FUNDAMENTALS OF MAGNETIC METHOD OF APPLED GEOPHYSICS

JAPAN INTERNATIONAL COOPERATION AGENOY

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BY

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Preface

In December 1975 I was appointed as Visiting Professor of Applied Geophysics in the Department of Geology, University of Ibadan, with the aid of Technical Assistance of the Government of Japan. Since then I have given a course of lectures on the gravitational and magnetic methods of applied geophysics to the final year students, in addition to the supervision of postgraduate research.

The lectures are intended to present the fundamentals of the geophysical methods in easily understandable form. This is the book printed from the cyclostyled volume for the magnetic method, the copies of which have been handed to the students as the 'handout'. The volume was written with reference to the books listed at the end of this volume, which were available in the Department. I owe a great deal to these books.

At this opportunity, I wish to express my sincere thanks to the staff members of the Department, who have been so friendly that I have worked here pleasantly without any difficulty, especially to Professor M.O. Oyawoye, former Head of Geology Department, who has not only helped me in doing my official work, but also privately considered my family so that we spend good time in Ibadan; to Prof. E.A. Fayose present Ag. Head of Geology Department, who has always been well disposed to me, and to Messrs O. Ofrey and C.I. Adighije, both Lecturers of Geophysics of the Department, who have assisted me in every respect of my duty, particularly, in the practical training of the students, and in guiding them for their B.Sc. theses.

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CONTENTS

MAGNETIC METHOD

BY

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1. Basic principles and definitions.

a) Magnetic poles.

One end of the magnetic needle of a compass points nearly to the north direction. The end is called the "north-seeking pole" or simply the "north pole: or the "positive pole: of the magnet, and the other end is called its "south-seeking pole: or "south pole" or "negative pole:. Exactly speaking the poles are always inside a little from the ends, because every magnet has a finite thickness.

The earth itself is a large magnet, whose south-seeking pole in near the geographical north pole and north-seeking pole near the geographical south pole (Fig. 1).

The magnetic poles of the earth in 1945 are at

76°N	102°W	South-seeking pole
68 S	146 E	North-seeking pole

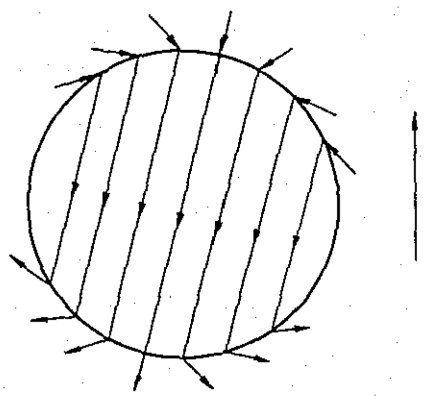


Fig. 1

The two poles are said respectively the north magnetic pole and the south magnetic pole of the earth.

b) Lines of force.

If a magnet, suspended at its centre of gravity, is displaced along the direction of its axis, the trace shows a curved line. The line is called a "line of force" of the north's magnetic field. Lines of force of a bar magnet start outwards at its N-pole and end at its S-pole, here such of the poles being considered to have an area. A line of force closes a loop a part of it running inside the magnet, where it runs from S to N (Fig. 2).

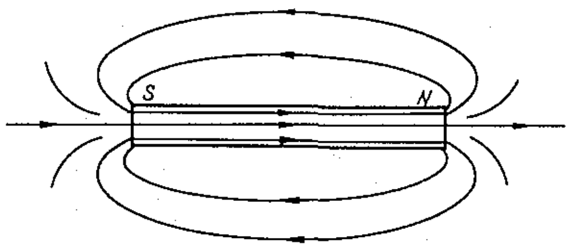


Fig.

c) Magnetic force.

The force F between two poles, having a strength P_1 and P_2 respectively and separated by a distance r, is expressed by

$$F = \frac{1}{\mu} \frac{P_1 P_2}{r^2}$$

where μ is called the magnetic permeability of the medium. When two equal strength of pole P_1 and P_2 are situated in a non-magnetic medium as air ($\mu = 1$), separated by 1 cm and F is 1 dyne, the pole strength of P_1 and P_2 is defined unit.

When the poles have the some sign, the feros is repulsive, and when apposite sign, it is attractive.

d) Magnetic field.

If a magnetic pole exists, lines of force are considered around it. The region is called the magnetic field.

When a unit pole is placed in this magnetic field, it will be exerted by the field and will move along the line of force on which it is placed.

