ARAB REPUBLIC OF EGYPT

TECHNICAL COOPERATION PROGRAM

TO

PLANNING AND RESEARCH DEPARTMENT,

SUEZ CANAL AUTHORITY

A SUPPLEMENTARY TEXT TO TRAINING PROGRAM DOCUMENT

SEPTEMBER 1978

JAPAN INTERNATIONAL COOPERATION AGENCY

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PURPOSE OF TRAINING AND GENERAL DISCUSSION

The following is an outline of the contents of the lecture given by Professor Yoshimi Nagao of Kyoto University, the Chairman of the Supervisory Committee of this Project, at the opening of the training course on September 27, 1978.

INTRODUCTORY REMARKS

It is expected that the world society will in the future inevitably become increasingly entangled in various kinds of problems and face many challenges calling for intimate understanding of our world, and thoughtfully prepared decisions will be required to meet them. These problems include the widening gap between rich and poor countries, the energy crisis, food shortages, the population explosion, conflicts of national interests, and the rampant crime in cities among others. Such problems will continuously present themselves as serious issues, and we must resolve them.

In the midst of this, human life must be kept going while we strive for betterment of the world community. While we can anticipate world troubles there is certainly one thing that we can never forget; that is the fact that in order to survive, our world society needs to secure a stable supply of energy, natural resources, industrial products and other important items which are indispenable to human life. The so-called "oil crisis" of 1973 made it abundantly clear how important it is to maintain a stable supply of natural resources for our world society.

There is no denying that the importance of the Suez Canal as one of the world's major maritime transport arteries can only increase in the future. It is hardly necessary to explain what kind of benefits the Suez Canal can bring in the future to Egyptian society as well as to the rest of the world.

At this crucial time then, the Suez Canal Authority has decided to establish the Economic Unit as research and planning organization within the Authority. In the future we will face, for better or for worse, a variety of changes, often unanticipated, complex and radical, which will exert widespread, longlasting effects throughout the world. In order to prepare itself for these changes, the Authority needs to have a 'nerve center' within its own organization which will be able to act as a 'brain,' an 'in-house think-tank,' or 'open feedback system.' The Economic Unit was planned to meet this need of the Authority. Given the increasingly complex and problematic world we can expect to face in the future, it should be abundantly clear to all of us that the role of the Economic Unit, its mission, can hardly be belittled and certainly not forgotten.

Those of you present today are the selected few destined to become the professional analyst, researchers and planners of the Suez Canal Authority. You will have to master such technical fields as maritime economics and transportation, project evaluation, transit analyses and forecasts, systems analysis, financial and cost analyses as well as the actual operation of the Canal itself. The expectations which have been placed in you are by no means small. I hope that you will devote as much of your time and effort as possible to meet the expectations of the Suez Canal Authority.

To make a little clearer the scope and purpose of the training program here in Japan, I would like to take this opportunity to briefly explain some of the fundamental problems faced by the Economic Unit and also point out the objectives of your training. Technical details will be explained later by other lecturers in their respective courses.

TRAINING OBJECTIVES

First of all, one must have a clear understanding of the purpose of the training, the subjects to be studied, and, finally, how the results of your training will be brought into play in the actual work of the Economic Unit.

The second important point one must always keep in mind is that the classroom and job site training that make up this technical training program are in many respects very different from the regular education one receives at a university. In this program the contents and the very results of the training must be directly linked with the actual work and needs of the Economic Unit. One's skills must become progressively specialized and advanced in line with the specific jobs of the Unit. As a result, it is constantly necessary to review and see if one's progress in training is in keeping with the actual work of the Unit.

Given these points about the nature of your training, I would like to discuss the following areas in this orientation lecture:

(1) Roles of the Economic Unit
(2) Specific Jobs of the Economic Unit
(3) Scope of Training

ROLES OF THE ECONOMIC UNIT

It goes without saying that in order to achieve the training objectives one must keep in mind what roles are expected of the Economic Unit.

Figure 1 shows in the most general terms a conceptual map or 'bird's eye' view of how the activities of the Economic Unit will be related and coordinated with other activities both within and without the Authority.

As can be clearly seen from the Figure, the Suez Canal offers its services primarily for the benefit of two kinds of clients or demand sectors. One is to benefit the world economy, and the other is to benefit Egyptian society and its economy. If the Canal operations were to be disrupted, this would give great shocks not only to the Egyptian economy but also to that of the rest of the world. Seen from this aspect the efficient operation of the Canal is a matter of concern not only to the Suez Canal Authority but also to all shipping companies and their operators as well as all firms and industries, reaching out to national and international economies. In order to operate the Canal efficiently, the following questions must be answered:

1) In the long run, what kinds of changes will take place in the Canal transit volume and fleet mix? What are the factors causing such changes in the Canal transit?
2) How much revenue can Egyptian society expect from Suez Canal tariffs ten years from now?
3) How would changes in natural, economic and social conditions in other parts of the world affect the Canal transit?
4) How would tariff increases affect world trade and maritime transport patterns?
5) If the Suez Canal is to be expanded in capacity, what would be the appropriate timing and scale of such a project?

This does not however exhaust the problems that will be faced by the Authority in the future, and other new ones are certain to emerge.

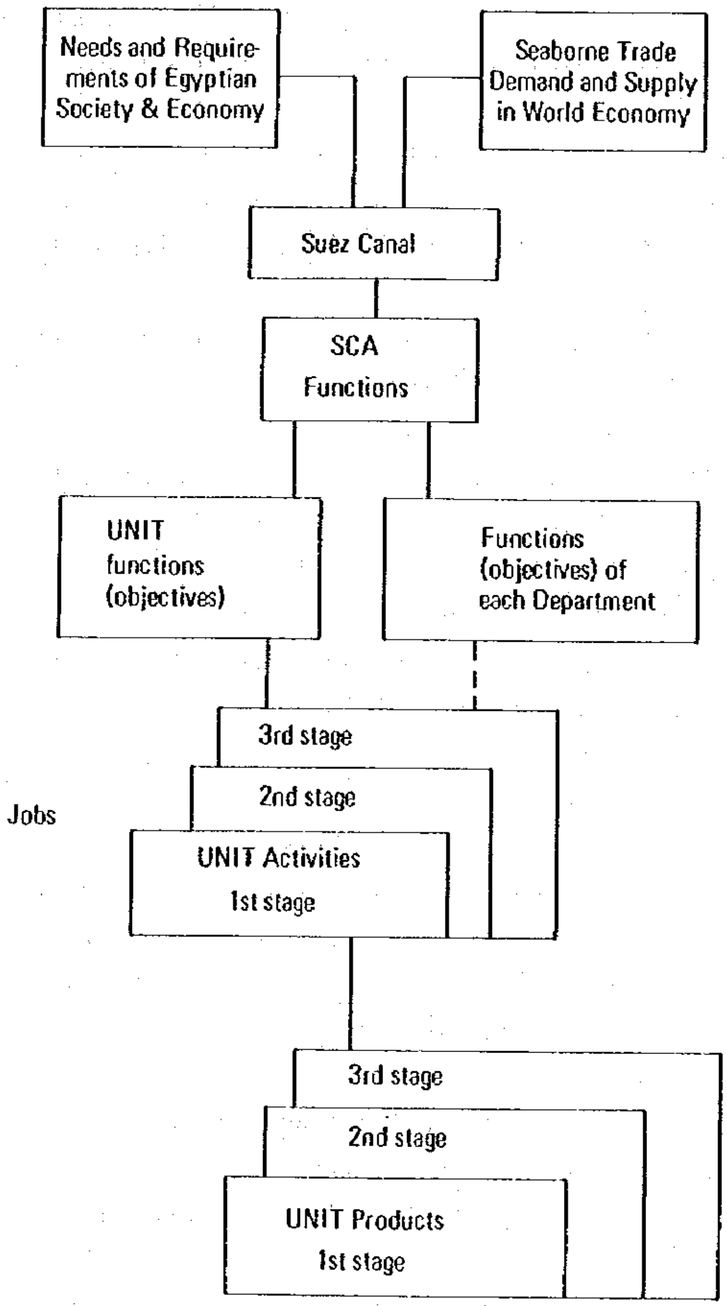

Fig. 1 Necessary Training for the UNIT Staff

In coming to grips with these questions the Authority's top management will find considerable amounts of information and alternative ideas available both from departments within the Authority and from outside sources. In order for the Suez Canal Authority's management to make the necessary decisions, it is necessary that there be an organization within the Authority where this supply of information and ideas can be organized, analyzed, and evaluated before being translated into sets of policy alternatives upon which the management can act.

In response to this need of the Authority, the Economic Unit was established as a 'research and planning' organization to assist the SCA management in long term planning and decision-making. The tasks of the Economic Unit are by no means inconsiderable, and to achieve its objectives the research capabilities of the Unit must be strengthened and expanded within the shortest possible time. The staff members must become able to:

1) Collect and analyze data and information on the various factors affecting Canal operations. Such factors will include international economics and trade, and the economics and planning of seaborne trade and maritime transportation.
2) Examine appropriate levels for Canal tariffs and the tariff structure so as to advise the SCA management on determination of tariff policy.
3) Evaluate feasibility studies and project surveys made by outside consultants and participate in such studies in the future.
4) Collect and analyze data on Canal operations and examine alternate methods.
5) Engage in financial and cost analyses of projects or operations pertinent to the SCA management's planning and decision-making.

<u>SPECIFIC JOBS OF THE ECONOMIC UNIT</u>

What are the specific jobs that must be carried out by the Economic Unit in order to most effectively fulfill the roles so far described?

It should be quite obvious that the Economic Unit cannot do everything all at once from the beginning. To play with the old saying "Rome wasn't built in a day," we might say that the Economic Unit will not be built in a day either; it must be built from a solid foundation and rise up like the Pyramids of the past.

Following this dictum we would like to suggest the following plan.

(1) After a detailed examination of the specific job requirements of the Economic Unit, annual plans will be drawn up each year in line with improvements in the research capability of the staff members.
(2) The work output of each job will gradually be upgraded in the hope that the Economic Unit will soon reach a point where it is fully operational and carrying out the jobs required of it by the SCA management.
(3) In the first stage (the first year), we feel that the Unit staff should concentrate on the following problems:
 1) Analysis and short term forecasting of Canal transit and revenues.
 2) Review of feasibility studies conducted on Canal expansion projects by outside consultants.
 3) Acquisition of knowledge and techniques concerning maritime transport and Canal transit data analysis.
 4) Technical training for the Economic Unit will start from fundamentals and gradually become specialized. Emphasis will be placed more on pragmatic approaches and application rather than upon abstract theories and textbook knowledge.

SCOPE OF TRAINING

Figure 2 shows the scope of technical training envisioned for the Economic Unit. The activities for each year are based on the actual organization of the Unit and the quantity and quality of the staff. Reports on each activity will be prepared.

The basic form of the training will be 'on-the'job' learning. Basic subjects required to carry out the jobs of the Unit will be chosen and reviewed; these will include economics, applied mathematics, management and engineering. During this review of basic subjects it is hoped that various information processing methods such as data collection, data arrangement, and data processing will be learned.

The present status of worldwide maritime transportation relative to the Suez Canal's transit will be introduced.

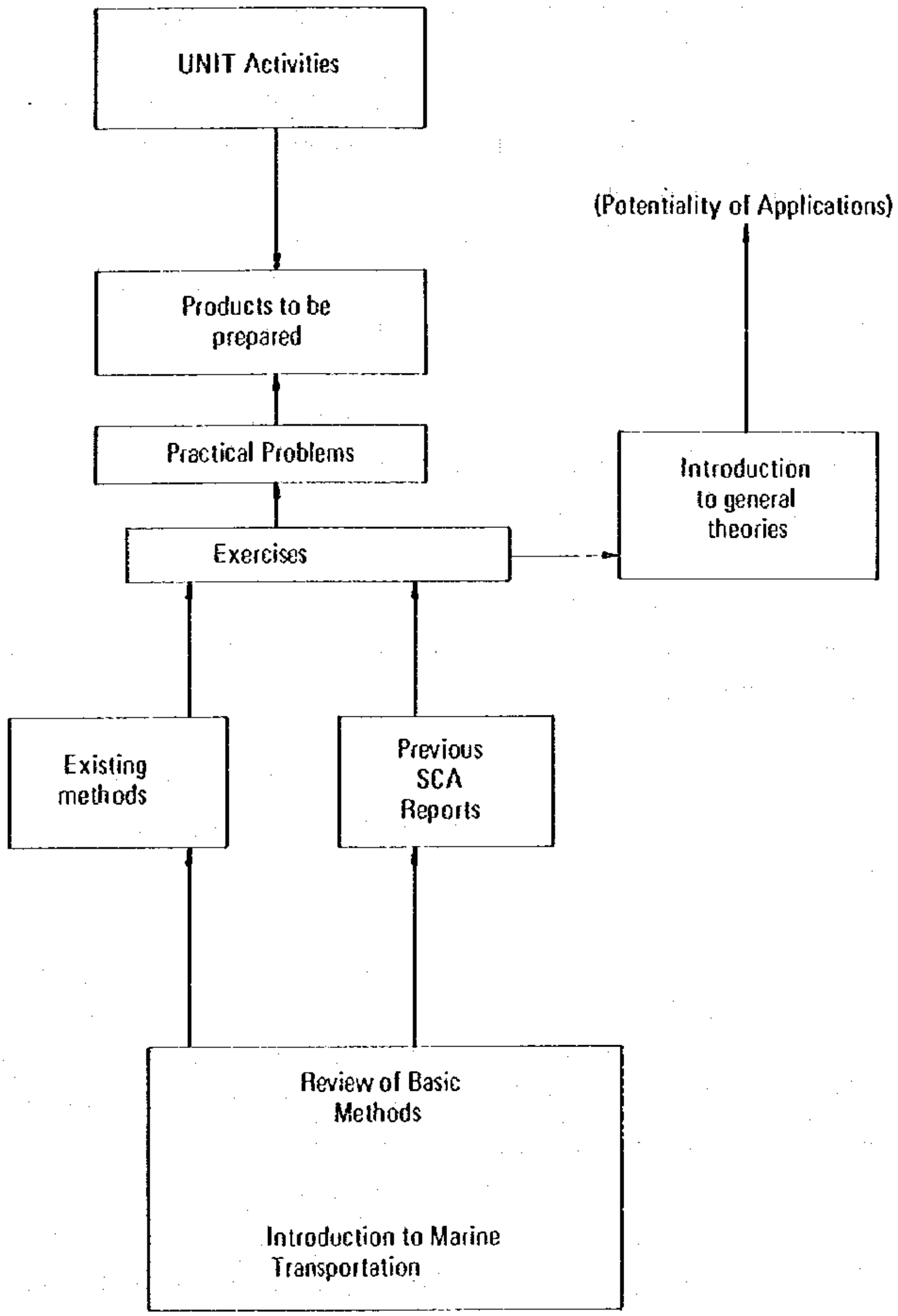

Fig. 2 Scope of Training

After the review of basic subjects, the training program will move on to methods of preparing reports on results. Theoretical problems will be solved using existing methods. Previous SCA reports will be used as texts.

In the process of working out these exercises, introductions to general theories such as transportation economy, maritime economy, forecasting methodology and concepts and the methodology of planning will be given. These lectures will be conducted not only for the purpose of providing keys to the solution of the exercises but also to expand the potential for practical application of the general theories.

However, while general theories will be introduced the training program will place its emphasis strictly on the handling of actual problems confronting the SCA, i.e. problems related to Unit jobs. This means that the training will be as close to actual situations as possible and will be based on the masses of data related to management of the Suez Canal, methods of data adjustment and analysis, and other types of information that may be obtained.

Results of the training program will be drawn up to include diagrams, tables and figures and will in part be presented as information and proposals by June 1979.

Training will be conducted at various locations, including the Port and Harbor Research Institute of the Ministry of Transport; Mitsubishi Research Institute; Keidanren (Federation of Economic Organizations); and the JICA Training Center. Many techniques of investigative information processing and planning will be learned through the careful observation and study of methods of data processing and computation.

Many tours will be scheduled, primarily to harbors and straits.

This activitity will be a success if you gain a better understanding of Japan and the Japanese through these tours.

FOR SCA INTERNAL USE ONLY

No. 1

AN INTRODUCTION TO MARITIME TRANSPORTATION

NO.1 AN INTRODUCTION TO MARITIME TRANSPORTATION

PREFACE

This No. 1 curriculum text is a compact compilation of the detailed texts (hereinafter simply called "TEXT") on the subject of "An Introduction to Maritime Transportation" to be distributed in advance to the participants of the Suez Canal Authority in the training which will be held on October 2-13, 1978 at the Japan Maritime Research Institute.

The report, therefore, is an introduction to the practical knowledge and concepts of maritime transportation necessary for undertaking the work of "Transit Forecasting and Project Evaluation" (TASK II-2) and of "Transportation Cost Analysis" (TASK II-3) on the basis of the "Inception Report" (July, 1978).

In the actual training, a lecture session is to be set up based on detailed data titled "Maritime Data and Reports" (see "TEXT") compiled by JMRI's Maritime Data Center, so that it can be of help in the tasks "Review Analysis" (TASK II-1) and "Information System" (TASK II-4).

In order to make it easier for the participants to acquire a basic knowledge of maritime transportation, introductory lectures titled "The Structure of Maritime Transportation" (reproduced briefly in Chapter I of this report) and "Basic Knowledge of Shipping" (also reproduced) are given at the start of this training schedule. Another lecture titled "International Shipping Problems (Intergovernmental Level)" (see "TEXT") is specially prepared to inform the participants of the political factors and their influence on maritime transportation which have become exceedingly conspicuous in recent years.

Lectures on the present state of and thinking on "World Economy and Trade", "Energy" and "LPG & LNG" (see "TEXT"), which have a close bearing on maritime transportation, are also incorporated in the training program. Moreover, a special lecture titled "Traffic in the Suez Canal and an Analysis" (see "TEXT") is given to explain the analytical method employed by JMRI to forecast and analyze the volume of traffic through the Suez Canal in reference to the Suez Canal Authority's monthly "Suez Canal Report" which gives a record of the traffic in the Canal. With respect to the operation and development of the Canal, we experiment with a free discussion on practical navigational matters with ship captains who are users of the Suez Canal. At the same time, we explain with a film the management of ships and the present state of shipbuilding technology. We believe the field trip to the "Ohi Container Terminal" will be valuable for the participants in understanding and observing with their own eyes how a container terminal and container ships are actually operated.

As regards the three important sectors of maritime transportation business -- "tramp shipping", "tanker shipping" and "liner shipping" -- special efforts are made in actual training to show as much as possible the practical approaches to cost analysis of transit through the Suez Canal. In Cost Analysis of Ships (I and II), time is provided for the participants themselves to get practice on making calculations based on a practical approach.

This text was prepared by Japan Maritime Research Institute. The staffs in charge of text compilation and the lecture are Mr. K. Akiba and S. Takamura. The lectures will also be given by Mr. S. Miyanaga, Dr. T. Ushijima, Mr. T. Ogawa, Mr. H. Nishiyama, Mr. S. Miyamoto, Mr. Y. Hirayama, Mr. S. Yoshida, Mr. H. Kikukawa, Capt. I. Usui and Prof. R. Hirono.

TABLE OF CONTENTS

CHAPTER 1 INTRODUCTION

1.1 Maritime Transportation Business

According to Lloyd's Register of Shipping, the world's total tonnage as of July 1977, was 67,945 ships aggregating 393,678,000 gross tons (including steel ships of more than 100 G/T, fishing boats and miscellaneous ships). These vessels are classified into many types according to the type of cargo and the volume of cargo they handle. Lloyd's Register of Shipping, for example, lists a great variety of ships, such as "oil tankers, liquefied gas carriers, chemical tankers, miscellaneous tankers, combination carriers (bulk/oil, ore & bulk), general cargo ships (single and multi-decks), passenger/cargo ships, container ships, lighter carriers, and vehicle carriers". Their sizes are varied, too. Maritime transport business, too, is varied and diversified. It can be roughly divided into three major sectors: (1) tramp shipping, (2) tanker shipping, and (3) liner shipping.

