
2° 40'	1.000518	1.000618	1.000718	1.000818	1.000918	1.001018
2° 50'	1.000649	1.000749	1.000849	1.000949	1.001049	1.001149
3° 0'	1.000789	1.000889	1.000989	1.001089	1.001189	1.001289
3° 10'	1.000936	1.001036	1.001136	1.001236	1.001336	1.001436
3° 20'	1.001092	1.001192	1.001292	1.001392	1.001492	1.001592
3° 30'	1.001255	1.001355	1.001455	1.001555	1.001655	1.001755
3° 40'	1.001425	1.001525	1.001625	1.001725	1.001825	1.001925
3° 50'	1.001601	1.001701	1.001801	1.001901	1.002001	1.002101

Note :  Mapping Area

0.9996 for the UTM projection  
 0.9999 for the state coordinate system (Japan)

### 3.1.2 Formulas for the Ellipsoidal Projection

The most practical equations is a set of series approximations which converge rapidly in a zone extending 3 to 4 of longitude from the central meridian.

The formulas are given by

$$\begin{aligned}
 x &= k_0 N [A + (1 - t + c) A^3/6 \\
 &\quad + (5 - 18t + t^2 + 72c - 58te^2) A^5/120 + \dots] \\
 y &= k_0 \{ (M - M_0) + N \tan \theta [A^2/2 + (5 - t + 9c + 4c^2) A^4/24 \\
 &\quad + (61 - 58t + t^2 + 270c - 330e^2) A^6/720] \} \\
 k &= k_0 \{ 1 + (1 + c) A^2/2 + (5 - 4t + 42c + 13c^2 - 28e^2) \\
 &\quad A^4/24 + (61 - 148t + 16t^2) A^6/720 \}
 \end{aligned}
 \tag{3-1.1}$$

where

$k_0$  = Scale factor on central meridian (e.g., 0.9996 for the UTM, 0.9999 for the state coordinate system less than 100 km from the central meridian, and 1.0000 for the local coordinates system, in general)

$$e'^2 = e^2 / (1 - e^2)$$

$$N = a / (1 - e^2 \sin^2 \theta)^{1/2}$$

$$t = \tan^2 \theta$$

$$c = e'^2 \cos^2 \theta$$

$$A = \cos \theta (\lambda - \lambda_0) \text{ with } \lambda \text{ and } \lambda_0 \text{ in radian}$$

and

$$\begin{aligned}
 M &= a \{ (1 - e^2/4 - 3e^4/64 - 5e^6/256 - 175e^8/16384) \theta \\
 &\quad - (3e^2/8 + 3e^4/32 + 45e^6/1024 + 105e^8/4096) \sin 2\theta \\
 &\quad + (15e^4/256 + 45e^6/1024 + 525e^8/16384) \sin 4\theta \\
 &\quad - (35e^6/3072 + 175e^8/12288 + \dots) \sin 6\theta \\
 &\quad + (315e^8/131072 + \dots) \sin 8\theta
 \end{aligned}$$

: Latitude in radians

(3-1.2)

On above formulas, M is the true distance along the central meridian from the Equator to  $\theta$ . M is expressed the following formulas for practical use.

$$M = A_1 \theta + A_2 \sin 2\theta + A_3 \sin 4\theta + A_4 \sin 6\theta + \dots \quad (3-1.3)$$

where  $A_1, A_2, \dots, A_n$  are the constants for the adopted ellipsoid.

The coefficients are determined from above coefficient's terms for the adopted ellipsoid.

A1	=	6366680.186709024
A2	=	- 15900.783137179
A3	=	16.546763164
A4	=	- 0.021428106
A5	=	0.0000300964
A6	=	- 0.000000451

$M$  is the meridian distance from equator on the datum of projection and can be computed from above equation for  $\theta_0$ , the latitude crossing the central meridian  $\lambda_0$ .

These transformations can be computed with micro-computer. Fig. 3-1.4 shows the general flow chart of above procedure, and additional details of computation is listed in Appendix I.