If the force which exerts the unit pole is 1 dyne, the magnetic field strength or the field intensity H at that point has a unit value, which is called 1 oersted, 1×10^{-5} oersted in called 1 gamma (= γ).

The magnetic field has a direction which is shown by a line of feros and has a magnitude expressed in oersted. Instead of oersted gauss is often used.

e) Magnetic moment.

Every magnet has two poles, such of which has an opposite sign and an equal pole strength P and is separated such other by a distance L. PL is called the magnetic moment and is usually expressed by M.

f) Intensity of magnetization.

When a magnetizable material is placed in a magnetic field, it will become a magnet due to the induction of the field. The induced magnetization, sometimes called polarization, is measured by the intensity of magnetization. The intensity of magnetization I is the magnetic moment per unit volume of the magnetized body.

If I has everywhere the same value and the direction of the magnetization is everywhere the same, the induced magnetization is called uniform. Assuming that a cylindrical body having an area of cross-section A and length L, is uniformerly magnetized, along its axis, its intensity of magnetization is as follows:

$$I = \frac{PL}{AL} = \frac{P}{A}$$

where P is the pole strength of the cylinder. That is, I is numerically equal to the pole strength of unit area.

We usually assume that the induced magnetization is uniform. In this case the direction of the magnetization at every point inside the body is parallel to that of the original field.

In the north hemisphere the upper part of the body is magnetized S (Fig. 3).

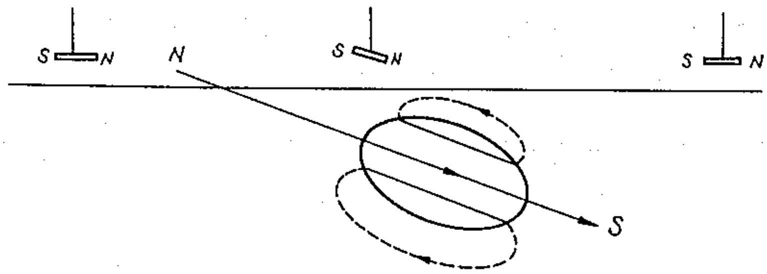


Fig. 3.

g) Susceptibility of a body.

If a body is placed in a magnetic field of strength H, the body is magnetized by the induction of the field, and the induced magnetization I is normally proportional to H:

$$I = kH.$$

k is called the magnetic susceptibility of the body.

For air, $k = 0$. When k is positive, the body is said para-magnetic; when k is a large positive value, it is said ferromagnetic as iron; and when k is negative, diamagnetic as rock salt and anhydrite.

In the case of the diamagnetic substance, the direction of the magnetization is opposite to the original field.

h) Magnetic induction.

The magnetic field in a magnetized body consists of two fields: the original external magnetic field H and the induced internal magnetic field H'. The sum $B = H + H'$ is called the magnetic induction of the body.

Since $H' = 4\pi I$ (from text book)

$$= 4\pi kH,$$

$$B = H + H' = (1 + 4\pi k)\mu$$

If we put $\mu = 1 + 4\pi k$,

$B = \mu H.$

If the body is non-magnetic on air, $k = o$.

$\therefore \mu = 1.$

μ is the permeability as in $F = \frac{pp'}{\mu r^2}$ (from text book) and can be calculated from k, which can be measured in the laboratory and usually expressed in 10^{-6} c.g.s.

i) Residual magnetism

When the magnetic character of a body is weak, that is, k is small, the relation

$$B = \mu H$$

holds. But for the ferromagnetic substance as iron, the relation does not hold.

If we put a ferromagnetic substance between two poles of an electro-magnet whose strength M can be changed, the change of induction B is absorbed on the change of applied field H.

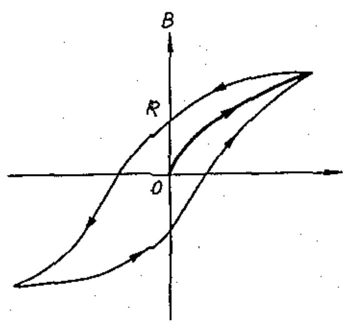


Fig. 4.

The result shows

$$B \neq \mu H,$$

i.e. when applied field H becomes rare, B does not vanish, and has a value which is called the residual magnetism.