It is necessary to study these three sectors separately as regards the Suez Canal. A ship's ability to bear the Canal dues, for example, differs in each sector according to the market conditions. Because the largest and the leading type of vessels greatly differ in each of the sectors, study should be conducted separately for each sector in connection with the Canal Development Program. It is necessary also to touch on the fact that shipping business in the three sectors has become segmented and specialized from the standpoint of their markets and systems. This is due to the fact that the economic, social and political environments of each sector differ greatly.

Traditionally, liner shipping is operated by the Liner Conferences. However, since many countries, particularly developing countries, are showing a strong interest in maritime transportation, particularly in liner shipping, the U.N. Convention on Code of Conduct for Liner Conferences was adopted in 1974. It is necessary also to take note of the flag preference policies of the developing countries and of the increase in tonnage of liners of East European countries, particularly, the USSR. On the other hand, it cannot be denied that some clash of interests exists among the developed countries as regards the operation of liner conferences. These factors may cause changes in the composition chart of liners. However, we must bear in mind that it is extremely difficult to obtain data concerning the operation of liner conferences.

Dry cargo shipping at present is operated on the free market principle, although there are some moves at the intergovernmental level to study some measures at UNCTAD (the sharing issue, for instance). Therefore, it is most important to understand accurately the free market principle and its special characteristics. In the field of dry cargo shipping, intergovernmental transactions are becoming of significant influence on the market as was seen in the case of the mass-purchase of grain by the USSR. Historically, dry cargo (tramp) shipping is the prototype of maritime transportation, and liner shipping is an offshoot of tramp shipping. Tankers, too, branched out from tramp shipping to form a sector of special-purpose ships.

Tanker shipping developed side by side with the energy revolution in which man began to use petroleum as the principal source of energy. With the oil problem assuming a political character as seen in the oil crisis of 1973, the tanker market has become very volatile. The closure and re-opening of the Suez Canal also greatly affected the tanker market. With regards to tanker shipping, it is necessary to know especially the background which gave birth to such big tankers as VLCC and ULCC. The closure in 1967 of the Suez Canal gave an impetus to the trend towards bigger tankers. These bigger sized tankers occupy an important position as regards the Suez Canal development program. It is of particular importance to make a practical study of the behavior of shipping concerns in reference to market conditions in order to conduct transportation cost analysis and transit forecasting.

It is necessary to understand thoroughly the trend of the measures taken to combat the tanker market recession of recent years and to control ocean pollution. Particularly important among measures relating to pollution are the structural requirements imposed on vessels (such as installation of Segregated Ballast Tank) and liability and insurance systems (P&I insurance, TOVALOP, CRISTAL and the convention of 1969 and FUND treaty of 1971). It is equally important to keep track of steps taken to combat the tanker market recession, such as the mooring and slow-streaming steps taken by the private sector, and efforts being made by governments.

1.2 Structure of Maritime Transportation

Vigorous economic activities not only accelerate the distribution of finished products through foreign trade channels but also increase demand for industrial raw materials and stimulate energy consumption. The demand for maritime transportation reflects economic activities. But there is always a time lag before invigorated economic activities are reflected in actual demand for maritime transportation. The demand for ships tonnage is expressed in ton-mile and is calculated by multiplying the volume of transportation required in economic activities by the distance from the place of production to the place of consumption; Volume of cargo (ton) x distance (mile).

Foreign trade cargo transported by ships is broken down into general cargo like industrial products, bulk cargo like iron ore and coal, and liquid cargo like crude oil. According to the type of cargo, a general cargo ship, a bulk carrier or tanker is employed. A vessel of the class and the type best suited for the special features of each type and volume of cargo is chosen to carry the cargo on a designated route. The size of a ship and its transport capacity usually are expressed in gross ton and deadweight ton.

The shipping market is the place where the exporter of industrial goods or raw material on the one hand and the supplier of the means of transportation on the other reach agreement on the freightage or charterage and where adjustments of these charges are carried out. In this context, the main commercial ports constitute shipping markets and freightage markets.

The tramp and tanker rates are normally fixed in these markets in the form of long-term or spot charters. Normally the charter rates fluctuate according to the demand and supply, with spot contract freight being affected most sensitively by the fluctuation.

The sudden slump of freightage since the oil shock is due to the big drop in the demand for transportation consequent to the stagnation of the economies of the principal countries. Moreover, just before the oil shock when the shipping business was booming a large number of shipbuilding orders were placed. In addition to the huge existing tonnage, these new ships were thrown into the market, there occurred a surplus of bottoms, adding to the worsening of the shipping market. If the world economy should recover, the supply pressure would relatively weaken and the freight rate would rise. However, the prospects of the world economic environment are not necessarily optimistic because of the energy problem and the currency instability.

1.3 Basic Knowledge of Shipping

The "TEXT" took up four subjects which were considered to be of the greatest importance in the work of the "unit". The four subjects were:

1) Types of cargo and ships
2) Tons (G/T, N/T, D/W, SCNRT, F/T)
3) Owners and charterers
4) Bills of lading

Regarding 1), it is important to understand the relationship between ships and cargo. Although an understanding of ships themselves is, of course, important. An explanation of ships was left out in view of the fact that the Suez Canal Authority has expert knowledge in this field. Therefore, it is desirable that the participants will continue their study of ships.

Regarding 2), an accurate knowledge of G/T, N/T, D/W, etc. is essential in dealing with all kinds of statistics. The training program tried to impart an understanding of the relationship of these terminologies with SCNRT, which is the source of revenue of the Suez Canal Authority, and with cargo tonnage. There do not exist any clear-cut formula showing the relationship among the values expressed by these terminologies. Therefore, care must be taken in using these terminologies.

Regarding 3), the main purport is to draw attention to the fact that the charter party constitutes the main pillar of the shipping mechanism, particularly with respect to tanker and tramp. Of the four items, this is the most difficult to be understood. It is hoped that the trainees will deepen their knowledge of this subject by gradual steps.
4) is important with respect to the liner trade.

1.4 Transportation Cost Analysis

After its reopening in 1975, the Suez Canal was confronted by the following problems.

The volume of oil exported from the Middle East region to Europe, East Coast of North America and Canada increased by 2.4 times from 242 million tons in 1966 to 581 million tons in 1977 (B.P. Statistics). However, the volume of northbound oil through the Canal dwindled to 1/5.4 from 167 million tons in 1966 to 31 million tons in 1977.

As for dry cargo, according to U.N. statistics, global movement by sea increased by 1.8 times from 855 million tons in 1966 to 1,555 million tons in 1976. As against this, the total of northbound and southbound goods traffic through the Suez Canal increased only by 1.27 times from 66.2 million tons in 1966 to 83.8 million tons in 1976. Here, too, we note a difference in the growth rates, although it is observed that the cargo involving Japan, which has comparatively little relation to the Suez Canal, has increased remarkably during this period and at the same time, Suez goods traffic increased to 93.8 million tons in 1977.

The causes of the above phenomenon have been analysed in various studies and reports which attributed the causes mostly to the increase in the size of tanker and stagnation of the tanker and dry cargo markets. The attributed causes, however, should be reviewed by the S.C.A. on its own judgement. In chapter 5, we conduct a basic study of transportation cost, which may help the S.C.A in its review.

CHAPTER 2 TRAMP SHIPPING

2.1 General

The word "Tramp" is used as opposed to "Liner" but Tramp shipping is, in fact, the prototype of modern shipping industry, with liner and tanker shipping having branched out and formed independent sectors in the process of the evolution of shipping in the 19th and 20th centuries. Tramp shipping can, therefore, be described as international trade of dry cargo in the form of non-regular operation. Although dry bulk trade has, nowadays, come to account for the greater portion of Tramp shipping, trade of packaged and fabricated goods still constitutes an important part in Tramp shipping and is of greater importance for the Suez Canal than dry bulk trade in view of its volumes in Suez Canal traffic.

In studying Tramp shipping particular attention should be paid to the following points:

1) The type and size of cargo and, correspondingly, those of ships are diversified.
2) The market mechanism works out the most efficient combination of cargo and ship.
3) The traditional contract system, which is unique and complex, supports the mechanism.

Of these three points, 2) and 3) are common to Tanker shipping to some extent.

In some specialized fields such as car carrier, wood chip carrier, ore carrier, combination carrier, etc., there exist very strong ties with specific big shippers, particular with respect to the larger vessels (In this respect, there is some similarity with tanker shipping). This means that in such areas the freedom of operation of the shipping companies tends to be restricted. This is somewhat different from the ordinary characteristics of the typical tramp. However, in most cases, such trade is generally dealt within the framework of tramp shipping because of the mutuality of cargo (dry cargo).

2.2 Dry Cargo Movement

Data on dry cargo movement can be obtained from the United Nations' "Monthly Bulletin of Statistics" and "Statistical Yearbook", and "World Bulk Trade" of Fearnley & Egars Chartering Co., Ltd. Also useable is OECD's "Maritime Transport" which is based on the aforementioned material.

"World Bulk Trade" lists in metric ton and in ton-mile the transport volume of iron ore, coal, grain, bauxite/alumina and phosphate (which constitute main dry bulk cargo) and also other dry bulk. The publication shows the volume by origination and destination areas. The main cargo movement affecting the Suez Canal can be ascertained from this.

The volume of cargo movement of this trade is relatively light and constitutes an extremely small percentage of the total world bulk trade, with iron ore being less than 1% and grain about 5%.

The difference between the via Suez and via Cape distances of Australia/UK, Continent trade is comparatively small, being about 2,200 miles, or roughly seven navigation days. At the present level of the market, this difference is not considered big enough to cover the Suez Canal toll. Consequently, in the actual 1977 Suez traffic, the bulk carrier's share of 26.2 million N/T was lower than those of tanker (75.6) and general cargo ship (66.6). The volume of goods traffic, too, was small, with iron ore 4.0 non-ferrous ore (3.3) and cereals (4.2) million metric tons.

There are no global statistics concerning the movement of packaged or fabricated cargo (iron & steel products, plants, general cargo, etc.). With respect to these cargo, there is considerable competition between tramp shipping and liner shipping, particularly the conventional type liner. The respective shares are not known. One can make an estimate only by using the various incomplete data.

2.3 Tramp Fleet

Statistics on the tramp fleet are available in Lloyd's Register of Shipping's "Statistical Table" and Fearnley & Egars Chartering Co., Ltd.'s "World Bulk Fleet".

The former publishes statistics on ships of more than 100 G/T as of July 1 every year. The listings are mainly in gross tonnage.

The latter takes single deck vessels of more than 10,000 D/WT as constituting the bulk fleet and publishes statistics annually as of January 1. Tramp vessels are of the two broad categories described below.

(1) Bulk Carrier
In Lloyd's, bulk carriers are more than 6,000 G/T while in Fearnley they are more than 10,000 D/WT. Bulk carriers are mainly in the 20,000-60,000 D/WT range, but there are many different sizes up to more than 150,000 D/WT ((*) The large bulk carriers are engaged mainly in trade connected with Japan). These carriers principally transport bulk cargo. However, those under about 30,000 D/WT in many cases transport iron & steel, plant, etc, and are sometimes called handy-sized bulk carriers.

Combination carriers are ships which perform the functions of the bulk carrier and tanker. They aim to lower the transportation cost by reducing the proportion of voyages in ballast and are engated in such trade as Arabian Gulf -- (crude oil) -- Europe -- (in ballast) -- South America -- (iron ore and/or coal) -- Japan -- (in ballast) -- A.G.

It can take advantage of movements of the freight market and engage in either the dry cargo or tanker trade, whichever is more profitable at the time.

Despite the fact that the shipbuilding cost of the combination carrier is higher than that of the same size tanker or bulk carrier, the combination carrier fleet expanded sharply from 8 million tons in 1970 to 26 million tons in 1977. In making a survey of the bulk carrier fleet, it is necessary to take into account the bulk carriers engaged in the dry bulk trade.

(2) General Cargo Ship
This category of ship is under 10,000 D/WT but about 15,000-20,000 D/WT ships with 'tween decks are also included.

In addition to transporting bulk cargo, these ships sometimes also carry general cargo, iron & steel, plant, containers, vehicles, etc.

According to Lloyd's "Statistical Table", the bulk carrier fleet (including bulk/oil carriers) increased 3.5 times in 10 years, from 29 million G/T in 1967 to 101 million G/T in 1977. This was an average annual increase of 13.3%.

The general cargo ship fleet expanded by 6.5% from 72 million G/T in 1970 to 77 million G/T in 1977, an average annual growth of 0.9%. It was in 1970 that the general cargo category was listed for the first time in Lloyd's "Statistical Table". The general cargo ship in the Table includes the conventional type liner and small vessels.

2.4 Dry Cargo Market Mechanism and Charter Parties

London, New York and Tokyo are the three big centers of the dry cargo market. The "Baltic Exchange" in London is Particularly well known.

Among the characteristics of the dry cargo market are:

1) It is where the best suited type and size of ship from among a great variety is found for any one of the diverse kinds and sizes of cargo.
2) It makes possible the best timing in marrying cargo to ship which is extremely important in the efficient operation of vessels.

The three main dry cargo markets exchange information via telex and function in rotation according to the time difference. The freightage is most sensitive to the demand and supply situation. The contents of concluded contracts are reported throughout the world at once and influences subsequent negotiations.

The forms of contract known as time charter and voyage charter enable the chartering of ships which occur unceasingly to be carried out smoothly. In order to achieve the best suited combination of cargo and ship mentioned in above, the transfer of a contract to a third party (known as sublet or relet) is permitted and takes place all the time as a matter of routine. The decision on whether a ship will use the Suez Canal will be determined in relation to the provisions of a charter contract such as the sharing of expenses, responsibility and who takes the profit or loss etc. Therefore, it is necessary to have an understanding of the general outline of the forms of contract.

2.5 Dry Cargo Market and Future Outlook

Affected by the global economic stagnation following the oil crisis of 1973, the tramp market is in a slump.

With iron are trade which accounts for roughly 45% of the total bulk trade not increasing because of sluggish steel production, there has been hardly any growth in bulk trade, the 1976 figure being 3,100 million ton-miles as against 2,900 million ton-miles in 1973. On the other hand, the bulk carrier fleet expanded from 109 million D/WT in 1973 to 150 million D/WT in 1976. This is the reason for the tramp market slump. Reflecting this situation, about 12.5 million D/WT (including combination carriers) were tied up as of September 30, 1978.

The increased imports of grain by the Soviet Union and China in 1978 have resulted in some rise in the freight rate but it is not enough to fully cover the transportation cost.

It is generally believed that the future outlook depends most on the world steel production, particularly that of Japan. The second large factor which influences the market every year is the amount of imports by grain-purchasing countries. As to the time when the current overtonnage of bulk carriers will be solved, the most widely held view is 1982 or later.

CHAPTER 3 TANKER SHIPPING

3.1 General

In June, 1975, the Suez Canal was reopened after eight years of closure and since then the volume of transit through the Canal has been gradually returning to the former level. The number of transits in 1977 reached 19,700 vessels, or 92% of the 1966 total of 21,250 ships. However, loaded tankers account for only 8.4% of the total transits (Tankers in ballast account for 30.2%). Thus, the Canal is greatly affected by the change occurred in the transportation structure while the Canal was closed and by the stagnation of the tanker market itself.

On the other hand, it must be judged that very large tankers have a vital effect on the Suez Canal development program. The volume of tankers transiting the Canal is closely related to the tanker market. The prospects of the tanker market hold an important position in implementing the Canal development program.

In order to forecast the tanker market, it will be necessary to have an understanding of the basic factors related to the mechanism of tanker shipping. As basic factors, we shall take up here such subjects as the volume of oil sea-borne trade, the tanker tonnage, the trend towards large-sized tankers which changed the transportation structure, the forms of ownership of tankers, market trends, and future prospects.

3.2 Oil Cargo Movement and Tanker Fleet

As the demand for energy shifted from coal to oil, it was only natural that tankers came on stage as an efficient means of transporting ever larger amounts of oil. (As Table 3.1 shows) the number of tankers increased rapidly in a short space of time. The world tanker tonnage was only about 44 million g/t in 1961, but had grown to 170 million g/t in 1977, a growth of roughly four times. During the same period, the proportion of tankers to the total volume of carriers rose from 32% to almost 45%.

The ocean movement of oil, too, expanded greatly during this period. Specifically, oil movement in 1961 was 480 million tons but in 1977 it was 1,695 m. tons, a growth of 3.5 times.

To cope with this increased demand for transportation, not only was the total tonnage of tankers increased but also bigger and bigger vessels were built in order to cut down gradually the transportation and shipbuilding costs.

Table 3.1 Merchant Fleet of the World

	Total Tonnage			Tanker Tonnage		
	No. of Ships	Tons Gross	Annual Growth Rate (%)	Tons Gross	Annual Growth Rate (%)	Tanker Tonnage among Total Tonnage
1960	36,311	129,770	3.9	41,465	9.4	32.0
61	37,792	135,916	4.7	43,849	5.7	32.3
62	38,661	139,980	3.0	45,304	3.2	32.4
63	39,571	145,863	4.2	47,121	4.0	32.3
64	40,859	135,000	4.9	50,563	7.3	33.0
1965	41,865	160,392	4.8	55,046	8.9	34.3
66	43,014	171,130	6.7	60,200	9.4	35.2
67	44,375	182,100	6.4	64,198	6.6	35.3
68	47,444	194,152	6.6	69,214	7.8	35.6
69	50,276	211,661	9.0	77,392	11.8	36.6
1970	52,444	277,490	7.5	86,140	11.3	37.9
71	55,041	247,203	8.7	96,141	10.4	38.9
72	57,391	268,340	8.6	105,129	9.3	39.2
73	59,606	289,927	8.0	115,365	9.7	39.8
74	61,194	311,323	7.4	129,491	12.2	41.6
1975		342,163		150,057		43.9
76		372,000		168,161		45.2
77		393,678		174,124		44.2

Source: Lloyd's Register of Shipping, statistical tables.

3.3 Increase in Size of Tankers

The history of ocean-going tankers, as seen from the point of view of type of vessel, can be divided into the following periods:

1) 1st Generation -- Up to the time of the Korean War (1950) when the 30,000 dwt super tanker appeared.
2) 2nd Generation -- The 100,000 dwt mammoth tanker era from the Suez War (1956-7).
3) 3rd Generation -- The age of the VLCC, which can be ordinarily defined as any crude oil tanker with a DWT in excess of 175,000, from 1965 to about the time of the second closure of the Suez Canal (1967).
4) 4th Generation -- The years after 1968, the age of the ULCC (Ultra Large Crude Carrier) of more than 300,000 dwt.

The factors, which caused tankers to become successively larger in size include, in addition to the expansion of maritime oil movement mentioned earlier, can be attributed to the fact that the transport distance became longer.

3.4 Factors Behind Increase in Tanker Size

The demand for oil transport is ordinarily arrived at by multiplying the volume of oil movement by the transport distance. The factors which determine the distance are the location where the oil is produced, the volume of production, the type of oil produced (such as low sulfur crude) and the volume of demand in the place of consumption. As is shown in Figure 3.1, the sudden increase in 1967 from 4,000 to 5,000 miles is due to the closing of the Suez Canal. The Sharp increase after 1970 was caused by the increase in American dependence on Middle East Oil.

This shows that a consideration of the distance is an important factor in measuring the demand for tankers.

In future, it will be necessary in determining the average transport distance to study the effect that would be caused by the development of oil resources in new areas.