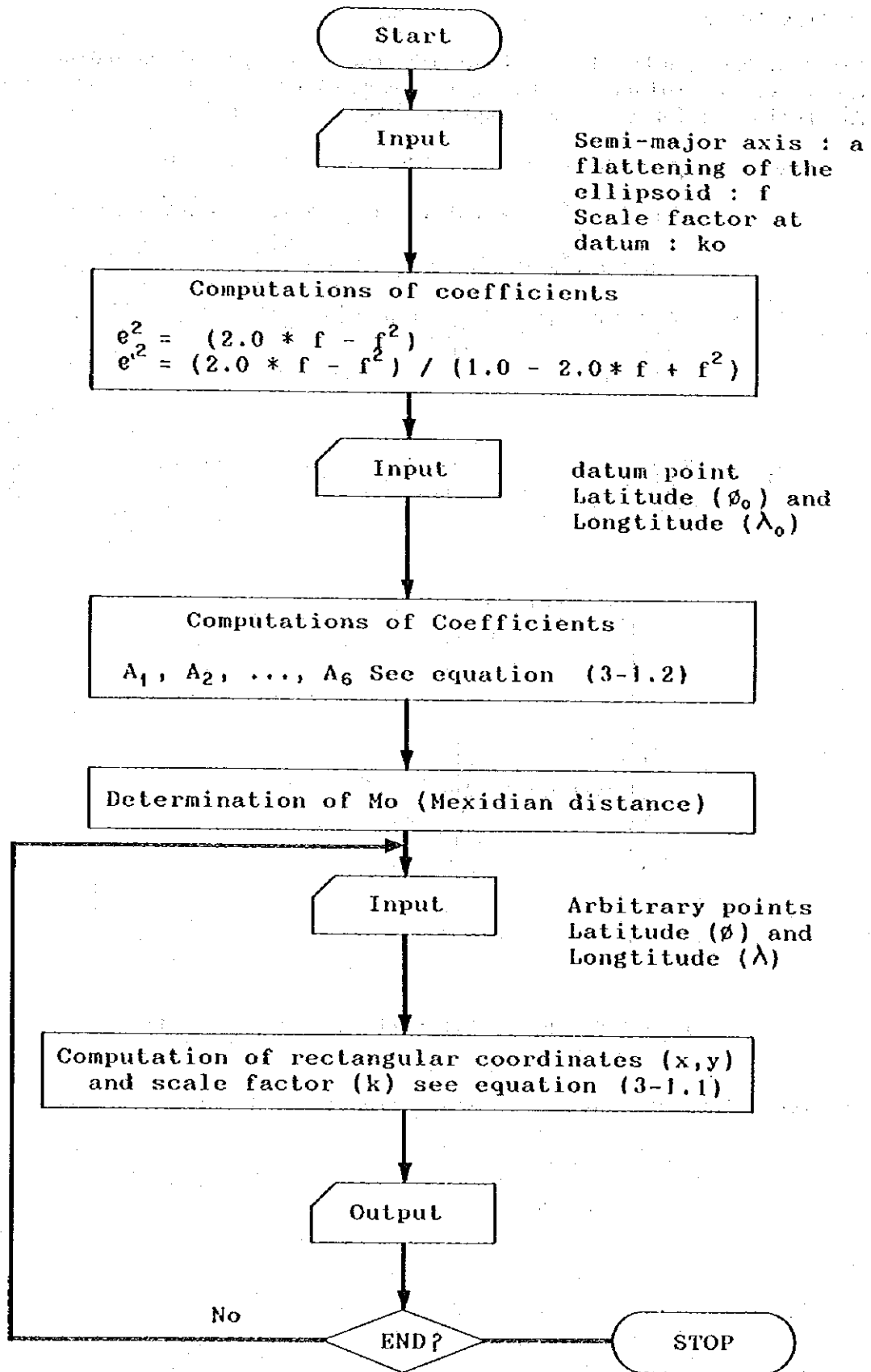


Fig. 3-1.4 COMPUTATION FOR TRANSVERSE MERCATOR PROJECTION

Inverse formulas

The Inverse problem is to determine the geographical coordinates from the ground coordinates. There are many kinds of formulas and methods. The following formulas were expressed by Thomas, 1952.