For instance, if a rock was once magnetized by the earth's field in an ancient geological age and after that, if the earth's field has changed, the magnetization of the rock has not usually followed the change, but has retained a part of the magnetization acquired in the ancient time, i.e. the rock has a residual magnetism. This residual magnetism is sometimes called the paleo magnetism. It depends on the coercive force of the rock.

j) Magnetic field and electric current.

Any electric current generates a magnetic field. The relation between the two is shown in Fig. 5.

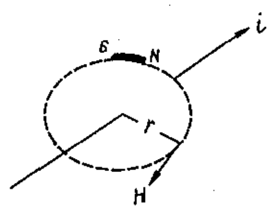


Fig. 5

If we place a magnet near a wire having electric current, the direction of its axis is as shown in Fig. 5. The field intensity H due to the current is proportional to the current i. When the wire is long, H is inversely proportional to the distance r from the wire.

2. The Earth's Magnetic Field

a) Magnetic elements.

If we suspend a bar magnet at its centre, its axis is along the direction of the line of force at that point.

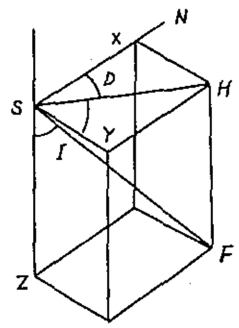


Fig. 6

The field intensity along the line of force is called the total intensity of the earth's magnetic field.

F = 0.25 oersted near the mg. equator

= 0.7 " " " poles

Average F = 0.5 oersted.

F is resolved into three components, X, Y, Z, or D, I, H or H, D, V(Z), each set of which are called three magnetic elements, where

X: north component

Y: east "

Z: vertically downward component

H: horizontal intensity or component

D: Declination, reckoned positive from the north to the east

I: dip or inclination, reckoned positive from the horizontal direction to downward.

The magnetic elements are connected by the relations:

$X = H \cos D$ $\qquad$ $H = F \cos I$

$$Y = H \sin D \qquad Z = F \sin I$$

$$\tan D = \frac{Y}{X} \qquad \tan I = \frac{Z}{H}$$

$$H^2 = X^2 + Y^2 \qquad F^2 = H^2 + Z^2 = X^2 + Y^2 + Z^2.$$

To determine the field intensity at a point, three elements should be known. Usually one of the three sets, mentioned above, is taken as the necessary elements.

The vertical plane through F and H is called the magnetic meridian passing the point.

b) Isomagnetic charts.

From the magnetic survey over the world, distributions of magnetic elements are known, and isomagnetic charts are drawn.

Isomagnetic lines for equal declination are called isogonic lines; for equal dip, isoclinic lines; for equal total intensity, isodynamic lines (Figs. 8, 9, 10).

Isomagnetic lines for H, Z, X, Y are simply called lines of equal horizontal, vertical, north or east force.

The isogonic map is useful for navigators to know the direction of the boat by means of a compass. Declination is a correction applied to the compass reading.

c) Resolution of the earth's field.

The earth's field may be considered to be composed of the internal field and the external field.

54% of the earth's field comes from the interior of the earth.

The magnetism of whole rocks is too small to cover the internal field. And the interior of the earth has high temperature. So the material of deep interior could not be magnetized. Therefore, to explain the internal field, we have to consider electric current in the interior.

The external field may be attributed to the electric current in the ionosphere, which is the ionized layer by ultraviolet ray and contains electrons omitted by the sun.

A very small part of the earth's field is originated from the surface. It may be considered the electric currents flow across the surface. But this field is less than 1% of the whole field. So it is almost negligible. This field is called the nonpotential field.

d) Variation with time.

The earth's magnetic field is always changing with time. The variations may be considered to consist of secular variations, diurnal variations, and magnetic storms.

i) Secular variations.

Changes in the earth's field which occur through many years are called secular variations (Fig. 7).

From continuous observations at many stations, lines of equal annual changes can be drawn on a map. The line is called the isopor, and the map the isoporic map. The isoporic map can be drawn for every magnetic element (Fig. 11).

If we look at a world isoporic map of total intensity, there are some points where extreme changes are found. These points are called the "foci of isopors". The positions of these foci are always changing.

Thus, the secular variations are not regular, and have various values according as the position and the epoch.

The cause may be attributed mostly to the internal phenomena of the earth, but has not yet been explained. However, what are to be considered as the cause are mechanical change of the earth, convention current, and a change of distribution of heat.

The secular variations have been found to be related to the 11 year cycle of sunspot activities (Fig. 13). Therefore, a small part of the variations must come from outside the earth.