In the past, it was necessary from the point of view of strength to use thick steel plates in order to build ships of larger size. This made fabrication

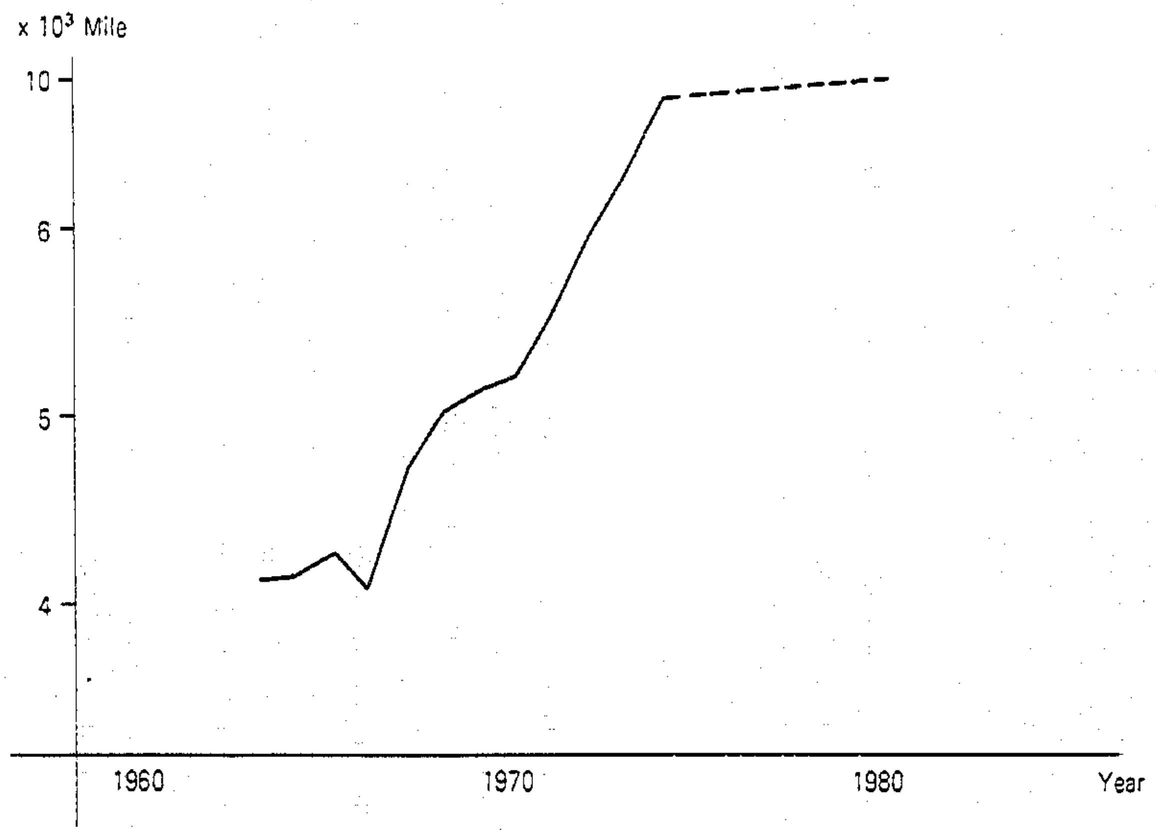

Fig. 3.1 Transitional Oil Transportation Average Distance

more difficult. But the development of high-tension steel which has great strength even when it is thin and the development of the welding method of joining steel plates made possible the construction of large-size vessels.

In addition, many other technical innovations were made it possible to reduce the construction time and cost. There emerged the block construction method in which several blocks, including some of the fittings, were first built on land and then assembled in the dock. There was also the adoption of marking by means of numerical control. All these contributed to the development of larger vessels.

Ordinarily when vessels become of bigger class, there is a tendency for the shipbuilding cost, seamen's cost, fuel cost and the voyage cost per ton to decrease. For instance, a 400,000 dwt vessel is eight times a 50,000 dwt vessel, but its total cost is said to be only 2.5 times.

And in the case of the operating cost, with the indices of a 30,000 dwt products tanker as 100, the figure for a 120,000 dwt tanker is 39 and that for a 250,000 dwt tanker is 22, showing that the indices diminish as the vessel becomes larger in size.

3.5 Ownership of Oceangoing Tankers

One of the distinctive features of the form of ownership of oceangoing tankers is that the oil company, which is the shipper, itself owns and operates tankers, functioning as a shipping company. This takes the form either of self-owned tankers or chartered tankers. Charters, moreover, are divided into spot and long-term charters.

In the case of the self-owned tanker, the oil company (or its direct subsidiary) builds a tanker and operates it. According to E. A. Gibson Report, as of December 31, 1977, 36% of the world's tanker tonnage consisted of self-owned vessels. Among the major oil companies, there are those which own more than 50% of the tankers they use. This high ownership rate was the result of the sharp rise in transport cost and the difficulty of arranging for vessels subsequent to the first Suez War in 1956. Self-ownership has the advantages of (1) stable transport, (2) lower cost of transport and (3) relative freedom in assigning and operating vessels.

Despite these advantages, no oil company will own all of the tankers it uses. In the first place, it is financially impossible. Then, there is also the danger of overtonnage arising from such factors as a suspension of the operation of an oil refinery or a change in the place of loading.

To cover the tanker space it needs over and above its self-owned ships, an oil company charters tankers from independent owners. Charters extending over one year are generally known as long-term charter. Actually, there is no stringent rule setting one year or more as long term. However, contracts today have become more complex, and seven-month charters and three-over-winter charters are also referred to as long-term charters.

Spot charters are those for one voyage or for two or three short trips. In the case of spot charter the cost of transport fluctuates greatly compared with self-owned or long-term charter. Thus, the usual pattern is to adjust the long-term charter to the summer demand and make up the winter shortage by spot charter.

3.6 Tanker Market

In this chapter, we shall limit the tanker market to the spot market.

In the spot market, ordinarily, if the tanker volume available is relatively large compared to the volume of cargo movement, the tanker market declines and vice versa. Its ups and downs are brought about by various factors.

The tanker market has shot up at times of the Korean War, the Middle East War, the closure of the Suez Canal, the closure of the tap line, speculation over U.S. energy policy -- all non-economic factors which brought about a sudden demand for tankers. Normally, however, it is economic or seasonal factors which cause a rise in the tanker market, such as the winter demand for heating oil. In a situation where the volume of oil movement and the supply of tankers is balanced, tanker rates show an upward tendency when some factor occurs which causes an increase in the volume of oil movement. On the contrary, if ships which were ordered at a time of high tanker rates are thrown one after the other into the market when there is already a surplus of vessels, the balance between the volume of tankers and volume of oil movement will be upset and the tanker market will start dipping. Even in such a situation, the above-mentioned non-economic factors may stimulate tanker prices, but such a boom would not last for long. This was the case with the first closure of the Suez Canal.

After the first closure of the Suez Canal, the balance was gradually re-established and after 1967-68, a stable market set in. But in 1970 a boom in tanker demand erupted with the growing dependence of the United States on Middle East oil. (The freight registered highest quotations and WS average was over 200, see "TEXT" "Tanker Shipping".)

It was a phenomenon which resulted from the addition of a demand factor to a situation which was gradually approaching a balanced level. The tanker market is characterized by economic and non-economic factors which readily affect its fluctuations and which are very difficult to grasp. In order to know the trend of the tanker market, one must very carefully analyze all kinds of information.

3.7 Standard Transport Rate for Tanker

At present, the World Scale is most generally used as the standard transport rate for Tankers. At one time in the part the USMC and ATRS of the United States and the MOT Scale and INTA Scale of Britain were being used, but they were integrated into the World Scale in 1969. The World Scale (WS) Rate is calculated based upon the transport cost of a 19,500 d.w.t. and 14 knot turbine vessel. The rate is made public every year which is revised by the fluctuations in the fuel cost and port dues. Some representative World Scale Rates are listed in (Table 3.2.)

The rates are shown for every main routes, and "WS60", for instance, means 60% of standard transport charge. This WS rate has the advantage of covering trade in the whole world with one rate. It is convenient in the event of long-term contracts and gives at a glance a comparison of rates among various routes.

Table 3.2 PG/Miscellaneous Main Route Worldscale Rate ('78.7.1)

LOADING PORT	RAS TANURA			M.A. AHMADI			KHARG ISLAND		
DISCHARGE PORT		S	CS		S	CS		S	CS
MILFORD HAVEN	17.04	10.30	13.64	17.41	10.67	14.01	17.71	10.97	14.31
FALMOUTH	16.87	10.14	13.47	17.24	10.51	13.84	17.54	10.81	14.14
LAVERA	16.71	8.07	12.33	17.08	8.44	12.70	17.38	8.74	13.00
VENICE	18.04	7.65	12.76	18.41	8.02	13.13	18.71	8.32	13.43
GENOA	16.70	7.79	12.18	17.07	8.16	12.55	17.37	8.46	12.85
ROTTERDAM	17.65	10.98	14.28	18.02	11.35	14.65	18.32	11.65	14.95
WILHELMSHAVEN	17.77	11.02	14.36	18.14	11.39	14.73	18.44	11.69	15.03
HAMBURG	18.02	11.27	14.61	18.39	11.64	14.98	18.69	11.94	15.28
STOCKHOLM	19.60	12.71	16.12	19.97	13.08	16.49	20.27	13.38	16.79
NEW YORK	17.48	12.74	15.09	17.85	13.11	15.46	18.15	13.41	15.76
NEW ORLEANS	17.58	14.18	15.87	17.95	14.55	16.24	18.25	14.85	16.54
SAN FRANCISCO	15.88			16.25			16.55		
LOS ANGELES	16.33			16.70			17.00		
TRINIDAD	15.19	12.92	14.05	15.56	13.29	14.42	15.86	13.59	14.72
SANTOS	12.60			12.97			13.27		
MELBOURNE	11.33			11.70			12.00		
KEELUNG	8.50			8.87			9.17		
YOKOHAMA	10.14			10.51			10.81		

Note: S = Via Suez in both direction
CS = Via Cape Horn in laden voyage and Via Canal in ballast
Ras Tanura/Rotterdam case, fare should be added $2.06 in S and $0.93 in CS.

CHAPTER 4 LINER SHIPPING

4.1 General

Liner shipping is the off-shoot of tramp shipping which is regarded as the prototype of the shipping business. Formerly, the sailing ship was entirely dependent on the weather. It was technical innovation, specifically the emergence of the steamship, which made it possible for ships to be operated on regular schedules. The most distinctive point of difference between liner shipping on the one hand and tramp and tanker shipping on the other is that the former has traditionally been operated by means of liner conference system. The liner conference system developed and was maintained as a means to ensure regular shipping service and a stable freight market. In general, it can be said that the conference is a method of operation of liner shipping based on an agreement among shipowners regarding freight rates and sailing rights and so on. In this respect, the conference could be considered of a cartel judged from the point of view of anti-monopoly or anti-trust laws. On the other hand, the liner operation by non-conference lines (outsiders) should be tolerated as a matter of course. However, among those countries approving the conference system there exist differences of viewpoints. For instance, the European countries which take a strict view of membership have the concept of "closed conferences". As against this, there are the "open conferences" in the U.S. trades because the strict U.S. Anti-Trust Law compels conference membership to be made open to all companies. Among the developed countries, furthermore, the restraints on liner conferences resulting from the state trading system of the East European countries and the flag preference of the developing countries have become issues.

The attention of all countries are today strongly focussed particularly on these issues as they concern liner shipping. The developing countries, for example, demanded a reform of the liner conference system with a two-front approach, one stressing shipper's interest in connection with the promotion of their export trade and the other stressing interest in connection with the development of their own merchant shipping. The efforts of the developing nations resulted ultimately in the adoption in 1974 of the UN Convention on a Code of Conduct for Liner Conferences. As for the USSR, its fleet expansion program in the liner field is spectacular. By mid-1977, the Soviet liner tonnage reached 7.52 million G/T, the second largest in the world. The deep interest of the developed countries in liner shipping is clearly evident in OECD's "1977 Maritime Transport".

A singular trend in liner shipping is the remarkable pace of containerization. According to "1977 Maritime Transport" (para. 28), as of December 31, 1977, as many as 27 liner trades were operating container services. Of these, those which are operating via the Suez Canal are, among others, Japan·Far East/Europe trade, Japan·Far East/Mediterranean trade, Europe·Mediterranean/Australia·New Zealand trade and Europe·Mediterranean/Middle East trade (See "TEXT", "Liner Shipping" for membership and number of ships in these trades.).

Regarding the Far East trade route to and from Europe, it will be necessary to take into account the effect of the Trans-Siberian Land-Bridge (TSLB) which consists of container transport via the Trans-Siberian Railroad between the Far East and Europe or the Middle East. The TSLB in 1977 handled 20% of the containerized transport from Japan to Europe while the eastbound TSLB container movement to Japan was estimated to be half of the Westbound traffic in terms of containers. In relation to the Suez Canal, attention must be paid to the remarkable pace of industrialization in the Middle East region. High load factors were reported in these general liner trades ("1977 Maritime Transport", page 41) but they gave rise to port congestion problems. The containerization of these trades may become one means of improving the port congestion situation.

4.2 General Cargo Movement and Liner Fleet

Liner shipping carries "general cargo", but there is no hard-and-fast definition of what constitutes "general cargo". The transportation of general cargo is a field in which liner ships compete with tramp ships. On the other hand, some liner ships carry bulk cargo which is generally regarded as the field of tramp ships. There are no statistics available on a worldwide basis relating to the type and volume of cargo transported by liner ships. It is extremely difficult to obtain data from the liner conferences. And it is even more difficult to obtain data of the business done by non-conference outsiders.

However, this is not to say that there are no ways to estimate the volume of general cargo moved by sea. An estimate, which is used in UNCTAD's "Review of Maritime Transport", can be obtained by subtracting the movements of the five major bulk cargoes and some of the minor cargoes listed by Fearnley & Egers from the total movement of dry cargoes listed in the "UN Statistics." But, it is difficult to distinguish liner cargoes from this estimated general cargo volume. Regarding liner cargoes movement, a suggestion is made (in the OECD's "1976 Maritime Transport" (para. 127)) that "liner trades benefit on the whole from the general economic uptrend". The OECD also supplied the figure (190 million M/T) of liner cargo moved in the OECD member countries in 1970. Therefore, a projection can be made by taking this figure if the annual economic growth rates are given.

As far as the Suez Canal is concerned, one approach is to compute from the record of transit through the Canal. But, this approach has a drawback in the data on Suez Canal goods traffic, the volume of cargo included in the "others" category is proportionately very large and it is not known what items are included in "others". This makes it extremely difficult to analyse.

As for the size of the liner fleet, one suggestion is given in the 1974 UN Liner Code Convention under Article 49 relating to the conditions for the convention to take effect: "Lloyd's Register of Shipping -- Table 2 'World Fleets -- Analysis by Principal Types' in respect to general cargo (including passenger/cargo) ships and container (full cellular)". In computing the liner fleet, it is probably necessary first to clarify the framework for categorization in accordance with the purpose of obtaining the liner fleet figure.

4.3 Freight Market and Future Outlook

In the wider framework, the liner shipping market is influenced by supply and demand. Although rates are fixed in consultation with shippers, the feature is that a freight rate is arrived at within the scheme of a sort of cartel known as "liner conference". The only source from which the trend of the liner freight market can be known is the "Liner Freight Index" (Bundesministerium für Verkehr; Hamburg) which is published monthly in the form of an index (1965=100). The liner rates of this index are for "cargoes via north continental port on German account (but not restricted to German flag)". (See Table XXIV of Statistical Annex, "1977 Maritime Transport").

The "1977 Maritime Transport" (pages 115-6) sees the liner freight market as follows: "The liner trades were the only part of the shipping industry which provided a reasonable return to their shipowners during 1977. In spite of the slowdown in economic growth, most liner companies were able to maintain fairly satisfactory load factors and their financial results have remained reasonably healthy. The extent to which they were able to recover their increased costs can to some degree be seen from the movement in the German Liner Sea Freight Index (referred to in preceding paragraph). Nevertheless, liner operations have been faced with severe competition from a number of directions, i.e. the competition from the incursions of Soviet shipping lines and legislative protection of developing countries."

4.4 1974 UN Liner Code Convention

The conditions for the entry into force of the Convention are that "the present Convention shall enter into force six months after the date on which not less than 24 States, the combined tonnage of which amounts to at least 25 per cent of world tonnage, ...". As of March 31, 1978, the number of countries which have signed the Convention is 28 and their combined tonnage is 4.73% of the world tonnage. Consequently, the tonnage condition for entry into force has not yet been met.

The principal points of this Convention are as follows: (1) It makes conference membership open, recognizing the right of shipping lines of the two countries engaged in trade to join the pertinent conference as members: (2) It sets, with respect to cargo shares, a guideline concerning the share of each nation participating in the trade (the so-called 40:40:20 formula, allocating 40 per cent each to the trading countries of both ends and the remaining 20 per cent to third countries): (3) It makes consultation between shippers and conference obligatory in regards to the operation of the conference: (4) It freezes general increase of rates for 15 months and (5) It establishes a system of international mandatory conciliation for the settlement of disputes. Thus, it can be seen that the Convention is of such character as will greatly change the liner shipping system in the future.

CHAPTER 5 TRANSPORTATION COST ANALYSIS

5.1 Purpose of Transportation Cost Analysis

The purpose of conducting a transportation cost analysis in this chapter is to obtain analytical data for examining the cause-and-effect relationship between maritime cargo movement and the actual volume of Suez Canal ship traffic.

In the event demand for transportation related to Suez Canal trade arises, the ships which transport that cargo can be divided into the following three groups:

1) The group of ships which cannot avoid passing through the Canal because of the type and size of ship and the loading and discharging ports.
2) The group of ships which cannot pass through the Canal because of limitations such as that of draft.
3) The in-between group of ships may or may not pass through the Canal depending on the trade and other conditions.

The pursposes of making a transportation cost analysis are

(a) to analyse the yardstick and the numerical value for deciding whether ships of the in-between group will transit the Canal or not:

(b) to analyse the transportation cost of the in-between and impossible-to-transit groups of ships.

The aim of the analysis is also to obtain an indication of how (a) and (b) would change in the future.

5.2 Analyses Which Can Be Conducted on the Basis of the Transportation Cost Analysis

Among the analyses which can be conducted on the basis of the transportation cost analysis, the principal one is the impact which the increase in the size of ships and the freight market will have on the Suez Canal.

Regarding the impact of the increase in the size of ships, the starting point of the analysis is to work out the transportation cost separately with respect to ship type, ship size, cargo, loading and discharging ports, laden or in ballast and via Suez or Cape.

The closure of the Canal for eight years is believed to have accelerated the increase in the size of tankers. It will be necessary to analyse, for example, the competitive position of 60,000 D/WT tankers now that the Canal has been reopened, the competitiveness of 150,000 D/WT as against VLCC and ULCC after the first Stage Development has been completed, and the Canal toll which the 260,000 D/WT tanker will be able to bear if the second Stage Development Project is executed.

From this analysis, it will be possible to obtain a comparison of costs of various sizes of tanker and bulk carrier in trades relating to the Suez Canal.

This comparison is based on the cost of maritime transportation, but it must be noted that from the long-range viewpoint of the oil companies, an additional matter for study will be the total cost, including port construction (i.e. berths for large ships, harbor dredging, etc.) and location of refineries. They will plan new shipbuilding, charters and berth construction on the basis of this. Cost comparison with other means of transportation such as pipeline and landbridge is also an item which naturally can be examined.

The freight market affects the Suez Canal because the freight revenue is an important element in the voyage accounts of the shipping companies which decide whether a ship will transit the Canal or not.