$$\begin{aligned} \phi &= \phi_1 - (N_1 \tan \theta_1 / R_1) \left[ \phi / 2 - (5 + 3t_1 + 10c_1 - 4c_1^2 - 8e_1^2) \phi^3 / 24 + (61 + 90t_1 + 298c_1 + 45t_1^2 - 252e_1^2 - 3c_1^2) \phi^5 / 720 \right] \\ \lambda &= \lambda_0 + \left[ \phi - (1 + 2t_1 + c_1) \phi^3 / 6 + (5 - 2c_1 + 28t_1) \phi^5 / 120 \right] \end{aligned} \tag{3-1.4}$$

where

$\phi_1$  = "footpoint latitude". The latitude at the central meridian which has the same y coordinate as that of the point  $(\phi, \lambda)$ . See below Fig. 3-1.5 .

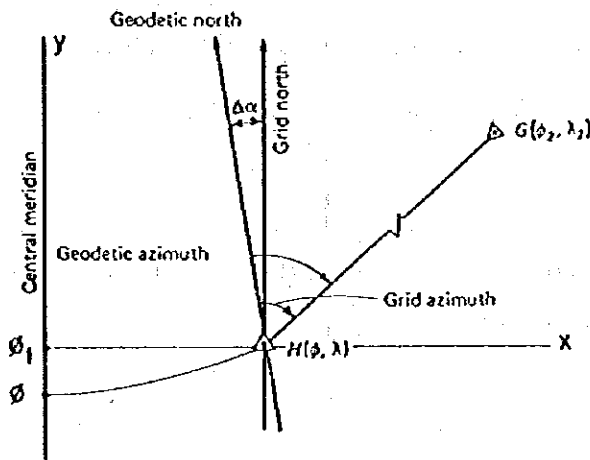


Fig. 3-1.5 FOOTPOINT LATITUDE AND TRUE LATITUDE

The footpoint latitude may be found from the following equation.

$$\begin{aligned} \phi &= u + (3e_1/2 - 27e_1^3/32 + \dots) \sin 2u \\ &+ (21e_1^2/16 - 55e_1^4/32 + \dots) \sin 4u \\ &+ (151e_1^3/96 + \dots) \sin 6u \end{aligned} \tag{3-1.5}$$

where

$$\begin{aligned} e_1 &= [ 1 - (1 - e^2)^{1/2} ] / [ 1 + (1 - e^2)^{1/2} ] \\ u &= M / [ a (1 - e^2/4 - 3e^4/64 - 5e^6/256 \dots) ] \\ M &= Mo + y/ko \end{aligned}$$

$M_0$  is the meridian distance from the equator and can be computed by equation (3-1.3) for  $\theta_0$ .

The other terms are obtained to following equations:

$$e_1^2 = e^2 / (1 - e^2)$$

$$c_1 = e^2 \cos^2 \theta_1$$

$$t_1 = \tan^2 \theta_1$$

$$N_1 = a / (1 - e^2 \sin^2 \theta_1)^{1/2}$$

$$R_1 = a / (1 - e^2) / (1 - e^2 \sin^2 \theta_1)^{3/2}$$

$$\theta = x / (N_1 k_0)$$

### Universal Transverse Mercator projection (UTM)

The Universal Transverse Mercator projection is the ellipsoidal Transverse Mercator to which specific parameters, such as central meridians, have been applied.

Each geographic location in the UTM projection is given x, and y coordinates, in meters, according to the Transverse Mercator projection, using the meridian halfway between the two bounding meridians as the central meridian, and reducing its scale to 0.9996 of true scale as shown in the Fig. 3-1.6.