The secular variations are so small that they are usually not taken into consideration for the magnetic prospecting carried out within a few months.

ii) Diurnal variations.

In addition to the secular variations magnetic elements change with a period of 24 hours, and the change depends on the local time. The variations are not regular for every day, but the period is almost constant. The whole range of the change for H is, say, 30γ, which depends on the position of the observation and also the day of observation. This change is called the diurnal variation.

Most of the variations depend on the latitude. Those of D, I, and Z normally occur in opposite phase in the Northern and the Southern hemispheres (Fig. 12).

The change may be originated mostly by the change of electric current in the ionosphere, which in turn caused by the effect of the sun. The variation caused by the sun is called the solar diurnal variation.

In addition to the solar change in the diurnal variation, thers is another small variation having a period of 25 hours and the amptitude of 1/15 of the solar. This must be originated by the moon. It is called the lunar diurnal variation. The cause may be due to the change of ionosphere by the lunar tide.

The diurnal variation including the solar and the lunar variation is usually corrected for the observed values in magnetic prospecting.

iii) Magnetic storms.

The earth's field sometimes changes suddenly, and the amount of change attains, say, 1000γ after one day, then gradually the change decreases to the normal value after several days. These changes are called magnetic storms.

The change is larger in the high latitudes than in the low latitudes.

The cause may be attributed to the change in the sun, as activities of sunspot. During the magnetic storms, radio is disturbed. The magnetic prospecting is sometimes compelled to be stopped. Because the elimination of its effects from the observed value is very difficult. The amount of the change depends on the position and the time.

3. Magnetic measurements.

a) Measurements.

Observation points are determined in advance of the measurements by ordinary surveying. The spacing of consecutive two points depends on the purpose, for instance, several hundred meters for oil field where relatively large geological anticlines are to be found, or sometimes 10m for mines.

At first a base point is selected in the area to be surveyed. All the other observation points are referred to this point.

Besides this fundamental base point, usually some local base points are taken and they are connected to the fundamental base.

Starting from a base point, the observations are carried on consecutively at field points on a loop and end at the base point. The time interval needed to close a loop is normally less than 2 hours. The difference between the two repeated observations at the base point is distributed to the observed values at the field points. By this method the effects of the diurnal variation of the earth's field and the change of reading due to the instrumental condition for the field points may be eliminated. The correction for the change of reading mentioned above is called the base correction.

For the measurement, we note the reading of the instrument a, the observed time t, and the temperature of the instrument T. t is used for the base correction and the diurnal correction. T is sometimes used as an instrumental correction.

b) Corrections to the observed reading.

i) Temperature correction.

Usually this correction is omitted. Because the change of temperature is not large when the loop measurements above described are carried out in a relatively short time. If this correction is necessary, it can be calculated with the temperature coefficient determined at the factory for each instrument. The temperature correction are usually made to 20°C in the temperature zone.

ii) Correction for diurnal variation and base corrections.

When the two kinds of corrections are not simply considered together the diurnal correction is first calculated from records at a near magnetic observatory or the continuous measurements made in parallel at a definite point in the surveyed area. Then the base correction is calculated.

iii) Normal correction.

The earth's magnetic field depends upon the latitude and the longitude of the observation point. The correction for the general magnetic field of a region including the survey area is called the normal correction. It can be calculated from the magnetic chart which shows the general field under question.