A look at Suez Canal traffic in the two years of 1976 and 1977 reveals the trend of the tanker traffic, particularly southbound VLCC in ballast, increasing when the tanker market rises. (See "TEXT", "Suez Canal Transit Analysis")

Another important data in this respect is the geographical distribution of the tanker's discharging ports. In other words, the interrelationship between ship's size, distribution of discharging ports and the tanker market combined with the cost analysis leads to a far more accurate analysis. The starting point of southbound VLCC (i.e. the discharging port of the previous voyage) is exceptionally important and should be studied by obtaining the data from the ship's declaration.

The forecast of the future freight market is the biggest point of interest also of all shippers and shipping companies, but because it involves not only economic but also political and even psychological elements it is extremely difficult to assess. It is generally considered that, in the long run, the tendency is for the freight market level to approach the cost level following a process of ups and downs. Thus, for the purpose of simplification, the transportation cost is sometimes construed as being the freight rate. But a historical examination of the 30 years since World War II shows that for most of the time the freight market moves at a level considerably below cost and, in cycles of 3-10 years, makes a sharp rise lasting for a very short time, say half a year, and then drops again. This has been the pattern of the past.

As a problem of the future, it will be necessary to pay attention to some factors which may accelerate overtonnage. These factors include surplus of shipbuilding capacity, the subsidizing policy for ship of each country, the speculative nature of the shipping industry, etc.

At this point, we would like to touch upon the relationship between transportation cost analysis and transit forecasting. Generally, these two are handled as independent problems. In the flow of the analysis and forecasting model, the former's position is set as the input into the latter. However, in actual practice, there are cases in which they are the two sides of the same thing. In other words, the analysis itself is the forecast. For instance, because the analysis of voyage accounts directly determines whether a ship will transit the Canal or not, it simultaneously serves as a forecast.

5.3 Scope of Study in This Curriculum

The scope of study in this curriculum covers a detailed examination of the nature and substance of the various factors which constitute transportation cost and a study of how voyage accounts are computed with these factors. The purpose is to acquire knowledge necessary for the collection and analysis of data concerning transportation cost.

The substance and nature of the transportation cost differs somewhat according to tramp, tanker and liner. Here, we shall study the general substance of each type of expense which is common to each of the three sectors. The substance peculiar to each sector and calculation according to ship size using concrete numerical values will be taken up in the next curriculum.

5.4 Classification of Costs

One of the most usual classifications of costs is to divide them into three groups, i.e.

1) Capital cost --- expense for purchasing and owing ship, which includes depreciation of ship price and interest on loan.
2) Ship cost --- expenditure for keeping ship in working condition, which includes cost of crew, stores, lubricating oil, repair & maintenance, insurance, miscellaneous items, and administration.
3) Voyage cost --- direct outlay for transporting cargo, which includes fuel cost, port charges, cargo expense and other miscellaneous expenses.

The above classification corresponds to divisions of payment shared by charterers and owners under three kinds of charter contract.

Under bare boat charter, charterers pay for ship cost and voyage cost, under time charter, they pay only for voyage cost and under voyage charter, the owners pay for all of capital, ship and voyage costs, as shown in Table 5.1.

5.5 Capital Cost

Depreciation of ship price is based on its purchase cost, which includes so-called fitting out expense in addition to contract price.

1) Contract price is the net price of purchasing a ship from shipbuilder and is paid to builder usually in four installments at the time of signing the building contract, keel laying, launching and delivery.
2) Fitting out expense is cost incurred before delivery of ship to owner, such as interest for three installments (C,K, and L), registration fee, rigging expense (supplies, supervisor and crew expense).

Table 5.1 Forms of Contract and Cost Sharing

Type of Charter / Cost	(Owned Vessel)	Bare Boat Charter	Time Charter	Voyage Charter
Capital Cost Ship price interest	O			
Ship Cost Crew expense stores lub. oil repair insurance misc. administration	O	O		
Voyage Cost Fuel cost port charges cargo exp.	O	O	O	
Charters' payment to owners	(No payment Charterers = owners)	Bare boat charter money	Time charter money	Freight

O = Charterers to pay
No mark = Owners to pay

One of the difficult points in transportation cost analysis is depreciation. Depreciation is primarily a tax matter. The principal shipping countries have their own regulations concerning the method of depreciation and the period of depreciation, usually in the 12-18 years range. There are even countries like the United Kingdom which leave depreciation to the free will of the owner.

From the point of view of cost analysis, however, depreciation is a matter of the shipowner to set a target of so many years in which to get back his investment.

It is not easy to generalize the attitude of investors. There will also be a difference in the attitude depending on whether the investor is optimistic or pessimistic about the future of the freight market. One method is to assume a fixed period of time, such as 10 years. However, the more closely realistic way is to postulate from the examples provided by long-term contracts reported in the market.

5.6 Interest

There are many forms of ship financing. The most general one is the one adopted by OECD and used by the shipbuilders of all OECD member countries. This is the arrangement concerning ship export credit conditions which has been effective as from July 1, 1974. It is as follows:

> For any contract relating to any new ship to be negotiated from 1st July, 1974 onwards, governments participating in this Understanding agree to abolish existing official facilities and to introduce no new official facilities for export credits for ships on terms providing:
> 1) a maximum duration exceeding 7 years from delivery and repayment other than by equal instalments at regular intervals of normally six months and a maximum of 12 months.
> 2) payment by delivery of less than 30 per cent of contract price.
> 3) an interest rate of less than 8 per cent, net of all charges.

The owner (buyer) arranges the financing for the remaining 30% of the cost. The interest on this will be decided in talks with the banking institution and is affected by the prevailing money situation.

The above are the contracted interest rates related to financing of ship purchase. Under an average freight market, repayment of 70% in seven years ordinarily results in a deficit. Thus, generally, to cover this deficit interest on a separate financing arrangement becomes necessary.

5.7 Ship Cost

Crew expense consists of wages, provisions, repatriation, welfare etc. Recent reports show that the yearly expense of a ship ranges from US$300,000 to US$1,500,000 according to nationality of officers and crew as shown in Table 5.2.

Table 5.2 Estimated Crew Expenses by Nationality

(Unit: $)

Nationality	Crew	No.	Salary O/T, etc.	Provisions & Repatriation	Total
U.S.A.	American	32	1,750,000		1,750,000
Japan	Japanese	30	1,150,000	100,000	1,250,000
Norway	Norwegian	38	850,000	150,000	1,000,000
England	English	36	700,000	400,000	1,100,000
Italy	Italian	38	780,000		780,000
Spain	Spanish		450,000		450,000
Greece	Greek, etc.	32	400,000		400,000
Hong Kong		28	325,000		325,000
Korea		28	300,000	100,000	400,000

Source: Japan Maritime Research Institute.
(Compiled from B.V. Bulletin, ILO report, etc.)

Stores consist of deck supplies, engine supplies, cabin and galley supplies, and include wires, hawsers, paints, spare parts, utensils, etc. Lubricating Oil is used for engines. Turbine consumes far less oil than diesel.

Repair and Maintenance

A ship must be inspected yearly (annual survey) in dry dock or afloat, and every four years (special survey) in dry dock. Special survey costs about 40-50% more than annual survey. Recent improvements in bottom cleaning under water have enabled shipowners to extend the intervals between dry docking from one year to one and a half or two years. Maintenance includes small repairs and spare parts. It is natural that these costs increase as a ship becomes older.

Insurance consists of hull insurance, P&I, off-hire insurance, etc. Hull insurance covers loss or damage due to sinking, stranding, burning, collision, etc. The insurance premium is decided by the insurance company (so-called underwriter) depending upon type, tonnage, age, insured amount of ship, volume and past achievement in loss ratio of owners or managers. P&I, standing for "protection" and "indeminity" pertaining principally today to oil pollution, covers risks not covered by hull insurance, including repatriation of ill seamen, damages to pier, buoy or other port equipment, injury to longshoremen, cleaning of and compensation for oil pollution, cargo claim, etc. Off-hire insurance is adopted by some shipowners chiefly for large sized vessels to cover any longtime off-hire caused by accidents. Miscellaneous charges include water and telegram charges, etc. Administration cost is the cost of managing the vessel.

5.8 Voyage Cost

Fuel cost usually comprises costs of C-F.O. which is used for main engine, and A-F.O. for generator. The generator supplies electric power for lights and motors of winches and ladder, etc.

The fuel expense is proportionate to the power of the engine and differs according to the type of engine, either diesel or turbine. Because of the rise in the fuel price, the diesel engine which consumes less fuel has become more advantageous than turbine. Because of the current practice of "slow steaming" to cut down the fuel consumption by reducing speed, the Canal transit of southbound VLCC in ballast has been affected.

Port charges include Canal dues, wharfage, buoyage, pilotage, towage, line handling, light dues, tonnage dues, launch hire, etc.

Cargo expense includes stevedorage, weighing, measuring, tallying, dunnage, etc.

There are cases in which the cargo expense becomes the responsibility of the shipper by the terms of the contract.

In dry cargo voyage charter, under "F.I.O" (free in and out) terms, the shipper bears both the loading and unloading expense and under "F.O" or "F.D." (free discharge), the shipper bears only the unloading expense.

In the case of liner, the expenses of both loading and unloading of general cargo are borne by the shipowner under what is known as "berth term". However, when dry bulk cargo is transported by voyage charter, the terms of the contract will decide which party pays.

5.9 Estimate of Yearly Cost Increase

Another vital point in transportation cost analysis is the forecast of future price rise. The violent inflation on the heels of the oil crisis of 1973 is still fresh in our memory. Because of this inflation, shipowners who had long-term contracts suffered tremendous losses and are even now incurring losses.

The rise in commodity prices at present is relatively moderate but it is difficult to forecast the future cost of fuel oil.

With regards to repair costs, it is necessary to take into consideration not only price increases but also the increase in expenses that accompany the aging of ships and that are necessitated by the tightening of various international regulations.

5.10 Trial Calculation of Transportation Cost (50,000 D/WT tanker)

Capital cost

Contract price	:	US$13,000,000
Fitting-out expense	:	5% of contract price
Loan	:	70% for repayment in 7 years
	:	30% for repayment in 8th, 9th and 10th years
Interest	:	8% per annum

Trial Calculation of Capital Cost is as follows:

(Unit: $)

	50,000 L/T (diesel)	
Contract Price	13,000,000	
Fitting out Exp. (5%)	650,000	
Total	13,650,000	
	1st year	10 years average
Repayment 10 years	1,365,000	1,365,000
Interest 8%	1,065,000	573,000
Capital Cost Total	2,430,000	1,938,000

Trial Calculation of bareboat charter hire on cost bais

Bareboat charterage is usually based on "per DWLT per month" or "daily".

1st year

$2,430,000 ÷ 12 month ÷ 50,000 L/T = $4.05

" ÷ 365 days = $6,658

10 years average

$1,938,000 ÷ 12 month ÷ 50,000 L/T = $3.23

" ÷ 365 days = $5,310

Ship cost

Trial Calculation of Ship Cost

Ship Cost	50,000 OWT – Diesel		Yearly Increase Rate
	1st Year	10 years Average	
Crew Expense	$1,250,000	$1,572,250	5%
Stores	50,000	62,890	5
Lubricating Oil	85,000	106,913	5
Maintenance & Repair	50,000	509,000	*
Insurance	200,000	200,000	
Miscellaneous	20,000	25,156	5
Administration	125,000	157,225	5
Total Ship Cost	$1,780,000	$2,633,434	

*: Yearly estimation

Trial calculation of time charter hire on cost basis

1st year

$2,430,000

$1,780,000

$4,210,000 ÷ 11.5 m. ÷ 50,000 L/T = $7.32

÷ 350 d. = $12,029

10 years average

$1,938,000

$2,633,434

$4,571,434 ÷ 11.5 m. ÷ 50,000 L/T = $7.95

÷ 350 d. = $13,061

350 d. : Total working days of a year. In this case 15 days are reserved for drydocking and repair.

Voyage Cost

Approximate voyage cost of 50,000 DWT of Ras Tanura/Rotterdam/Ras Tanura via Suez is as shown in voyage estimate. The rate of W80 is shown as an example.

VOYAGE ESTIMATE

No.
Date

M/S, S/S Suez Canal
(D/W 50,000 L/T) Speed 15.k't (laden)
16.k't (in ballast)

Cargo	Tons	Rate	Freight	
Crude oil	46,883	$8.816 (W80)		
		$2.06 (toll)		
		Des/Dem –		$509,900

Port Suez/Suez	Distance (mile)	at Sea (day)	in Port (day)	Port charges $
Ras Tanura			1.5	12,000
	6,755	18.8		
Rotterdam			2.0	24,000
	6,755	17.6		
Ras Tanura				
		1.0		50,000
Suez				
		1.0	1.0	40,000
				Cargo expense $
				Commission $
(spare)			1.0	Sundries
Total		38.4	5.5	$ 5,000
	Total	43.9 days		

Fuel consumption					
C-F.O.	at Sea	56 T/d	2,151 L/T		
	in Port	28 T/d	154 T		
	Total		2,305 T	@$80	$ 184,400
A-F.O.	at Sea	1 T/d	T		
	in Port	1 T/d	44 T		
	Total		44 T	@$130	$ 5,720

D/W	50,000 L/T	Total expense	$ 321,121
Fuel	2,349	Net proceed	$ 188,780
Spare	168	Hire cost	$ 528,073
Water } Others }	600	Net profit	$(-)339,293
		(C/B)	$ 2.58
Cargo	46,883	(H/B)	$ 7.22

Trial calculation of freight on cost basis, Freight rate is usually based on long ton of cargo loaded.

1st year

$321,120 (voyage cost per voyage)
$528,073 ($12,029 x 43.9 days)
849,193 ÷ 46,883 L/T = $18.11

10 years average

$403,969 (voyage cost, 5% yearly increase)
$573,378 ($13,061 x 43.9 days)
977,347 ÷ 46,883 L/T = $20.85

Charter base (C/B) and Hire base (H/B), which represent profitability and Hire cost per D/W:L/T 30 days, are now widely used in Japan.

$$C/B = \frac{\text{Freight - voyage cost (per voyage)}}{\text{Deadweight (L/T)}} \cdot \frac{\text{30 days}}{\text{voyage days}}$$

$$H/B = \frac{\text{Capital cost + Ship cost (per year)}}{\text{Deadweight (L/T)}} \cdot \frac{\text{30 days}}{\text{Working days per year}}$$

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No. 2

AN INTRODUCTION TO BASIC MATHEMATICS, STATISTICS AND COMPUTER PROGRAMMING

PART I. BASIC MATHEMATICS

TABLE OF CONTENTS

CHAPTER 1 REVIEW

1.1 Numbers

ELEMENTARY MATHEMATICS is concerned mainly with certain elements called numbers and with certain operations defined on them.

The unending set of simbols 1,2,3,4,5,6,7,8,9,10,11,12,13,..... used in counting are called natural numbers.

In order that subtraction be always possible, it is necessary to increase the set of numbers. We prefix each natural number with a + sign (in practice, it is more convenient not to write the sign +) to form the positive integers, we prefix each natural number with a - sign (the sign must always be written) to form the negative integers, and we create a new symbol 0, read zero. On the set of integers,

$$\ldots,-6,-5,-4,-3,-2,-1,0,+1,+2,+3,+4,+5,+6,\ldots$$

the operations of addition and subtruction are possible without exception.

THE RATIONAL NUMBERS. The set of rational numbers consists of all numbers of the form m/n, where m and n ($\neq 0$) are integers.

If a and b are rational numbers, $a+b$, $a-b$ and $a\cdot b$ are rational numbers. Moreover, if a and b are $\neq 0$, there exists a rational number x such that

$$ax=b.$$

When a or b or both are zero, we have the following situations

- $b=0$ and $a\neq 0$: $ax=b$ becomes $ax=0$, and $a\neq 0$, then $x=0$, that is, $0/a=0$ when $a\neq 0$.
- $a=0$ and $b\neq 0$: $ax=b$ becomes $0x=b$; then $b/0$, when $b\neq 0$, is without meaning since $0x=0$.
- $a=0$ and $b=0$: $ax=b$ becomes $0x=0$; then $0/0$ is indeterminate since every number x satisfies the equation.

In brief: $0/a=0$ when $a\neq 0$, but division by o is never permitted.

THE IRRATIONAL NUMBERS. Any rational number is expressed as m/n where m and n are both integers, but we have numbers which are never expressed as above expression such as $\sqrt{2}=1.41421356\cdots$, and $\pi=3.14159\cdots$. These numbers are called irrational numbers. Endless decimals without any repetition are always irrational numbers.

THE REAL NUMBERS. The set of real numbers consists of the rational and irrational numbers. For any real number x, $x^2\geqq 0$ holds, the equality holds if and only if $x=0$.

THE IMAGINARY NUMBERS AND COMPLEX NUMBERS will play no role here and the term will be used only in pointing out their exclution. Since no confusion can result, the term number will be used hereinafter to mean a real number.

1.2 Expansion and Factoring of expressions

In order to apply mathematics to actual problems, easy handling of expansion and factoring of expressions is one of the very important work. Everyone has to be familiar with the following expansion equalities.

$(x+y)^2=x^2+2xy+y^2$ $\qquad$ $(x-y)^2=x^2-2xy+y^2$

$(x+y)(x-y)=x^2-y^2$ $\qquad$ $(x+a)(x+b)=x^2+(a+b)x+ab$

$(a+b)^3=a^3+3a^2b+3ab^2+b^3$ $\qquad$ $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

$(a+b)(a^2-ab+b^2)=a^3-b^3$ $\qquad$ $(a-b)(a^2+ab+b^2)=a^3-b^3$

$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

$(a+b+c)(a^2+b^2+c^2-ab-bc-ca)=a^3+b^3+c^3-3abc$

For verification of above equalities, use the equality $a(b+c)=ab+ac$ or $a(b+c+d)=ab+ac+ad$.

EXAMPLE: Verify $(x+y)(x-y)=x^2-y^2$.

Let $x+y=a$, then $(x+y)(x-y)=a(x-y)=ax-ay$.

Now $a=x+y$, $ax-ay=(x+y)x-(x+y)y=xx+yx-(xy+yy)=xx-yy=x^2-y^2$.

1.3 Exponents

POSITIVE INTEGER EXPONENTS. If a is any number and n is any positive integer, the product of the n factors $a\cdot a\cdot a\cdots a$ is denoted by a^n. To distinguish between the letters, a is called the base and n is called the exponent.

If a and b are any bases and m and n are any positive integers, we have the following laws of exponents:

$a^m\cdot a^n = a^{m+n}$ $\qquad$ $(a^m)^n = a^{mn}$

$a^m/a^n = a^{m-n}$, $a\neq 0$, $m>n$; $\quad a^m/a^n = 1/a^{n-m}$, $a\neq 0$, $m<n$.

$(a\cdot b)^n = a^n b^n$ $\qquad$ $(a/b)^n = a^n/b^n$, $b\neq 0$

THE nth ROOT OF a. Let n be a positive integer and a and b be two numbers such that $b^n=a$, then b is called an nth root of a.

If a is real number and n is odd, then exactly one of the nth roots of a is real.
If a is real and n is even, then there are exactly two real n th roots of a when $a>0$, but no real nth roots of a when $a<0$.

THE PRINCIPAL nth ROOT OF a is the positive real nth root of a when a is positive, and the real nth root of a, if any, when a is negative. The principal nth root of a is denoted by $\sqrt[n]{a}$, called a <u>radical</u>. The integer n is called the <u>index of the radical</u> and a is called the <u>radicand</u>. For example, $\sqrt{9}=3$, $\sqrt[6]{64}=2$, $\sqrt[5]{-243}=-3$.

ZERO, FRACTIONAL, AND NEGATIVE EXPONENTS. When r and s are positive integers and p is any rational number, the following extend the definition of a^n in such away that the laws of exponent are satisfied when n is any rational number.