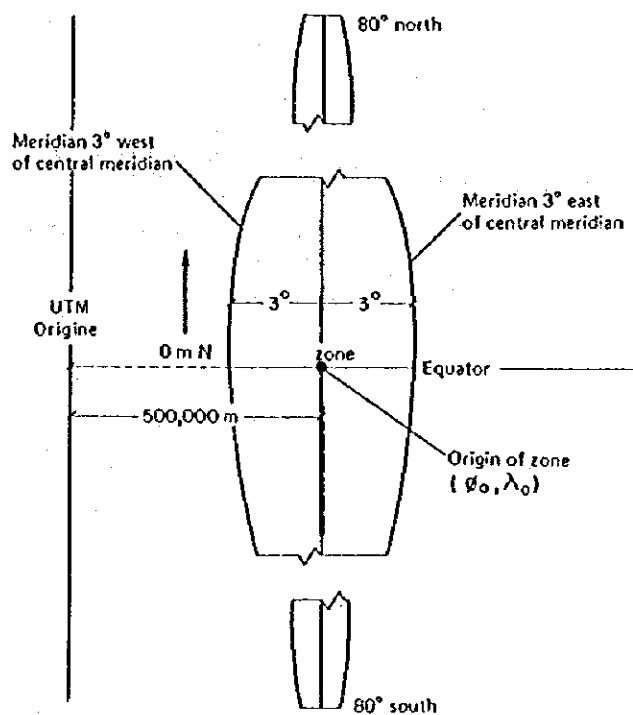


Fig. 3-1.6 THE ORIGIN OF A UTM GRID ZONE

In the Northern Hemisphere, the Equator at the central meridian is considered the origin, with an x coordinates of 500,000.0 m and a y of 0.

Numbers increase toward the north and east, and the UTM coordinates are obtained from equation ( 3-1.1 ) after little modified as shown in the following.

No	:	Origin is on the equator, and Mo = 0
ko	:	Scale factor at central meridian ko = 0.9999
N	:	} UTM coordinates
E	:	

Note : Some official Ellipsoids in use throughout the world are shown below.

Name	Date	Equatorial Radius, $a$ , meters	Polar Radius $b$ , meters	Flattening $f$	Use
GRS 1980 <sup>a</sup>	1980	6,378,137 <sup>*</sup>	6,356,752.3	1/298.257	Newly adopted
WGS 72 <sup>b</sup>	1972	6,378,135 <sup>*</sup>	6,356,750.5	1/298.26	NASA
Australian	1965	6,378,160 <sup>*</sup>	6,356,774.7	1/298.25 <sup>*</sup>	Australia
Krasovsky	1940	6,378,245 <sup>*</sup>	6,356,863.0	1/298.3 <sup>*</sup>	Soviet Union
Internat'l	1924	6,378,388 <sup>*</sup>	6,356,911.9	1/297 <sup>*</sup>	Remainder of the world.†
Hayford	1909				
Clarke	1880	6,378,249.1	6,356,514.9	1/293.46 <sup>**</sup>	Most of Africa; France North America; Philip- pines.
Clarke	1866	6,378,206.4 <sup>*</sup>	6,356,583.8 <sup>*</sup>	1/294.98	
Airy	1849	6,377,563.4	6,356,256.9	1/299.32 <sup>**</sup>	Great Britain
Bessel	1841	6,377,397.2	6,356,079.0	1/299.15 <sup>**</sup>	Central Europe; Chile; Indonesia, Japan.
Everest	1830	6,377,276.3	6,356,075.4	1/300.80 <sup>**</sup>	India; Burma; Paki- stan; Afghan.; Thai- land; etc.

Values are shown to accuracy in excess significant figures, to reduce computational confusion.

<sup>a</sup> Maling, 1973, p. 7; Thomas, 1970, p. 84; Army, 1973, p. 4, endmap; Colvocoresses, 1969, p. 33; World Geodetic, 1974.

<sup>b</sup> Geodetic Reference System. Ellipsoid derived from adopted model of Earth.

<sup>c</sup> World Geodetic System. Ellipsoid derived from adopted model of Earth.

<sup>\*</sup> Taken as exact values. The third number (where two are asterisked) is derived using the following relationships:  $b = a(1 - f)$ ;  $f = 1 - b/a$ . Where only one is asterisked (for 1972 and 1980), certain physical constants not shown are taken as exact, but  $f$  as shown is the adopted value.

<sup>\*\*</sup> Derived from  $a$  and  $b$ , which are rounded off as shown after conversions from lengths in feet.