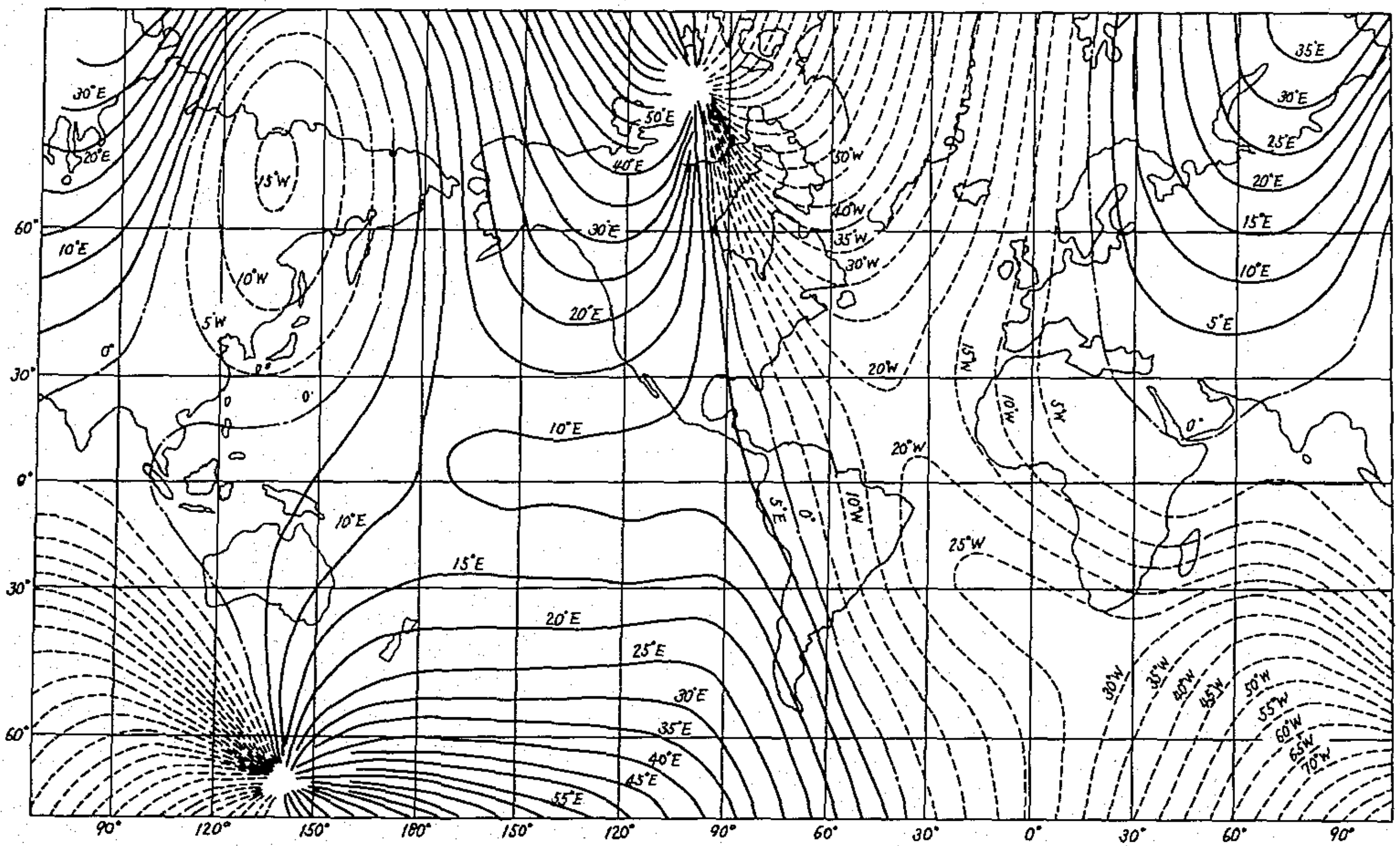


Fig. 8 Lines of equal declination, 1965. 0. (Rikanenpyo, 1971)