DEFINITIONS	
$a^0=1,\ a\neq 0$	$3^0=1,\ (1/20)^0=1,\ (-5)^0=1$
$a^{r/s}=\sqrt[s]{a^r}=(\sqrt[s]{a})^r$	$5^{1/2}=5,\ (64)^{5/6}=(\sqrt[6]{64})^5=2^5=32$
$a^{-p}=1/a^p$	$4^{-1}=1/4,\ 3^{-1/2}=1/\sqrt{3}$

IRRATIONAL EXPONENTS. Without attempting to define them, we shall assume the existence of numbers as $a^{\sqrt{2}}$, a^{π},..., in which the exponent is irrational. We shall also assume that these numbers have been defined in such a way that the laws of exponents are satisfied.

CHAPTER 2 FUNCTION AND GRAPH

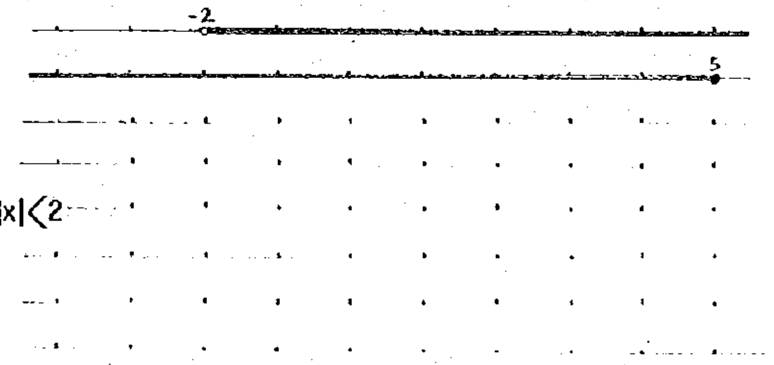
2.1 Functions

A VARIABLE IS A SIMBOL selected to represent any one of a given set of numbers. If the set consists of just one number, the simbol represending it is called a constant.

The range of a variable consists of the totality of numbers of the set which it represents. For example, if x is a day in October, the range of x is the set of positive integers 1,2,3,4,···,31.

EXAMPLES: Represent graphically each of the following ranges,

a) $x > -2$
b) $x \leq 5$
c) $x > -1$
d) $-3 \leq x \leq 4$
e) $-2 < x < 2$ or $|x| < 2$
f) $x < 3$
g) $-3 < x \leq 5$
h) $x < -3$, $x \geq 4$



FUNCTION. A correspondence (x,y) between two sets of numbers which pairs to an arbitrary number x of the first set y of the second is called a function. In this case, it is customary to speak of y as a function of x. The variable x is called the independent variable and y is called the dependent variable.

A function may be defined

a) by a table of correspondents or table of values, as

x	1	2	3	4	5	6	7	8	9	10	11	12
y	4	5	6	7	8	9	10	11	12	13	14	15

b) by a equation or formula, as $y=x+3$.

For each value assigned to x, the above relation yields a correspondents. Note that the table above is a table of values for this function.

A function is called single-valued if, to each value of x in its range, there corresponds just one value of y; otherwise, the function is called multi-valued. For example, $y=x^2-3x+2$ defines y as a single-valued function of x, while $x^2+y^2=1$ or $y=\pm\sqrt{1-x^2}$ defines y as a multi-valued (here, two-valued) function of x.

At times it will be more convenient to label a given function of x as f(x), to be read "f function of x" or simply "f of x".
If there are two functions one may be labeled f(x) and the other g(x). Also, if $y=f(x)=x^2-3x+2$ the statement "the value of the function is 2 when x=3" can be replaced by "f(3)=2".

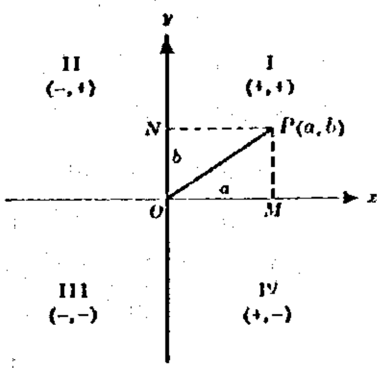
Let y=f(x). The range of independent value x is called the domain of definition of the function, while the range of the dependent variable is called the range of the function. For example, $y=x^2$ defines a function whose domain of definition consists of all (real) numbers and whose range is all non-negative numbers, that is, zero and the positive numbers; f(x)=3/(x-2) defines a function whose domain of definition consists of all numbers except 2(why?) and whose range is all numbers except 0.

2.2 Graph

A function y=f(x), by definition, yields a collection of number pairs (x,f(x)) or (x,y) in which x is any element in the domain of definition of the function and f(x) or y is the corresponding value of the function.

THE RECTANGULAR CARTESIAN COORDINATE SYSTEM in a plane is a device by which there is established a one-to-one correspondence between the points of the plane and ordered pairs of real numbers (a,b).

Consider two real number scales intersecting at right angles in O, the origin of each (see adjoining diagram), and having the positive direction on the horizontal scale (now called the x-axis) directed to the right and the positive direction on the vertical scale (now called the y-axis) directed upward.



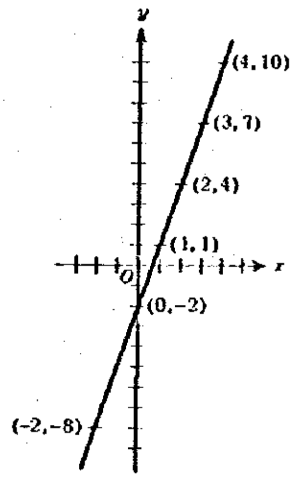
Let P be any point distinct from O in the plane of the two axes and join P to O by a straight line. Let the projection of OP on the x-axis be OM=a and the projection of OP on the y-axis be ON=b. Then the pair of numbers (a,b) in that order are called the plane rectangular cartesian coordinates (briefly, the rectangular coordinates) of P. In particular, the coordinates of O, the origin of the coordinate system, are (0,0).

The first coordinate, giving the directed distance of P from the y-axis, is called the <u>abscissa</u> of P while the second coordinate, giving the directed distance of P from the x-axis, is called the <u>ordinate</u> of P. Note carefully that the point (4,3) and (3,4) are distinct points.
The axes divide the plane into four sections, called <u>quadrants</u>.The figure in the previous page shows the customary numbering of the quadrants and the respective signs of the coordinates of a point in each quadrant.

THE GRAPH OF A FUNCTION f(x) consists of the totality of points (x,y) whose coordinates satisfy the relation y=f(x).

EXAMPLE: Sketch the graph of the function $y=3x-2$.

x	-2	-4/3	-1/2	0	1/3	1	2	5/2	3	4
y	-8	-6	-7/2	-2	-1	1	4	11/2	7	10



After plotting the points whose coordinates (x,y) are given the above table, it appears that they lie on a straight line. The adjoining sketch is not the complete locus since (1000,2998) is one of its points and is not shown. Moreover, although we have joined the points by a straight line, we have not proved that every point on the line has as coordinates a number-pair given by the function. These matters as well as such questions as: What values of x should be chosen? How many values of x are needed? will become clearer as we proceed with the study of functions. At present,

1) built a table of values,
2) plot the corresponding points,
3) pass a smooth curve through these points, moving flom left to right.

Any value of x for which the corresponding value of a function f(x) is zero is called a <u>zero</u> of a function. Such values of x are also called <u>roots</u> of the equation f(x)=0. The real roots of an equation f(x)=0 may be approximated by estimating from the graph of y=f(x) the abscissas of its points of intersection with the x-axis.

Algebraic methods for finding the roots of equations will be treated in later chapters.

CHAPTER 3 LOGARITHMS

If $N=b^a$, then a is called logarithm of a positive numver N to a given base b denoted by $a=\log_b N$. $(b\neq 1, b>0, N>0)$

EXAMPLE:

$9=3^2 \longrightarrow \log_3 9=2$

$64=4^3 \longrightarrow \log_4 64=3$

$64=2^6 \longrightarrow \log_2 64=6$

$1000=10^3 \longrightarrow \log_{10} 1000=3$

$0.01=10^{-2} \longrightarrow \log_{10} 0.01=-2$

Fundamental Laws of Logarithms

1. $\log_b xy=\log_b x+\log_b y$
2. $\log_b x/y=\log_b x-\log_b y$
3. $\log_b x^n=n \log_b x$
4. $\log_b \sqrt[n]{x}=(1/n)\log_b x$

(for laws 1–4: $x>0,\ y>0$; $b>0,\ b\neq 1$)

5. $\log_b C=\log_d C/\log_d b$ $(b>0, C>0, d>0, b\neq 1, d\neq 1)$

Numerical Computations

In numerical computations the most useful base for a system of logarithms is 10. Such logarithms is called common logarithms.

The common logarithm of a positive number N consists of 2 parts: an integer (positive,negative,or 0) called the characteristics and a positive decimal function called the mantissa.

In the case $N=b^a$ or $a=\log_b N$,
when we know the value of a, N is called the antilogarithm(antilog) of the given logarithm.

The <u>cologarithm</u> of a positive number N(written, colog N) is the logarithm of its reciprocal 1/N.

Thus $\operatorname{colog} N=\log(1/N)=\log 1-\log N=-\log N$

Exponential Equation is an equation involving one or more unknowns in an exponent.

EXAMPLE: 1. Solve $2^x=7$

2. Solve $(1.03)^{-x}=2.5$

In the calculus the most useful system of logarithms is the natural system in which the base is a certain irrational number e=2.71828 approximately.

$(e=\lim_{n\to\infty}(1+1/n)^n)$

The natural logarithm of N, ln N, and the common logarithm of N, $\log_{10}N$ are replaced by the formula

$\ln N \fallingdotseq 2.3026 \times \log N \ (2.3026 \fallingdotseq 1/\log e)$.

CHAPTER 4 LINEAR EQUATIONS AND LINEAR FUNCTIONS

4.1 Equation

An equation is a statements, such as

(a) $5x-4=2x+5$

(b) $x^2-3x=-2$

(c) $3x+5y= 4xy+3$

that two expressions are equal.
The (a) is a linear equation in one unknown, the (b) is a quadratic in one unknown, the third is linear in each of the two unknowns but is of degree two in the two unknowns.

4.2 Solution of Equation

Any set of values of unknowns for which the two members (left hand side and right hand side) of an equation are equal is called a solution of the equation.
Thus, $x=3$ is a solution of (a) since $5(3)-4=2(3)+5$; $x=1$ is a solution of (b) (and also $x=2$ in (b)); and $x=2$, $y=1$ is a solution of (c).
A solution of an equation in one unknown is also called a roof of the equation.

4.3 Linear Function

An equation with 2 unknowns x, y usually has unlimited number of solutions (x, y), and y is taken to be a function of x.
If the equation is linear, this function y is called a linear function.

4.4 A pair of numbers (x, y) satisfying a linear equation lies on some straight line on the plane, and vice versa.

a. When given a linear function $y=f(x)$ or $f(x, y)=0$,

1) we find two pairs of (x, y)s, i.e. (x_1,y_1), (x_2, y_2) such that $y_1=f(x_1)$, $y_2=f(x_2)$ or $f(x_1, y_1)=0$, $f(x_2,y_2)=o$,

2) plot these two points (x_1, y_1), (x_2, y_2) on the plane,

3) and then draw a straight line passing through these two points.

b. How to find linear functions of given straight lines on the plane?

1) Slope-intercept form

The equation of the straight line having slope m and y-intercept b is

$$y=mx+b$$

2) Point-slope form

The equation of the line having slope m and passing through the point (x_1,y_1) is

$$y-y_1=m(x-x_1)$$

3) Two point form

The equation of the line passing through the points (x_1,y_1) and (x_2,y_2) is

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1) \qquad (\text{if } x_1\neq x_2)$$

$$x=x_1(=x_2) \qquad (\text{if } x_1=x_2)$$

or $(x_2-x_1)(y-y_1)=(y_2-y_1)(x-x_1)$

4) Intercept form

The equation of the line whose x-intercept is a and whose y-intercept is b, where $ab\neq0$, is

$$\frac{x}{a}+\frac{y}{b}=1$$

CHAPTER 5 SOLVING EQUATIONS

1. To Solve a Linear Equation in one unknown.

a) $ax + b = 0$

b) $ax + b = cx + d$

c) $(ax + b)/c = (dx + e)/f$

Case b) and c) finaly are induced to a) by arranging the terms by the equality law.

The solution of a) is $-b/a$ (if $a \neq 0$)
any number (if $a = 0$, $b = 0$)
non-exist (if $a = 0$, $b \neq 0$)

$ax + b = 0$

$\therefore ax = -b$

If $a \neq 0$, then $x = -b/a$

If $a = 0$, then if $b \neq 0$ x doesn't exist.
if $b = 0$ x is any number.

b) $ax - cx = b - d$

$(a-c)x = b-d$

If $a \neq c$, then $x = (b-d)/(a-c)$

If $a = c$ and $b \neq d$, then x doesn't exist.

If $a = c$ and $b = d$, then x is any number.

c) $(ax + b).f = (dx + e).c$

$afx - cdx = ce - bf$

$(af - cd)x = ce - bf$

If $af \neq cd$, then $x = (ce - bf)/(af - cd)$

If $af = cd$ and $ce \neq bf$, then x doesn't exist.

If $af = cd$ and $ce = bf$, then x is any number.

2. Two Linear Equations in Two Unknowns

Let the system of equations be

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

Each Equation has unlimited number of solutions (x, y) corresponding to the unlimited number of points on the Locus (Straight line) which it represents.

Our Problem is to find all solutions common to the two equations or the co-ordinates of all points common to the two lines.

3 The Solution of
$$\begin{cases} a_1x + b_1y + c_1 = 0 \text{ -------- } (1) \\ a_2x + b_2y + c_2 = 0 \text{ -------- } (2) \end{cases}$$

(1) x a_2 $\quad a_1a_2x + a_2b_1y + a_2c_1 = 0$ ------------- (1)'

(2) x a_1 $\quad a_1a_2x + a_1b_2y + a_1c_2 = 0$ ------------- (2)'

(1)' - (2)' $\quad (a_2b_1 - a_1b_2)y = a_2c_1 - a_1c_2$ --------- (3)

(1) x b_2 $\quad a_1b_2x + b_1b_2y + b_2c_1 = 0$ ------------- (1)'

(2) x b_1 $\quad a_2b_1x + b_1b_2y + b_1c_2 = 0$ ------------- (2)'

(1)' - (2)' $\quad (a_1b_2 - a_2b_1)x = b_1c_2 - b_2c_1$ --------- (4)

In (3) and (4), if $a_1b_2 \neq a_2b_1$

then $x = (b_1c_2 - b_2c_1)/(a_1b_2 - a_2b_1)$ and

$y = (a_1c_2 - a_2c_1)/(a_1b_2 - a_2b_1)$

if $a_1b_2 = a_2b_1$ and ($a_1c_2 \neq a_2c_1$ or $b_1c_2 \neq b_2c_1$), then we have no solution.

if $a_1b_2 = a_2b_1$ and $a_1c_2 = a_2c_1$ and $b_1c_2 = b_2c_1$, that is, $a_1/a_2 = b_1/b_2 = c_1/c_2$, then (1) and (2) are the s me equations.

In this case, we have many solution x, y satisfying (1) and (2), such that $x = k$, $y = -(1/b_1)(a_1k + c)$ for any k if $b \neq 0$.

The graph of the quadratic function $y = ax^2 + bx + c$ $(a \neq 0)$ is a parabola. If $a>0$, the parabola opens upward. If $a<0$, the parabola opens downward.
The lowest point of the parabola $(a>0)$ or the highest point of the parabola (a 0) is called a vertex.

The Vertex of a given parabola $y = ax^2 + bx + c$ $(a \neq 0)$

As
$$\begin{aligned}
y &= ax^2 + bx + c \\
&= a(x^2 + (b/a)x) + c \\
&= a(x^2 + 2(b/2a)x) + c \\
&= a[x^2 + 2(b/2a)x + (b/2a)^2 - (b/2a)^2] + c \\
&= a[x^2 + 2(b/2a)x + (b/2a)^2] - a(b/2a)^2 + c \\
&= a[x + (b/2a)]^2 - a(b^2/4a^2) + c \\
&= a[x + (b/2a)]^2 - (b^2/ 4a) + (4ac/4a) \\
&= a[x + (b/2a)]^2 - (b^2 - 4ac)/4a,
\end{aligned}$$

the coordinates of the vertex are $(-(b/2a), -(b^2 - 4ac)/4a)$.

$\because$) If $a>0$, then $a[x + (b/2a)]^2 \geqq 0$ equality holds if and only if $x = -(b/2a)$.

Therefore, $y = 0-(b^2-4ac)/4a \geqq -(b^2 - 4ac)/4a$

Therefore, if $a>0$, $x = -(b/2a)$ gives the minumum value of y. which is equal to $-(b^2 - 4ac)/4a$.

If $a<0$, then $a[x + (b/2a)]^2 \leqq 0$ equality holds if and only if $x = -(b/2a)$.

So that $y = a[x + (b/2a)]^2 -(b^2-4ac)/4a \leqq -(b^2-4ac)/4a$

A quadratic equation in one unknown x is the form

$$ax^2 + bx + c = 0 \qquad (a \neq 0)$$

Frequently a quadratic equation may be solved by factoring.

General Method to find solutions.

$$ax^2 + bx + c = 0$$

$$\therefore \quad a[x + (b/2a)]^2 - (b^2 - 4ac)/4a = 0$$

$$\therefore \quad [x + (b/2a)]^2 - (b^2 - 4ac)/4a^2 = 0$$

If $b^2 - 4ac \geqq 0$, then

$$[x + (b/2a)]^2 - (\sqrt{b^2 - 4ac}/2a)^2 = 0$$

Now, by applying the factoring formula $p^2-q^2=(p+q)(p-q)$ the above equation is factored.

$$[(x+\frac{b}{2a})+(\frac{\sqrt{b^2-4ac}}{2a})][(x+\frac{b}{2a})-(\frac{\sqrt{b^2-4ac}}{2a})]=0$$

$$x+\frac{b}{2a}+\frac{b^2-4ac}{2a}=0 \quad \text{or} \quad x+\frac{b}{2a}-\frac{b^2-4ac}{2a}=0$$

$$x=\frac{-b+\ b^2-4ac}{2a} \quad \text{or} \quad x=\frac{-b-\ b^2-4ac}{2a}$$

If $b^2-4ac<0$, then

$$(x+\frac{b}{2a})^2\geq 0 \quad \text{and} \quad -\frac{b^2-4ac}{4a^2}>0$$

So that any number x can never satisfy $ax^2+bx+c=0$

If we define i to be $i^2=-1$, then, in the case $b^2-4ac<0$

$$ax^2+bx+c=(x+\frac{b}{2a})^2-\frac{b^2-4ac}{4a^2}$$

$$=(x+\frac{b}{2a})^2-\frac{(4ac-b^2)i^2}{4a^2}$$

$$=(x+\frac{b}{2a})^2-(\frac{\sqrt{4ac-b^2}i}{2a})^2$$

$$=[(x+\frac{b}{2a})+\frac{\sqrt{4ac-b^2}i}{2a}][(x+\frac{b}{2a})-\frac{\sqrt{4ac-b^2}i}{2a}]$$

and we obtain

$$x=\frac{-b+\sqrt{4ac-b^2}i}{2a} \quad \text{or} \quad x=\frac{-b-\sqrt{4ac-b^2}i}{2a}$$

Discriminant of the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ is, by definition, the quantity $b^2 - 4ac$.

When a, b, and c are rational numbers, the roots of the equation are

real and unequal if and only if $b^2 - 4ac > 0$

real and equal if and only if $b^2 - 4ac = 0$

rational if and only if $b^2 - 4ac$ is the square of a rational number

imaginary (complex) and unequal if and only if $b^2 - 4ac < 0$.

Sum and Product of the roots.