† Other than regions listed elsewhere in column, or some smaller areas.

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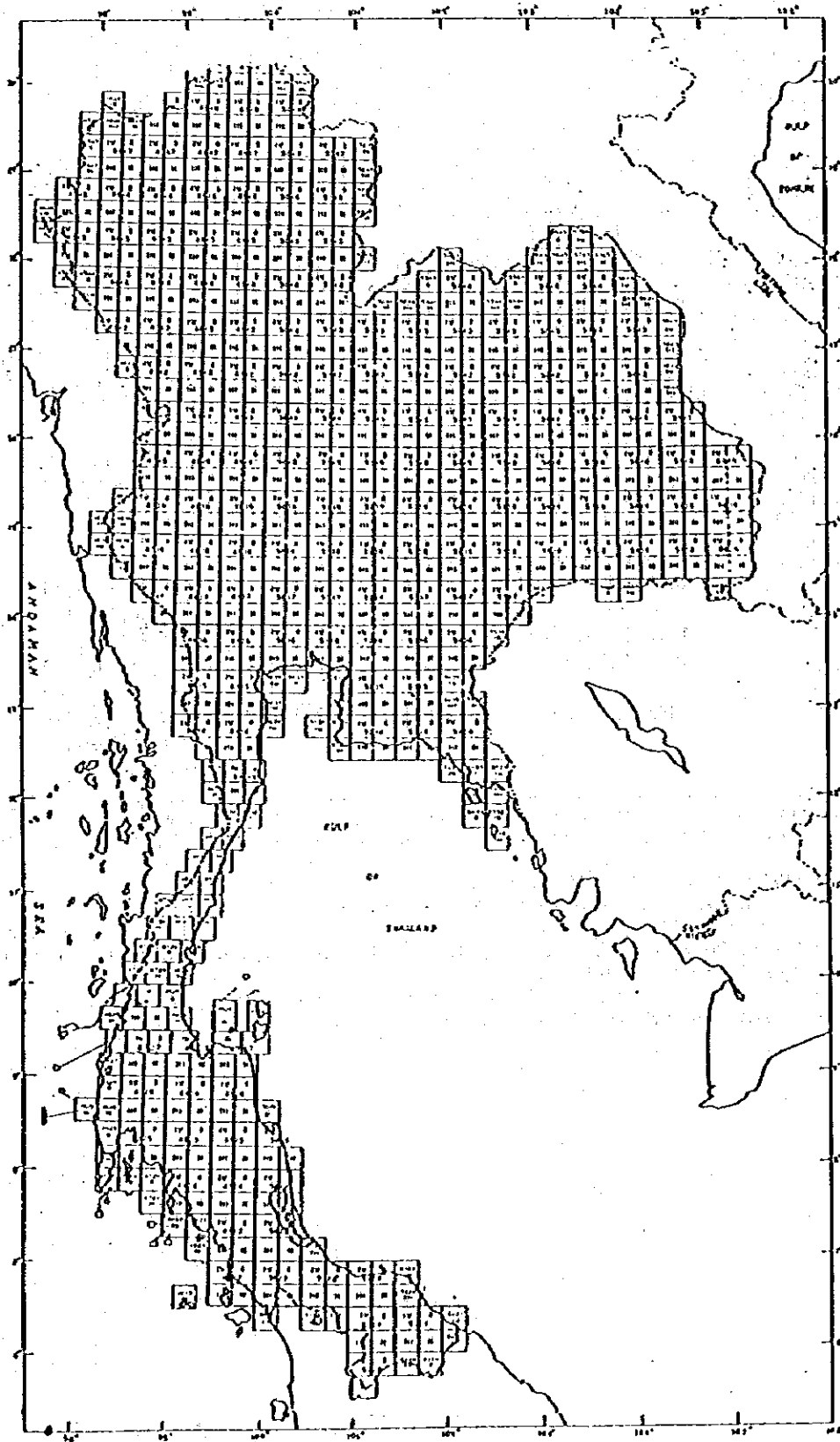


Fig. 3-1.7 MAP INDEX 1:50,000