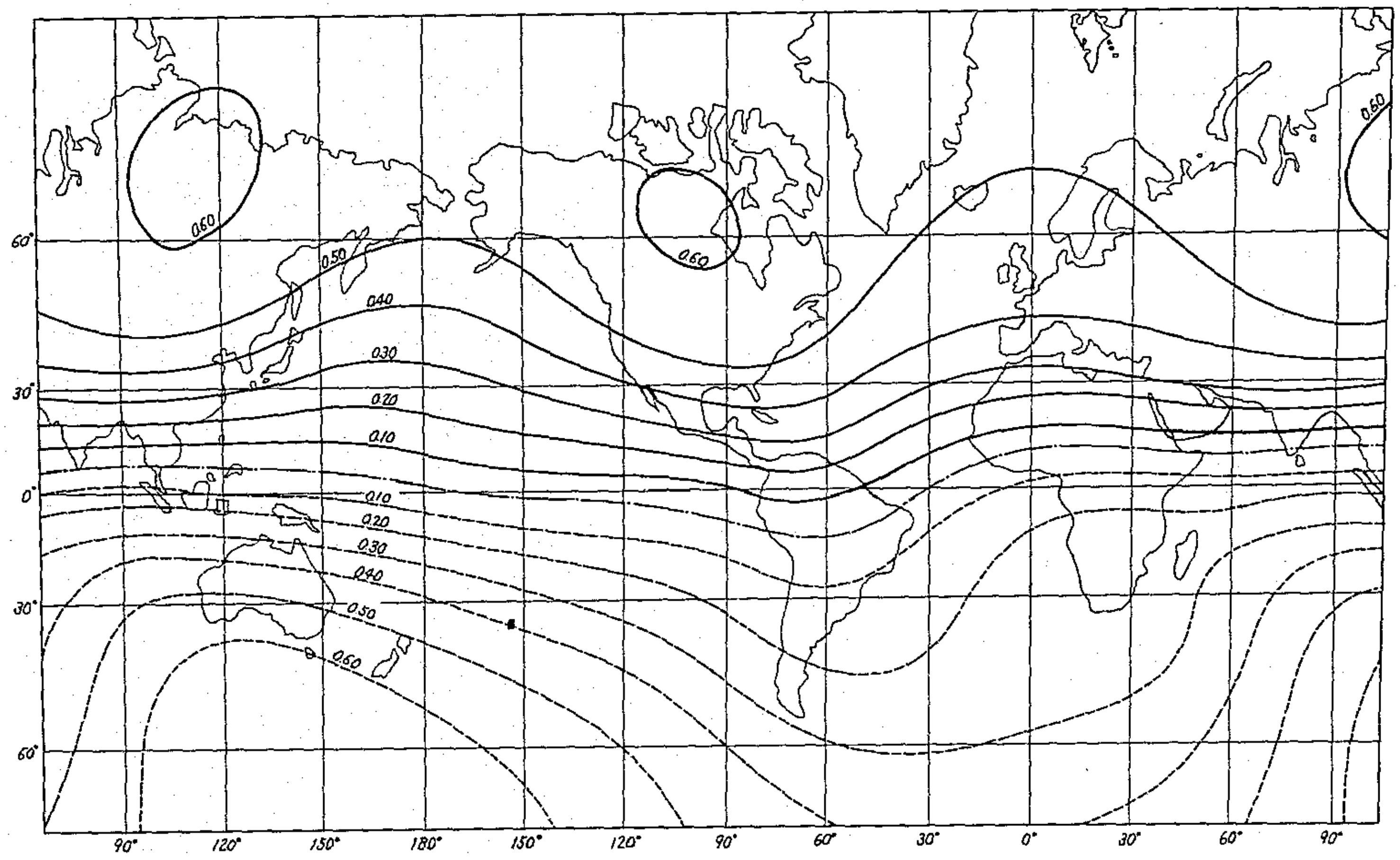


Fig. 9 Lines of equal vertical intensity, 1965, 0. (Rikanenpyo, 1971)