If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then

$$\alpha + \beta = -b/a \qquad \alpha\beta = c/a.$$

A quadratic equation whose roots are α and β may be written in the form

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0.$$

Any Arrangement of a set of objects in a definite order is called PERMUTATIONS of the set of letters all at a time.

The number of permutations ${}_nP_r$ of n different objects taken r n at a time is given by

$$ {}_nP_r = n(n-1)---(n-r+1) $$

$$ = \frac{n(n-1)---(n-r+1)(n-r)---2\cdot 1}{(n-r)\cdot -------2\cdot 1} = \frac{n!}{(n-r)!} $$

n!: the symbol n! (read factorial n) denotes the product of the positive integers from 1 to n inclusive.
for example, $1!=1$, $3!=1\cdot 2\cdot 3=6$.
(we define $0!=1$ for many laws hold.)

The COMBINATIONS of n objects taken r at a time consists of all possible sets of r of the objects, without regard to the order of arrangement.

The numbers of combinations of n objects taken r at a time will be denoted by ${}_nC_r$.

$$ {}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{(n-r)!r!} \qquad ({}_nC_n = {}_nC_0 = 1) $$

1) ${}_nC_r = {}_nC_{n-r}$ holds.

2) ${}_{n+1}C_r = {}_nC_r + {}_nC_{r-1}$ holds.

Binomial Thorem

$$ (a+b)^n = {}_nC_0 a^n + {}_nC_1 a^{n-1} b + {}_nC_2 a^{n-2} b^2 + --- = {}_nC_k a^{n-k} b^k + --- $$

$$ --- + {}_nC_{n-2} a^2 b^{n-2} + {}_nC_{n-1} a b^{n-1} + {}_nC_n b^n $$

$$ = \sum_{k=0}^{n} {}_nC_k a^{n-k} b^k $$

By Binomial Theorem

${}_nC_0 + {}_nC_1 + {}_nC_2 + --- + {}_nC_{n-1} + {}_nC_n = 2^n$ is shown since $(1+1)^n = 2^n$

Pascal's Triangle

$${}_0C_0$$
$${}_1C_0 \quad {}_1C_1$$
$${}_2C_0 \quad {}_2C_1 \quad {}_2C_2$$
$${}_3C_0 \quad {}_3C_1 \quad {}_3C_2 \quad {}_3C_3$$
$${}_4C_0 \quad {}_4C_1 \quad {}_4C_2 \quad {}_4C_3 \quad {}_4C_4$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

Multinomial Theorem

In $(a+b+c)^n$, the coefficient of $a^{n_1}b^{n_2}c^{n_3}$ $(n_1+n_2+n_3=n)$ is given as $\frac{n!}{n_1!n_2!n_3!}$, so that $(a+b+c)^n = \sum_{n_1+n_2+n_3=n} \frac{n!}{n_1!n_2!n_3!} a^{n_1}b^{n_2}c^{n_3}$.

More than three letters, the same type of multinomial theorem holds:

$$(a_1+a_2+\cdots+a_k)^n = \sum_{n_1+n_2+n_3=n} \frac{n!}{n_1!n_2!n_3!\cdots n_k!} a_1^{n_1}a_2^{n_2}\cdots a_k^{n_k}.$$

CHAPTER 8 POLYNOMIAL

The polynomial of the nth degree in x has the form

$$f(x)= a_0x^n+ a_1x^{n-1}+ a_2x^{n-2}+\cdots+a_kx^{n-k}+\cdots+a_{n-2}x^2+ a_{n-1}x +a_n$$

in which n is a positive integer and the a's are constants with $a_0\neq 0$.

The term a_0x^n is called the leading term, a_n is called the constant term, and a_0 is called the leading coefficient.

Remainder Theorem

If a polynomial f(x) is divided by $(x-\alpha)$ until a remainder free of x is obtained, this remainder is $f(\alpha)$.

EXAMPLE: Let $f(x)=x^3+ 2x^2- x +3$.

Now, we try to divide f(x) by (x-2),

By actual division

$$(x^3+ 2x^2-x+3)/(x-2)=x^2+ 4x +7+ 17/(x-2),$$

or $x^3+ 2x^2-x+3=(x-2)(x^2+ 4x +7)+17$,

and the remainder is 17.

By the remainder theorem,

the remainder is $f(2)=2^3+2\cdot 2^2-2+3=17$.

Factor Theorem

If $(x-\alpha)$ is a factor of a polynomial f(x), then $f(\alpha)=0$, and conversely.

EXAMPLE: Let $f(x)=x^3+2x^2-4x +1$.

Now $f(1)=1^3+2\cdot 1^2-4\cdot 1+1=0$,

then f(x) has a factor (x-1).

$$\begin{array}{r|l} & x^2+3x-1 \\ \hline x-1 & x^3+2x^2-4x+1 \\ & x^3-x^2 \\ \hline & 3x^2-4x \\ & 3x^2-3x \\ \hline & -x+1 \\ & -x+1 \\ \hline & 0 \end{array}$$

By dividing f(x) by (x-1), we get quotient $(x^2+3x -1)$ and remainder 0. Thus, $f(x)=(x-1)(x^2+3x-1)$

Synthetic Division

By a process known as synthetic division, the necessary work in dividing a polynomial $f(x)$ by $(x-\alpha)$ may be displayed in three lines, as follows;

$$(a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n)/(x-\alpha)$$

$$\begin{array}{r|lllll}
 & a_0 & a_1 & a_2 \text{ ----------} & a_{n-1} & a_n \\
\alpha & & \alpha a_0 & \alpha(a_1+\alpha a_0) \text{ ----------} & & \\
\hline
 & a_0 & a_1+\alpha a_0 & a_2+\alpha a_1+\alpha a_0^2 \text{ -----} & & \text{-----}
\end{array}$$

(1) Arrange the devided $f(x)$ in descending powers of x, and set down in the first line the coefficients, supplying zero as coefficient whenever a term is missing.

(2) Place α, the synthetic divisor, in the second line to the left of the a_0.

(3) Recopy the leading coefficient a_0 directly below it in the third line.

(4) Multiply a_0 by α; place the product $a_0\alpha$ in the second line under a_1(in the first line), add to a_1, and place the sum $a_1+\alpha a_0$ in the third line under a_1.

(5) Multiply the sum in step(4) by α; place the product in the second line under a_2, add a_2, and place the sum in the third line under a_2(in the first line).

(6) Repeat the process of step(5) until a product has been added to the constant term a_n.

The first n numbers in the third line are the coefficients of the quotient, a polynomial of degree $(n-1)$, and the last number of the third line is the remainder $f(\alpha)$.

EXAMPLES: (1) Divide x^3+2x^2-x+3 by $x-2$.

$$\begin{array}{r|rrrr}
 & 1 & 2 & -1 & 3 \\
2 & & 2 & 8 & 14 \\
\hline
 & 1 & 4 & 7 & 17
\end{array}
\qquad
\begin{array}{ll}
\text{Quoutient} & x^2+4x+7 \\
\text{Remainder} & 17(=f(2))
\end{array}$$

(2) Divide x^3+2x^2-4x+1 by $x-1$.

$$\begin{array}{r|rrrr}
 & 1 & 2 & -4 & 1 \\
1 & & 1 & 3 & -1 \\
\hline
 & 1 & 3 & -1 & 0
\end{array}
\qquad
\begin{array}{ll}
\text{Quoutient} & x^2+3x-1 \\
\text{Remainder} & 0(=f(1))
\end{array}$$

Theorem

Finding rational roots of a polynomial equation.

In a polynomial $f(x)=a_0x^n+a_1x^{n-1}+\cdots+a_{n-2}x^2+a_{n-1}x+a_n$,
where all a's are integers and $a_0 \cdot a_n \neq 0$,
if the polynomial equation $f(x)=0$ has a rational root m/n,
where m and n are integers, then m is a divisor of the constant term a_n and n is a divisor of the leading coefficient a_0.

EXAMPLE: Find the roots of the polynomial equation

$$f(x)=6X^3+11X^2-3X-2=0.$$

If we have a rational root m/n,

then m: divisor of -2, that is, ±1, ±2,

n: divisor of 6, that is, ±1, ±2, ±3, ±6.

Try to compute $f(m/n)$, and find m/n such that $f(m/n)=0$.

CHAPTER 9 INEQUALITIES

An Inequality is a statement that one (real) number is greater than or less than another.

Two inequalities are said to have the same sense if their signs of inequality point in the same direction.

Thus $\begin{cases} 3>-2 \\ -5>-10 \end{cases}$ have the same sense; $\begin{cases} 3>-2 \\ -10<-5 \end{cases}$ have opposite sense.

The sense of inequality is not changed:

a) if the same number is added or subtracted from both sides.

b) if both sides are multiplied or divided by the same positive number.

The sense of inequality is changed if both sides are multiplied or divided by the same positive number.

An Absolute Inequality is one which is true for all real values of the letters involved.

For example, $x^2+1>0$ is an absolute inequality.

A Conditional Inequality is one which is true for certain value of the letters involved.

For example, $x+2>5$ is a conditional inequality since it is true for $x=4$ but not for $x=1$.

Solution of Conditional Inequality.

The solution of a conditional inequality in one letter, say x, consists of all values of x for which the inequality is true. These values line on one or more intervals of the real number scale.

To solve a linear inequality, proceed as in solving a linear equality keeping in mind the rules for keeping or reversing the sense.

EXAMPLE:

$$5X+4>2X+6$$
$$5X-2X>6-4$$
$$3X>2$$
$$X>2/3$$

To solve a quadratic inequality $f(x)=ax^2+bx+c>0$, solve the equality $f(x)=0$, locate the roots r_1 and r_2 on a number scale, and determine the sign of $f(x)$ on each of the resulting intervals.

EXAMPLE: $3X^2-8X+7>2X^2-3X+1$

$f(x)=X^2-5X+6>0$

$(x-2)(x-3)>0$

$f(x)>0$ $f(x)<0$ $f(x)>0$

——②——③——

$X<2$ or $3<X$.

General Solution for $f(x)=ax^2+bx+c>0$

1. $a>0$

1 $b^2-4ac>0$

In this case, $f(x)=0$ has two distinct real roots and by $a>0$ the parabola opens upward, so that the solution of $f(x)>0$ is

$$x<(-b-\sqrt{b^2-4ac})/2a \quad \text{or} \quad (-b+\sqrt{b^2-4ac})/2a<x.$$

Note Never express

$$(-b-\sqrt{b^2-4ac})/2a<x<(-b-\sqrt{b^2-4ac})/2a,$$

because

$$(-b+\sqrt{b^2-4ac})/2a>(-b-\sqrt{b^2-4ac})/2a \text{ holds always.}$$

2 $b^2-4ac=0$

In this case, $f(x)=0$ has only one real roots $-(b/2a)$, and by $a>0$ the parabola opens upward, so that the solution of $f(x)>0$ is

$$x<-(b/2a) \quad \text{or} \quad -(b/2a)<x,$$

that is, x is any real number except $x=-(b/2a)$.

3 $b^2-4ac<0$

In this case, $f(x)=0$ has no real roots which implys the graph of $y=f(x)$ never touches the x-axis, and by $a>0$ the parabola opens upward, so that solution of $f(x)>0$ is

x is any real number.

2. $a<0$

1 $b^2-4ac>0$

In this case, $f(x)=0$ has two distinct real roots and by $a<0$ the parabola opens downword, so that the solution of $f(x)>0$ is

$$(-b-\sqrt{b^2-4ac})/2a<x<(-b+\sqrt{b^2-4ac})/2a$$

2 $b^2-4ac=0$

In this case, $f(x)=0$ has only one real root $-(b/2a)$ and the parabola opens downword by $a<0$, so that the solution of $f(x)>0$ is

none, that is, any real number x satisfying $f(x)>0$ never exists.

3 $b^2-4ac<0$

In this case, $f(x)=o$ has no real roots, and the parabola opens downword by $a<0$, so that the solution of $f(x)>0$ is none.

3. $a=0$

In this case, $f(x)=ax^2+bx+c>0$ is $f(x)=bx+c>0$, and the graph of $f(x)$ is a straight line not a parabola.

So, we have following cases.

1 $b>0$

In this case, $f(x)=0$ has a root $x=-(c/b)$, and the slope of the line is positive, that is, right up, so that the solution of $f(x)>0$ is

$$-(c/b)<x$$

2 $b<0$

In this case, $f(x)=0$ has a root $x=-(c/b)$, and the slope of the line is negative, that is, right down, so that the solution of $f(x)>0$ is

$$x<-(c/b).$$

3 $b=0$

In this case, $f(x)>0$ is only $0\cdot x^2+0\cdot x+c>0$, so that

i) $c>0$ any real number x satisfies $f(x)>0$.
ii) $c\leqq 0$ no real number x satisfies $f(x)>0$.

Conclusion of Solution for $ax^2+bx+c>0$

1. $a>0$

1	$b^2-4ac>0$	$x<(-b-\sqrt{b^2-4ac})/2a$ or $(-b+\sqrt{b^2-4ac})/2a<x$
2	$b^2-4ac=0$	$x<-(b/2a)$ or $-(b/2a)<x$ (x is any real number except $x=-(b/2a)$
3	$b^2-4ac<0$	x is any real number

2. $a<0$

1	$b^2-4ac>0$	$(-b-\sqrt{b^2-4ac})/2a<x<(-b+\sqrt{b^2-4ac})/2a$
2	$b^2-4ac\leqq 0$	no solution

3. $a=0$

1	$b>0$	$-(c/b)<x$
2	$b<0$	$x<-(c/b)$
3	$b=0$	i) $c>0$ any real number ii) $c\leqq 0$ no solution

CHAPTER 10 THE DERIVATIVE

Continuous Function

f(x) is called continuous at x=c, provided

(1) f(c)is defined

(2) $\lim_{x \to c} f(x)$ exists

(3) $\lim_{x \to c} f(x)=f(c)$

The Derivative of y=f(x) at $x=x_0$

$$\lim_{\Delta x \to 0} \frac{y}{x} = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$ provided the limit exists.

If the subscript o is deleted, we obtain a function of x called the derivative with respect to x of the given function. The derivative with respect to x of the function y=f(x) is denoted by some one of the symbols y', $\frac{dy}{dx}$, f'(x), or $D_x y$.

The value of the derivative for any given value of x, say x_0, will be denoted by $y'\big|_{x=x_0}$, $\frac{dy}{dx}\Big|_{x=x_0}$, or $f'(x_0)$.

Higher order of derivatives.

The process of finding the derivative of a given function is called <u>differentiation.</u>

By differentiations, we obtain y'=f'(x), first derivative of f(x), y"=f"(x), second derivative of f(x), and so on.

Differentiation Fomulas

1.	$y=f(x)=x^n$	$y'=f'(x)=nx^{n-1}$
2.	$y=f(x)=ag(x)$	$y'=ag'(x)$
3.	$y=f(x)+g(x)$	$y'=f'(x)+g'(x)$
4.	$y=f(x)-g(x)$	$y'=f'(x)-g'(x)$
5.	$y=f(x)\cdot g(x)$	$y'=f'(x)g(x)+f(x)g'(x)$
6.	$y=\frac{f(x)}{g(x)}$	$y'=\frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$
7.	$y=u^n$ (u:function of x)	$y'=nu^{n-1}u'$
8.	$y=f(u)$	$y'=f'(u)\frac{du}{dx}$
9.	$y=e^x$	$y'=e^x$
10.	$y=\log_e x$	$y'=\frac{1}{x}$

(In 9 and 10, e is defined as $e=\lim_{x\to\infty}(1+\frac{1}{x})^x=\lim_{x\to 0}(1+x)^{\frac{1}{x}}$)

Applications of Derivatives

1. Relative maximum and minimum values
2. Velocity and acceleration
3. Proof of inequalities
4. Inflection point of a curve

<u>5 step rule</u> to find the derivative of f(x) at $x=x_0$

1 $y_0=f(x_0)$

2 $y_0+\Delta y=f(x_0+\Delta x)$

3 Δy

4 $\frac{\Delta y}{\Delta x}$

5 $\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$

CHAPTER 11 INTEGRATION

If $F(x)$ is a function whose derivative $F'(x)=f(x)$, then $F(x)$ is called an <u>integral of f(x)</u>.
If $F(x)$ is an integral of $f(x)$, then $F(x)+C$ is also an integral of $f(x)$ where C is a constant.

The <u>indefinit integral</u> of $f(x)$, denoted by $\int f(x)dx$, is the most general integral of $f(x)$, that is,

$$\int f(x)dx=F(x)+C$$

where $F(x)$ is any function such that $F'(x)=f(x)$ and C is an arbitrary constant.

EXAMPLE: Let $F(x)=x^4$, then $F'(x)=4x^3$

Thus x^4 is an integral of $4x^3$.

And $\int 4x^3dx=x^4+C$

Integration Formula

1. $\int af(x)dx=a\int f(x)dx$ (a is a cOnstant)

2. $\int (f(x)+g(x))dx=\int f(x)dx+\int g(x)dx$

3. $\int f(x)g'(x)dx=f(x)g(x)-\int f'(x)g(x)dx$

4. $\int f(x)dx=\int f(u(t))u'(t)dt$

Indefinit Integral of Algebraic Expression

1. $\int x^n dx=\frac{x^{n+1}}{n+1}+C$ $(n\neq -1)$

2. $\int x^{-1}dx=\int \frac{1}{x}dx=\log_e x+C$ ($\log_e x$ is natural logarithm of x)

3. $\int e^x dx=e^x+C$

The definite integral of f(x) between x=a and x=b, denoted by $\int_a^b f(x)dx$, is as follows

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a) \quad \text{where } F'(x) = f(x).$$

Thus, $\int_a^b f(x)dx$ gives the area bounded by $y=f(x)\geq 0$, the x-axis, and the ordinate x=a and x=b.

Application of Intergration

1. Area, Volume, and the Length of the curve on the plane.
2. Average of continuous values.
3. Distance, velocity.
4. Probability (continuous distribution case).

PART II. STATISTICS

TABLE OF CONTENTS

CHAPTER 1 VARIABLES AND GRAPHS

1.1. Variables

1.1.1. Population and Sample

In collecting data concerning characteristics of a group of individuals or objects, such as

(1) heights and weights of students in a university

(2) numbers of defective and non-defective bolts produced in a factry on a given day.

Population; { finite
infinite

A variable is a symbol, such as X,Y,H,x,B, which can assume any of a prescribed set of values, called "the domain" of the variable.

Variable { continuous variable
discrete variable

1.2. Scientific Notation

When writing number, especially those involving many zeros before or after the decimal point, it is convenient to employ the scientific notation using powers of 10.

Example: $10^1 = 10$, $10^8 = 100{,}000{,}000$

$10^0 = 1$, $0.0003416 = 3.416 \cdot 10^{-5}$

The rules: $(10^p) \cdot (10^q) = 10^{p+q}$ $10^p/10^q = 10^{p-q}$

Example: $(10^3) \cdot (10^2) = 10^{3+2} = 10^5$

$$\frac{(0.006) \cdot (80{,}000)}{0.04} = \frac{(6 \cdot 10^{-3}) \cdot (8 \cdot 10^4)}{4 \cdot 10^{-2}} = \frac{6 \cdot 8}{4} = 10^{(-3+4)-(-2)} = 12 \cdot 10$$

1.3. Functions

If to each value which a variable X can assume there corresponds one or more values of a variable Y, we say that Y is a function of X and write Y = F(X)

Example: The total population P of Japan is a function of time t, and we write $P = F(t)$.

Rectangular Coordinates.

1.4. Graphs

A graph is a pictorial presentation of the relationship between variables. Many types of graphs are employed in statistics, depending on the nature of the data involved and the purpose of which the graph is intended. Among these are "bar graphs", "pie graphs", "pictographs", etc. These graphs are sometimes reffered to as chart or diagrams. Thus we speak of bar charts, pie diagrams, etc. (see Problems)

1.5. Problems

1. How many significant figures are in each of the following, assuming the numbers are accurately recorded ?

a) 149.8 cm	d) 0.00280 meters	g) $4.0 \cdot 10^3$ kg
b) 149.80 cm	e) 1.00280 meters	h) $7.58400 \cdot 10^{-5}$ dynes
c) 0.0028 meters	f) 9 grams	

2. Locate on a rectangular coordinate system the points having coordinates

a) (5,2)	d) (1,-3)	g) (0,-2.5)
b) (2,5)	e) (3,-4)	h) (4,0)
c) (-3,1)	f) (-2.5,-4.8)	

Assume all given numbers are exact.

3. Table 1-1 gives Gross National Product of Japan (in millions dollar). Graph the data with some method, such as

1) line graph, 2) bar graph, 3) pictograph

Table 1-1

Year	1960	1965	1966	1967	1968	1969
G.N.P.	43	88	100	119	142	167

4. The areas of the various continents of the world in millions of square kilometers are presented in Table 1-2 Graph the data with bar graph and pie graph.

Table 1-2

Continent	Area (millions of Square kilometers)
Africa	30.3
Asia	27.5
Europe	4.9
North America	24.2
Oceania	8.5
South America	17.8
U.S.S.R.	22.4
Total	135.6

CHAPTER 2 FREQUENCY DISTRIBUTION

2.1. Arrays

2.1.1. Raw data and Arrays

Raw data are collected data which have not been organized numerically. An example is the set of heights of 100 male students obtained from an alphabetical listing of university records.

An array is an arrangement of raw numerical data in ascending or descending order of magnitude. The difference between the largest and smallest numbers is called the range of the data. For example, if the largest height of 100 male students is 74 inches and the smallest height is 60 inches, the range is 74 - 64 = 14 inches.

2.1.2. Frequency Distributions

When summarizing large masses of raw data it is often useful to distribute the data into "classes" and to determine the number of individuals belonging to each class, called the "class frequency".

Table 2.1
Heights of 100 Male Students at XYZ University

Height (inches)	Number of Students
60 – 62	5
63 – 65	18
66 – 68	42
69 – 71	27
72 – 74	8
	Total 100

A tabular arrangement of data by classes together with the corresponding class frequencies is called a "frequency distribution".

Table 2-1 is a frequency distribution of heights of 100 male students at XYZ University.

Class Intervals

Class Limits

Class Boundaries

The Size of Class Interval

The Class Mark

2.2. Frequency Distribution

2.2.1. General Rules for Forming Frequency Distributions

(1) Determine the largest and smallest numbers in the raw data

(2) Divide the range into a convenient number of class intervals

having the same size. The number of class intervals is usually taken between 5 and 20, depending on the data.

(3) Determine the number of observation falling into each class interval, i.e. find the class frequencies. This is best done by using a tally or score sheet.

Histograms and Frequency Polygons are two graphical representations of frequency distributions.

(1) A histogram consists of a set of rectangles having
   a) bases on a horizontal axis (the X axis) with centers at the class marks and lengths equal to the class interval sizes,
   b) areas proportional to class frequencies.

(2) A frequency polygon is a line graph of class frequency plotted against class mark. It can be obtained by connecting midpoints of the tops of rectangples in the histogram.

Relative Frequency of a class is the frequency of the class divided by the total frequency of classes and is generally expressed as a percentage. For example, the relative frequency of the class 66 - 68 in Table 2-1 is 42/100 = 42 %. The sum of the relative frequencies of all classes is clearly 1 Or 100 %.

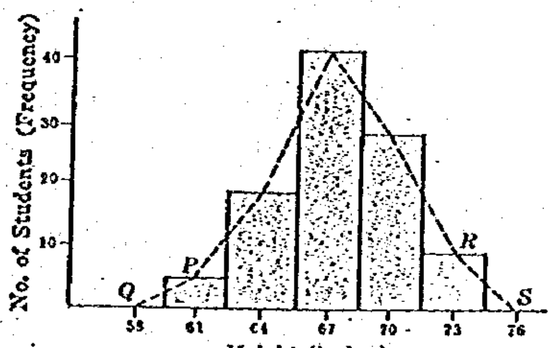


Fig. 2-1

2.2.2. Cumulative Frequency Distribution. Ogives

The total frequency of all values less than the upper class boundary of a given class interval is called the "cululative frequency" up to and including that class interval.

A table presenting such cululative frequencies is called cumulative distribution (table), and is shown in Table 2-2 for the student height distribution.

Table 2.2

Height (inches)	Number of Students
less than 59.5	0
less than 62.5	5
less than 65.5	23
less than 68.5	65
less than 71.5	92
less than 74.5	100

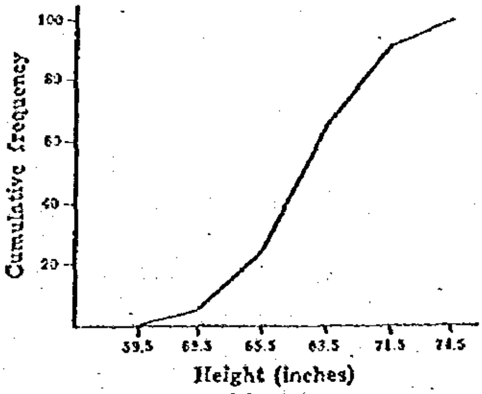


Fig. 2-2

2.3. Problems

1. Table 2-3 shows a frequency distribution of the weekly wages in dollars of 65 employees at PR Company. With reference to this table determine

Table 2.3

Wages (dollars)	Number of Employees
$50.00 – $59.99	8
60.00 – 69.99	10
70.00 – 79.99	16
80.00 – 89.99	14
90.00 – 99.99	10
100.00 – 109.99	5
110.00 – 119.99	2
Total	65

a) The upper limit of the fourth class
b) The class mark of the third class
c) The class boundaries of the fifth class
d) The size of the fifth class interval
e) The frequency of the third class
f) The relative frequency of the third class
g) The class interval having the largest frequency
(This is sometimes called modal class frequency)
h) The percentage of employees earning less than $80.00 per week.

Construct

i) A relative frequency distribution
j) A histogram
k) A relative frequency histogram
l) A frequency polygon
m) A relative frequency polygon
n) An "or more" cumulative frequency distribution
o) An "or more" ogive from the frequency distribution

3.1. Summation Notations

3.1.1. Index or Subscript Notation

Let the symbol Xj denote any of the N values $X_1, X_2, X_3, \cdots X_N$ assumed by a variable X. The letter j in Xj, which can stand for any of the numbers 1, 2, 3, $\cdots$ N is called a subscript or index. Clearly any letter other than j, such as i, k, p, q and s could have been used as well.

3.1.2. Summation Notations

The symbol $\sum_{j=1}^{N} Xj$ is used to denote the sum of all the Xj's from j = 1 to j = N, i.e. by definition.

$$\sum_{j=1}^{N} Xj = X_1 + X_2 + X_3 + \cdots + X_N$$

When no confusion can result we shall often denote this sum simply by $\sum X$, $\sum Xj$ or $\sum_{j} Xj$.

An "average" is a value which is typical or representive of a set of data. Since such typical values tend to lie centrally within a set of data arranged according to magnitude, averages are also called "measures of central tendency".

Several types of averages can be defined, the most common being the "arithmetic mean" or briefly "the mean". "the median", "the mode", "the geometric mean", and "the harmonic mean".

Each has advantages and disadvantages depending on the data and the intended purpose.

3.2. The Arithmetic Mean

he arithmetic mean or the mean of a set of N numbers X_1 , X_2 ,$\cdots$ X_N is denoted by X and is defined as

$$\overline{X} = \frac{X_1 + X_2 + \cdots + X_N}{N} = \frac{1}{N}\sum_{j=1}^{N} Xj = \frac{1}{N}\sum X$$

If the numbers $X_1, X_2, \cdots, X_K$ occur f_1 , f_2, $\cdots$, f_K times respectively (i.e. occur with frequency f_1 , f_2 , $\cdots$, f_K), the arithmetic mean is

$$\bar{X} = \frac{f_1X_1+f_2X_2+ --- +f_kX_k}{f_1+f_2+ --- +f_k} = \frac{1}{\sum f}\sum fX = \frac{1}{N}\sum fX$$

where $N = \sum f$ is the total frequency, i.e. the total number of cases.

3.3. The Median and The Mode

3.3.1 The median

The median of a set of numbers arranged in order of magnitude (i.e. in an array) is the middle value or the arithmetic mean of the two middle values.

For grouped data the median, obtained by interpolation, is given by

$$\text{Median} = L_1 + \left(\frac{N/2 - (\sum f)_1}{f\ \text{median}}\right) C$$

where L_1 = lower class boundary of the median class

N = total frequency

$(\sum f)_1$ = sum of frequencies of all classes lower than the median class

f median = frequency of median class

C = size of median class interval

This value of X is sometimes denoted by $\tilde{X}$.

3.3.2. The mode

The mode of a set of numbers is that value which occurs with the greatest frequency, i.e. it is the most common value.

The mode may not exist, and even if it does exist it may not be unique.

This value of X is sometimes denoted by $\hat{X}$.

A distribution having only one modal is called unimodal.

For unimodal frequency curves which are moderately skewed (asymmetrical), we have the empirical relation

Mean - Mode = 3 (Mean - Median)

In Figures 3-1 right are shown the relative positions of the mean, median and mode for frequency curves which are skewed to the right. For symmetrical curves the mean, mode and median all coincide.

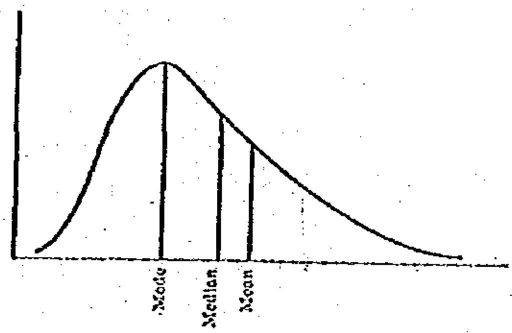


Fig.3-1

3.4. Other Measures

1) The Geometric Mean G

The geometric mean G of a set of N numbers X_1, X_2, ..., X_N is the Nth rood of the product of the numbers:

$$G = \sqrt[N]{X_1 \cdot X_2 \cdot \cdots\cdots \quad X_N}$$

2) The Harmonic Mean

The harmonic mean H of a set of N numbers X , X , ..., X is the reciprocal of the arithmetic mean of the reciprolals of numbers.

$$\frac{1}{H} = \frac{1}{N} \Sigma \frac{1}{X}$$

3) The Rood Mean Square (R.M.S.)

The root mean square or "quadratic mean" of a set of Numbers X_1 , X_2 , ..., X_N is sometimes denoted by $\sqrt{\overline{X^2}}$ and is defined by

$$R.M.S. = \sqrt{\overline{X^2}} = \sqrt{\frac{\Sigma X^2}{N}}$$

4) Quartiles, Deciles, and Percentiles

If a set of data is arranged in order of magnitude, we can think of the value which divide the set into four equal parts. These values, denoted by Q_1, Q_2 and Q_3 are called the first, second and third quartiles respectively, the value Q_2 being equal to the median. Similarly the values which divide the data into ten equal parts are called deciles and are denoted by D_1 , D_2 , ... , D_9 , while the values dividing the data into one hundred equal parts are called percentiles and denoted by P_1 , P_2 , ... , P_{99} .

The 5th decile and the 50th percentile correspond to the median. The 25th and 75th percentiles correspond to the first and third quartiles respectively.

3.5. Problems

1. If $\sum_{j=1}^{6} Xj = -4$ and $\sum_{j=1}^{6} Xj^2 = 10$, calculate

a) $\sum_{j=1}^{6} (2Xj + 3)$ b) $\sum_{j=1}^{6} (Xj - 1)$ c) $\sum_{j=1}^{6} (Xj - 5)$

2. Use the frequency distribution of height in the Table 2-1 to find the mean height of 100 male students at XYZ University.

3. Find the median weight of 40 male college students at State University (see Problem 2 (TEST)) by using

a) frequency distribution of Problem

b) the original data

4. Find the mean, median and mode for the set of numbers;

a) 3, 5, 2, 6, 5, 9, 5, 2, 8, 6

b) 51.6, 48.7, 50.3, 49.5, 48.9

5. Find the arithmetic mean, geometric mean, harmonic mean and R.M.S. of the numbers 3, 5, 6, 6, 7, 10, 12.

6. Determine

a) the quartiles Q_1, Q_2, Q_3

b) the deciles D_1, D_2, $\cdots$, D_9

c) the 35th and 60th percentile

by using the data (Problem 1 , Page 6) and show the results can be obtained from a percentage ogive.

CHAPTER 4 THE STANDARD DEVIATION AND OTHER MEASURES OF DISPERSION

4.1. The Range and The Mean Deviation

4.1.1. Dispertion or Variation

The degree to which numerical data tend to spread about an average value is called the "variation" or "dispersion" of the data.
Various measures of dispersion or variation are available, the most common being the range, mean deviation, semi-interquartile range 10 - 90 percentile range, and the standard deviation.

4.1.2. The Range, The Mean Deviation (or Average Deviation)

The range of a set of numbers is the difference between the largest and smallest numbers in the set.
The mean deviation of a set of N numbers $X_1, X_2, \cdots X_N$ is defined by

$$\text{Mean Diviation} = \text{M.D.} = \frac{1}{N}\sum_{j=1}^{N}|X_j - \bar{X}| = \overline{|X - \bar{X}|}$$

where $\bar{X}$ is the arithmetic mean of the numbers and $|X_j - \bar{X}|$ is the absolute value of the deviation of X_j from $\bar{X}$.
If $X_1, X_2, \cdots, X_K$ occur with frequencies $f_1, f_2, \cdots, f_K$ respectively, the mean deviation can be written as

$$\text{M.D.} = \frac{1}{N}\sum_{j=1}^{K} f_j\ |X - \bar{X}| = \overline{|X - \bar{X}|}$$

where $N = \sum_{j=1}^{K} f_j = \Sigma f$. This form is useful for grouped data where the X represent class marks and the f 's are the corresponding class frequencies.

Semi-interquartile Range = $Q = (Q_3 - Q_1)/2$

10 - 90 Percentile Range = $P_{90} - P_{10}$

4.2. Standard Deviation

The standard deviation of a set of N numbers $X_1, X_2, \cdots X_N$ is denoted by s and defined by

$$s = \sqrt{\frac{1}{N}\sum_{j=1}^{N}(X_j - \bar{X})^2} = \sqrt{\frac{1}{N}\Sigma x^2} = \sqrt{\overline{X^2} - \bar{X}^2}$$

where x represents the deviations of each of the numbers X_j from the mean $\bar{X}$.

Thus s is the root mean square of the deviations from the mean or, as it is sometimes called, "the root mean square deviation".

If $X_1, X_2, \cdots X_k$ occur with frequencies $f_1, f_2, \cdots, f_K$ respectively, the standard deviation can be written as

$$s = \sqrt{1/N \sum_{j=1}^{K} f_j \ (X_j - \bar{X})^2} = \sqrt{1/N \Sigma f x^2} = \sqrt{\overline{X^2} - \bar{X}^2}$$

wher $N = \sum_{j=1}^{K} f_j = \Sigma f$. In this form it is useful for grouped data.

4.2.2. The Variance

The variance of a set of data is defined as the square of the standard deviation and is thus given by s^2 .

When it is necessary to distinguish the standard deviation of a population from the standard deviation of a sample drawn from this population, we often use the symbol s for the latter and σ for the former. Thus s^2 and σ^2 would represent the "sample variance" and "population variance" respectively.

4.3. Properties of The Standard Deviation

(1) For normal distributions (see Chapter 6) it turns out that ;

a) 68.27 % of the cases are included between $\bar{X}$ - s and $\bar{X}$ + s, (i.e. one standard deviation on either side of the mean)

b) 95.45 % of the cases are included between $\bar{X}$ - 2s and $\bar{X}$ + 2s,

c) 99.73 % of the cases are included between $\bar{X}$ - 3s and $\bar{X}$ + 3s,

as indicated in Figure 4-1

For moderately skewed distributions the above percentages may hold approximately.

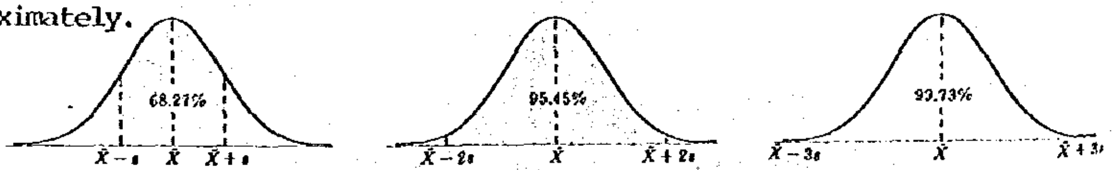


Fig. 4-1

(2) Suppose that two sets consisting of N_1 and N_2 numbers (or two frequency distributions with total frequencies N_1 and N_2) have variances given by s_1^2 , s_2^2 respectively and the same mean $\bar{X}$. Then the comnined or pooled variance of both sets is given by

$$s^2 = \frac{N_1 S_1^2 + N_2 S_2^2}{N_1 + N_2}$$

Note that this is a weighted arithmetic mean of variance.
This result can be generalized to 3 or more sets.

(3) Relative Dispertion

Corfficient of Variation $= V = s/\bar{X}$

4.4. Standardized Variable and Standard Score

The variable $z=(X - \bar{X})/s$
which measures the deviation from the mean in units of the standard deviation is called a standardized variable (which have $\bar{z} = 0$ and $s_z^2 = 1$) and is a dimensionless quantity (i.e. is independent of units used).

If deviations from the mean are given in units of the standard deviation, they are said to be expressed in standard units or standard scores. These are of great value in comparison of distribution.

4.5. Problems

1. Find
   a) the range of each set of numbers
   b) the mean deviation of the sets of numbers
   c) the standard deviation of the sets of numbers

   the data
   1) 12, 6, 7, 3, 15, 10, 18, 5
   2) 9, 3, 8, 8, 9, 8, 9, 18
2. Find the mean deviation and the standard deviation of the heights of the 100 male students at XYZ University. (See Table 2 - 1)
3. Table 4 - 1 shows the I.Q's of 480school children at a certain elementaly school. Find
   a) the mean
   b) the standard deviation
   c) the percentage of student's I.Q. which fall within the range $\bar{X} \pm s$, $\bar{X} \pm 2s$, $\bar{X} \pm 3s$
   d) convert the I.Q. into standard scores
   e) construct a graph of relative frequency vs. standard score.

Table 4 - 1

X	70	74	78	82	86	90	94	98	102	106	110	114	118	122	126
f	4	9	16	28	45	66	85	72	54	38	27	18	11	5	2

CHAPTER 5 ELEMENTARY PROBABILITY THEORY

5.1. Fundamental Rules of Probability

5.1.1. Classical Definition of Probability

Suppose an event E can happen in h ways out of a total of n possible equally likely (equally probable) ways. Then the probability of occurrence of the event (called its success) is denoted by

$$p = P_r\{E\} = h/n$$

The probability of non-occurrence of the event (called its failure) is denoted by

$$q = P_r\{\text{not} E\} = (n-h)/n = 1-h/n = 1 - P = 1 - P_r\{E\}$$

Thus $p + q = 1$ or $P_r\{E\} + P_r\{\text{not } E\} = 1$

The event "not E" is sometimes denoted by $\bar{E}$ or $\tilde{E}$.

5.1.2. Conditional Probability. Independent and Dependent Events.

If E_1 and E_2 are two events, the probability that E_2 occurs given that E_1 has occurred is denoted by $P_r\{E_2 | E_1\}$ or Pr E_2 given E_1 and is called the conditional probability of E_2 given that E_1 has occurred.