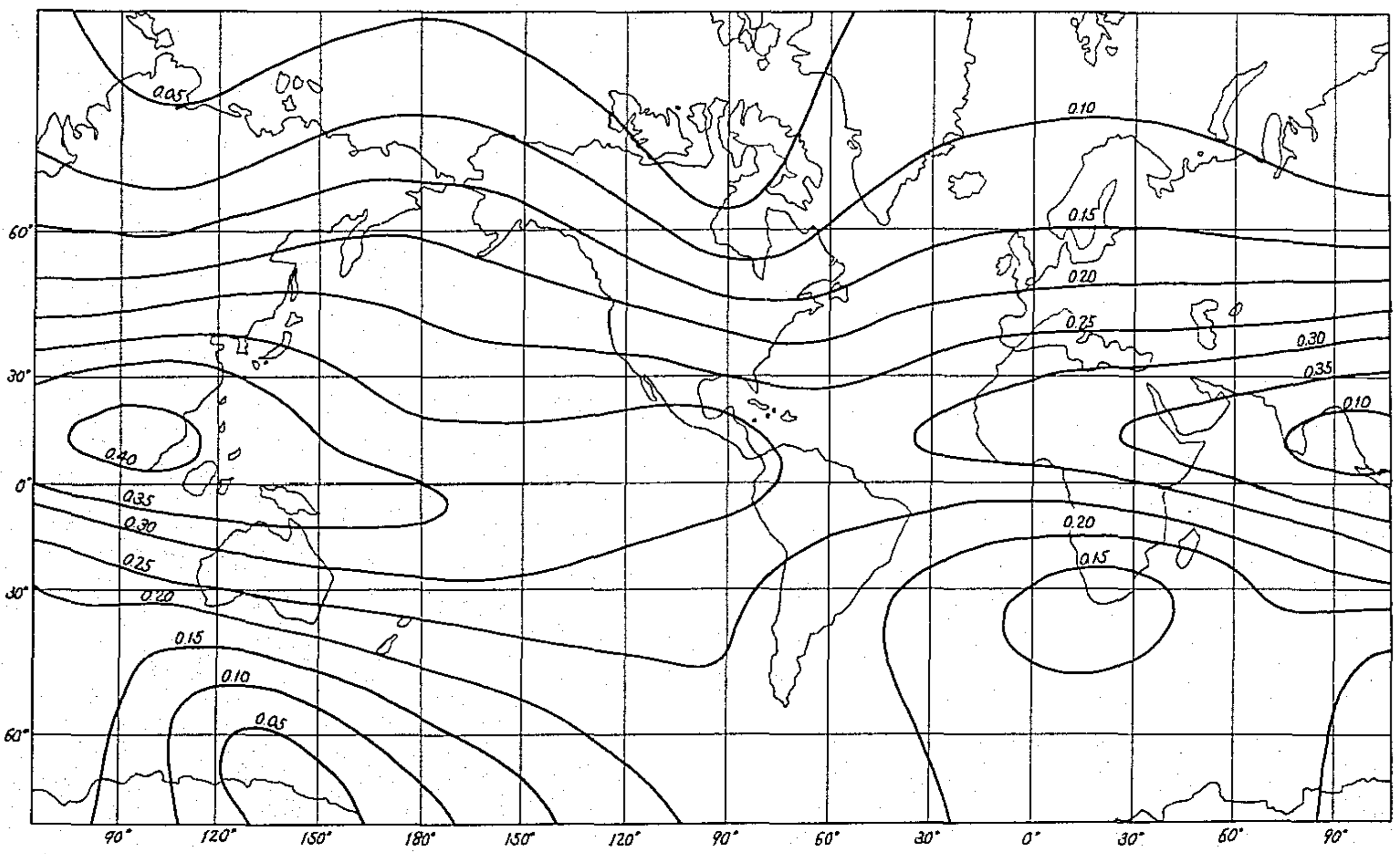


Fig. 10 Lines of equal horizontal intensity, 1965, 0. (Rikanenpyo, 1971)