If the occurrence or non-occurrence of E_1 does not affect the probability of occurrence of E_2, then $P_r\{E_2 | E_1\} = P_r\{E_2\}$ and we say that E_1 and E_2 are independent events;
otherwise they are dependent events.

If we denote by $\{E_1 \; E_2\}$ the event that "both E_1 and E_2 occur", sometimes called a "compound event" then

$$\Pr E_1E_2 = P_r\{E_1\} \, P_r\{E_2 | E_1\}$$

In particular

$$\Pr E_1E_2 = P_r\{E_1\} \cdot P_r\{E_2\} \quad \text{for independent events}$$

5.1.3. Mutually exclusive Events

Two or more events are called mutually exclusive if the occurrence of any one of them excludes the occurrence of the others. Thus if E_1 and E_2 are mutually exclusive events,

$$\Pr E_1E_2 = 0$$

If $E_1 + E_2$ denotes the event that "either E_1 or E_2 or both occur", then

$$P_r\{E_1 + E_2\} = P_r\{E_1\} + P_r\{E_2\} - P_r\{E_1 \, E_2\}$$

In particular,

$$P_r\{E_1 + E_2\} = P_r\{E_1\} + P_r\{E_2\} \text{ for mutually exclusive events.}$$

5.1.4. Discrete Probability Distributions

If a variable X can assume a discrete set of values $X_1, X_2, \cdots, X_k$ with respective probabilities $p_1, p_2, \cdots, p_k$ where $p_1 + p_2 + \cdots + p_1 = 1$ we say that a discrete probability distribution for X has been defined. The function p(X) which has the respective values $p_1, p_2, \cdots, p_k$ for $X = X_1, X_2, \cdots, X_k$, is called the probability function or frequency function of X. Because X can assume certain values with given probabilities, it is often called a discrete random variable. A random variable is also known as a chance variable or stochastic variable. The probability distribution can be represented graphically by plotting p(X) against X, as for relative frequency distributions.

By cumulating probabilities we obtain"cumulative probability distributions" which are analogous to cumulative relative frequency distributions. The function associated with this distribution is sometimes called a distribution function.

5.1.5. Continuous Probability Distributions

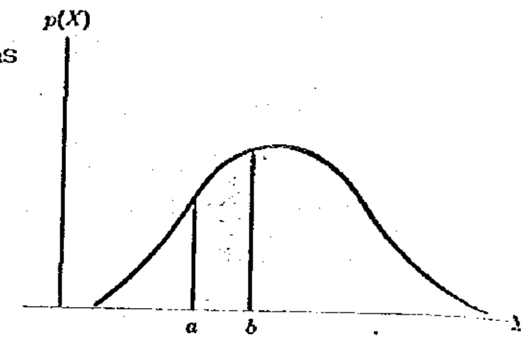


Fig. 5-1

The above ideas can be extended to the case where the variable X may assume a continuous set of values. The relative frequency polygon of a sample becomes, in the theoretical or limiting case of a population, a continuous curve such as a shown in Fig. 5-1, whose equation is $Y = p(X)$. The total area under this curve bounded by the X axis is equal to one, and the area under the curve between lines X = a and X = b gives the probability that X lies between a and b, which can be denoted by $P_r\{a < X < b\}$.
We call p(X) a "probability density function", or briefly a "density function", and when such a function is given we say that a "continuous probability distribution" for X has been defined.
The vriable X is then often called a continuous random variable.

5.2. Mathematical Expectation

If p is the probability that a person will receive a sum of money S, the "mathematical expectation", is defined as pS. The concept of expectation is easily extended. If X denotes a discrete random variable which can assume the values $X_1, X_2, \cdots, X_k$ with respective probabilities $p_1, p_2, \cdots, p_k$ where $p_1 + p_2 + \cdots + p_k = 1$, the mathematical expectation of X or simply the expectation of X, denoted by E(X), is defined as

$$E(X) = p_1X_1 + p_2X_2 + \cdots + p_kX_k = \Sigma pX$$

5.3. Combination

In obtaining probabilities of complex events an enumeration of cases is often difficult, tedious, or both. To facilitate the labor involved, use is made of basic principles studied in a subject called combinatorial analysis

Factrial n

$$n! = n(n-1)(n-2)\cdots 1 \qquad 0! = 1$$

Permutations ${}_nP_r$

$${}_nP_r = n(n-1)(n-2)\ldots\ldots(n-r+1) = n!/(n-r)!$$

In particular

$${}_nP_n = n!$$

The number of permutations of n objects consisting of groups of which n_1 are alike, n_2 are alike, $\cdots$ is

$$\frac{n!}{n_1!\,n_2!\cdots} \text{ where } n = n_1 + n_2 + \cdots$$

Combinations ${}_nC_r$

$${}_nC_r = \frac{n(n-1)\cdots(n-r+1)}{r!} = \frac{n!}{r!(n-r)!} = \frac{{}_nP_r}{r!}$$

$${}_nC_r = {}_nC_{n-r}$$

The number of combinations of n objects taken 1 or 2 or $\cdots$ or n at a time is

$${}_nC_1 + {}_nC_2 + \cdots\cdots + {}_nC_n = 2^n - 1$$

Relation of Probability to Point Set Theory

sample space, set, Euler diagram, null set

$\{E_1+E_2\}$ → either in E_1 or E_2 or both → union

$\{E_1E_2\}$ → common to both E_1 and E_2 → intersection

5.4. Problems

1. A ball is drawn at random from a box containing 6 red balls, 4 white balls and 5 blue balls. Determine the probability

a) red b) white c) blue

d) not red e) red or white.

Three balls are drawn successively from the box.
Find the probability that they are drawn in the order red, white and blue if each ball is

f) replaced g) not replaced

2. Find the probability of boys and girls in families with 3 children, assuming equal probabilities for boys and girls.
And represent graphically the distribution.

3. Find (a) $E(X)$, (b) $E(X^2)$, (c) $E\left[(X-\bar{X})^2\right]$ for the following probability distribution.

X	8	12	16	20	24
p(X)	1/8	1/6	3/8	1/4	1/12

4. Evaluate ${}_8P_3$, ${}_6P_4$, ${}_{15}P_1$, ${}_3P_3$, ${}_5C_3$, ${}_8C_4$, ${}_{10}C_8$

5. Determine the probability of three 6's in 5 tosses of a fair die.

6. A survey of 500 students taking one or more courses in algebra, physics and statistics during one somster revealed the following numbers of students in the indicated subjects.

Algebra	329	Algebra and physics	83
Physics	186	Algebre and statistics	217
Statistics	296	Physics and statistics	63

How many students were taking

a) all 3 subject
b) algebra but not statistics
c) physics but not algebra
d) statistics but not physics
e) algebra or statistics but not physics
f) algebra but not physics or statistics

CHAPTER 6 THE BINOMIAL, NORMAL AND POISSON DISTRIBUTIONS

6.1. The Binomial Distribution

If p is the probability that an event will happen in any single trial and $q = 1 - p$ is the probability that it will fail to happen in any single trial then the probability that the event will happen exactly X times in N trials (i.e. X successes and N - X failures will occur) is given by

$$p(X) = {}_NC_X\, p^X q^{N-X} = \frac{N!}{X!(N-X)!}\, p^X q^{N-X}$$

where $X = 0, 1, 2, \cdots, N$

The discrete probability distribution above is often called the binomial distribution since for X it corresponds to successive terms in the binomial formula or binomial expantion

$$(q + p)^N = q^N + {}_NC_1 q^{N-1} p + {}_NC_2 q^{N-2} p^2 + \cdots\cdots + p^N$$

where 1, ${}_NC_1$, ${}_NC_2$ ··· are called the binomial coefficients.

Some properties of the binomial distribution

Mean $\mu = Np$

Variance $\sigma^2 = Npq$

Standard deviation $\sigma = \sqrt{Npq}$

6.2. The Normal Distribution

One of the most important examples of a continuous probability distribution is the normal distribution defind by the equation

$$Y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(X-\mu)^2/\sigma^2}$$

when the variable X is expressed in terms of standard units, $z = (X - \mu)/\sigma$, equation above is replaced by the so-called "standard form"

$$Y = \frac{1}{\sqrt{2}} e^{-\frac{1}{2}z^2}$$

Some properties of the normal distribution given by equation

Mean μ

Variance σ^2

Standard deviation σ.

6.3. Poisson Distribution

The discrete probability distribution is given by

$$p(X) = \frac{\lambda^x e^{-\lambda}}{\lambda!}$$

where $e = 2.71828 \cdots\cdots$ and λ is a given constant, is called the "Poisson Distribution", after Poisson who discovered it in the early part of the 19 century.

The values of p(X) can be computed by using the table which gives values of $e^{-\lambda}$ for various values of λ, or by using logarithms.

Some properties of the Poisson distribution

Mean $\mu = \lambda$

Variance $\sigma^2 = \lambda$

Standard deviation $\sigma = \sqrt{\lambda}$

The Multinomial Distribution

If events $E_1, E_2, \ldots, E_k$ can occur with probabilities $P_1, P_2, \ldots P_k$ respectively, then the probability that $E_1, E_2, \ldots, E_k$ will occur $X_1, X_2, \ldots, X_k$ times respectively is

$$\frac{N!}{X_1!\, X_2! \cdots X_k!} P_1^{X_1} P_2^{X_2} \cdots P_k^{X_k}$$

where $X_1 + X_2 + \ldots\ldots + X_k = N$

6.4. Fitting of Data by Theoretical Distribution

When one has some indication of the distribution of a population by probabilistic reasoning or otherwise, it is often possible to fit such theoretical distributions (also called "Model" or "expected" distributions) to frequency distributions obtained from a sample of the population. The method used in general consists of employing the mean and standard deviation of the sample to estimate the mean and sample of the population. See Problems.

In order to test the "goodness of fit" of the theoretical distributions, use is made of the "chi-square test" which is given after.

CHAPTER 7 THE METHOD OF LEAST SQUARES AND CORRELATION THEORY

7.1. Relationship between Variables

Very often in practice a relationship is found to exist between two (or more) variables. It is frequently desirable to express this relationship in mathematical form by determining an equation connecting the variable.

To aid in determining an equation connecting variables, a first step is the collection of data showing corresponding values of the variables under consideration.

For example, suppose X and Y denote respectively the height and weight of adult males. Then a sample of N indivisuals would reveal the heights X_1, X_2, ---, X_N and the corresponding weights Y_1, Y_2, ---, Y_N.

A next step is to plot the points (X_1, Y_1), (X_2, Y_2), ---, (X_N, Y_N) on a rectangular coordinate system. The resulting set of points is sometimes called a "scatter diagram".

From the scatter diagram it is often possible to visualize a smooth curve approximating the data. Such a curve is called an "approximating curve".

The general problem of finding equations of approximating curves which fit given sets of data is called "curve fitting".

where Y: dependent variable
X: independent variable

Practical functions are follows

(1) $Y= a_0 + a_1X$		Straight line
(2) $Y= a_0 + a_1X + a_2X^2$		Palabola
(3) $Y= a_0 + a_1X + ---- + a_nX^n$		n-th degree Curve
(4) $Y= 1/a_0+a_1X$		Hyperbola
(5) $Y= ab^X + g$	(Modified)	Exponential Curve
(6) $Y= aX^b + g$	(")	Geometric Curve
(7) $Y= pq^{b^X} + h$	(")	Gompertz Curve
(8) $Y= 1/ab^X + g$		Logistic Curve

7.2. The Method of Least Squares

To motivate a possible definition, consider Fig 7-1 in which the data points are given by (X_1, Y_1), (X_2, Y_2), ---- (X_N, Y_N).

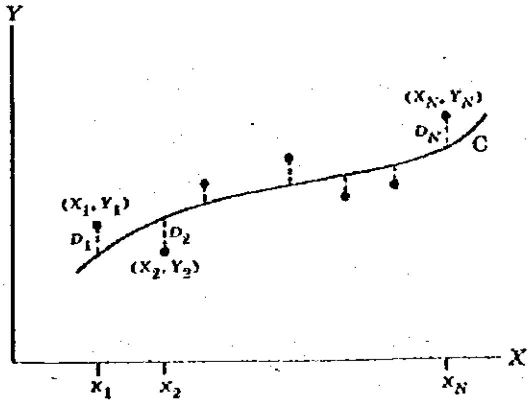


Fig. 7-1

For a given value of X, say X_1, there will be a difference between the value Y_1 and the corresponding value as determined from the curve C. As indicated in the figure we denote this defference by D_1, which is sometimes referred to as a "deviation", "error" or "residual" and may be positive, negative or zero.

Of all curves approximating a given set of data points, the curve having the property that

$$F = D_1^2 + D_2^2 + \text{-----} + D_N^2 \quad \text{is a minimum}$$

is called a "best fitting curve".

Thus a line (or curve) having this property is called a "least square line" (or curve).

The least square line approximating the set of points (X_1, Y_1) , (X_2, Y_2) ---, (X_N, Y_N) has the equation

$$Y = a_0 + a_1 X$$

where the constants a_0 and a_1 are determined by solving simultaneously the equations

$$Y = a_0 N + a_1 \Sigma X$$

$$XY = a_0 \Sigma X + a_1 \Sigma X^2$$

which are called the "nomal equations for the least square line".

The constants a_0 and a_1 can, if desired, be found from the formulas

$$a_0 = \frac{(\Sigma Y)(\Sigma X^2) - (\Sigma X)(\Sigma XY)}{N\Sigma X^2 - (\Sigma X)^2} \qquad a_1 = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{N\Sigma X^2 - (\Sigma X)^2}$$

The labor involved in finding a least square line can sometimes be shortened by transforming the data so that $x = X - \bar{X}$ and $y = Y - \bar{Y}$.

The equation of the least square line can then be written

$$y = \frac{\Sigma xy}{\Sigma x^2} x \quad \text{or} \quad y = \frac{\Sigma xy}{\Sigma x^2} x$$

From these equations it is at once evident that the least square line passes through the point $(\bar{X}, \bar{Y})$, called the "centroid" or "center of gravity" of the data.

If the variable X is taken as dependent instead of independent variable, we write as $X = b_0 + b_1 Y$. Then, the resulting least square line, however, is in general not the same as that obtained above.

Non-linear relationships can sometimes be reduced to linear relationships by appropriate transformation of variables.

Another Least Square Curve

1) Parabola

the least square parabola

$$Y = a_0 + a_1 X + a_2 X^2$$

the normal equations for the least square parabola

$$\Sigma Y = a_0 N + a_1 \Sigma X + a_2 \Sigma X^2$$

$$\Sigma XY = a_0 \Sigma X + a_1 \Sigma X^2 + a_2 \Sigma X^3$$

$$\Sigma X^2 Y = a_0 \Sigma X^2 + a_1 \Sigma X^3 + a_2 \Sigma X^4$$

2) Multiple Variable

the least square plane

$$Z = a_0 + a_1 X + a_2 Y$$

the normal equations

$$\Sigma Z = a_0 N + a_1 \Sigma X + a_2 \Sigma Y$$

$$\Sigma XZ = a_0 \Sigma X + a_1 \Sigma X^2 + a_2 \Sigma XY$$

$$\Sigma YZ = a_0 \Sigma Y + a_1 \Sigma XY + a_2 \Sigma Y^2$$

7.3 Regression and Correlation

When only two variables are involved we speak of simple correlation and simple regression. When more than two variables are involved we speak of multiple correlation and multiple regression.

If X and Y denote the two variables under consideration and Y tends to increase as X increases, the correlation is called "positive" or "direct correlation". If Y tends to decrease as X increases, the correlation is called "negative" or "inverse correlation".

7.3.1. Standard Error of Estimate

If we let Yest represent the value of Y for given values of X, measure of the scatter about the regression line of Y on X is supplied by the quantity

$$S_{YX} = \sqrt{\sum(Y - Yest)^2/N}$$

which is called the "standard error of estimate of Y on X. In general, $S_{YX} \neq S_{XY}$. Equation can also be written

$$S_{YX}^2 = (\sum Y^2 - a_0 \sum Y - a_1 \sum XY) / N$$

The "total variation" of Y is defined as $\sum(Y - \bar{Y})^2$, i.e. the sum of the squares of the deviation of the values of Y from the mean $\bar{Y}$. This can be written

$$\sum(Y - \bar{Y})^2 = \sum(Y - Yest)^2 + \sum(Yest - \bar{Y})^2$$

The first term on the right of equation is called the "unexplained variation" while the second term is called the "explained variation", so called because the deviations $(Yest - \bar{Y})$ have a definite pattern while the deviations $(Y - Yest)$ behave in a random or unpredictable manner. Similar results hold for the variable X.

7.3.2. Coefficient of Correlation

The ratio of the explained variation to the total variation is called the coefficient of determination . If there is zero explained variation, i.e. the total variation is all unexplained, this ratio is zero. If there is zero unexplained variation, i.e. the total variation is all explained, the ratio is one. In other cases the ratio lies between zero and one. Since the ratio is always non-negative. we denote it by r^2. The quantity r, called the "coefficient of correlation", is given by

$$r = \pm\sqrt{\frac{\text{explained variation}}{\text{total variation}}} = \pm\sqrt{\frac{\sum(Yest - \bar{Y})^2}{\sum(Y - \bar{Y})^2}}$$

If a linear relationship between two variables is assumed, equation becomes

$$r = \sum xy \Big/ \sqrt{(\sum x^2)(\sum y^2)}$$

where $x = X - \bar{X}$ and $y = Y - \bar{Y}$. This formula, which automatically gives the proper sign of r, is called the "product moment formula" and cleary shows the symmetry between X and Y.

If we write

$$S_{XY} = \sum xy/N, \quad S_X = \sqrt{\sum x^2/N}, \quad S_Y = \sqrt{\sum y^2/N}$$

then S_X and S_Y will be recognized as the standard deviations of the variable X and Y respectively, while S_X^2 and S_Y^2 are their variances. The new quantity S_{XY} is called the "covariance of X and Y. Formula can be written in the equivalent form

$$r = \frac{N\sum XY - (\sum X)(\sum Y)}{\sqrt{[N\sum X^2 - (\sum X)^2][N\sum Y^2 - (\sum Y)^2]}}$$

which is often used in computing r.

7.4. Problems

1. Table 7.1 shows the respective heights X and Y of a sample of 12 fathers and their oldest sons.

(a) Construct a scatter diagram.

(b) Find the least square regression line of Y on X, and X on Y.

(c) Compute the standard error of estimate, S_{YX}.

(d) Construct two lines parallel to the regression line and having vertical distance S_{YX} from it.

(e) Determine the percentage of data points falling between these two lines.

(f) Compute the total variation, the unexplained variation and the explained variation.

(g) Find the coefficient of determination and the coefficient of correlation.

(h) By using the product-moment formula, obtain the linear correlation coefficient.

Table 7.1

Height X of Father	65	63	67	64	68	62	70	66	68	67	69	71
Height Y of Son	68	66	68	65	69	66	68	65	71	67	68	70

PART III. COMPUTER PROGRAMMING

TABLE OF CONTENTS

CHAPTER 1 INTRODUCTION

FORTRAN is an automatic coding language. It closely resembles the ordinary language of mathematics and provides the facility for expressing any problem requiring numerical computation. In particular, problems imbolving large sets of equations and containing many variables may be handled easily. FORTRAN is especially suited for solving scientific and engineering problems, and it is also suitable for many business applications.

The FORTRAN language consists of words and symbols arranged into statements. A set of FORTRAN statements, describing each step in the solution of a problem, is a FORTRAN program (a source language program). Each computing system that has a FORTRAN compiler translates FORTRAN programs into its own machine language. So FORTRAN language sometimes has a little difference between machines.

This textbook of basic computer program is the guide of FORTRAN language for beginners of computer programing. Therefore the content doesn't cover all of FORTRAN language, but the content is prepared adequately to make any computer program. Only computation technic, for solving actual problem is left as next stage.