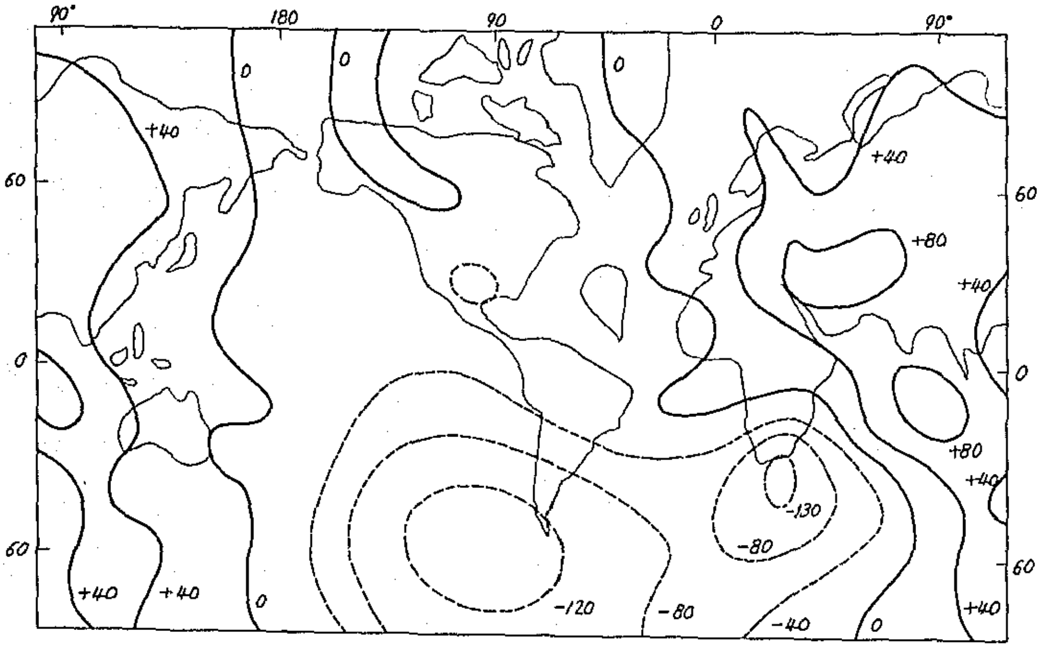


Fig. 11 Lines of equal annual change of total intensity F, 1940-5, in γ

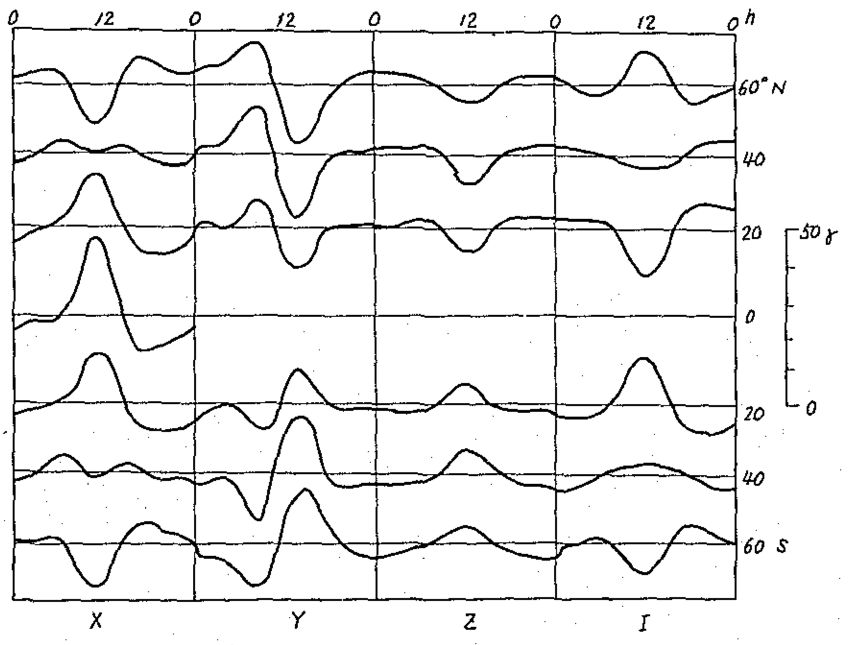


Fig. 12 Solar diurnal variation at the equinoxes.

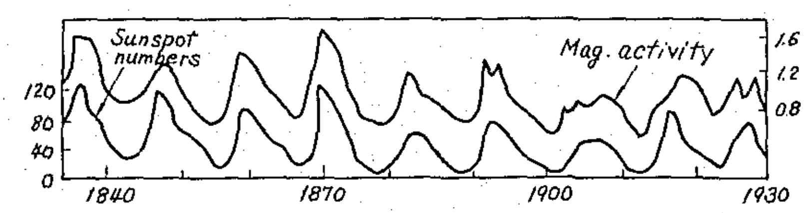


Fig. 13 Changes of magnetic activity and sunspot numbers.

4. Magnetic Instruments.

a) Absolute measurements.

The measurements of the absolute values of three elements D, I, H and described here.

In principle the declination at a point is measured with the use of a compass needle and a meridian mark. Instead of a compass needle, a bar magnet suspended horizontally by a fine wire is also used. With the aid of a telescope and a horizontal circle the angle between the direct one of the magnet and the meridian mark is determined. Under favourable condition the accuracy attains to 0.'1.

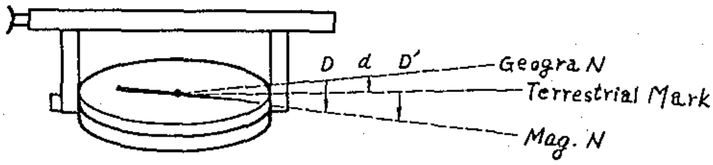
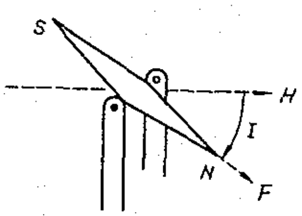


Fig. 14

For the inclination I, a dip circle is used. Let magnetic needle, placed in the magnetic meridian, be supported freely at its centre by a support. Then the axis of the needle points to the line of force of the earth's field. The inclination is read at once from the vertical circle Fig. 15).



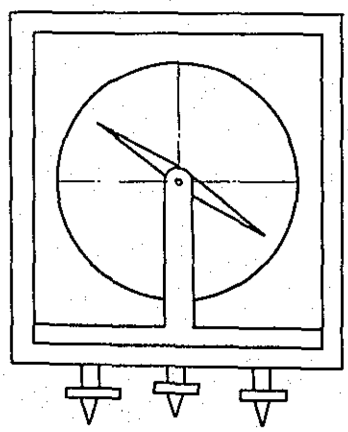
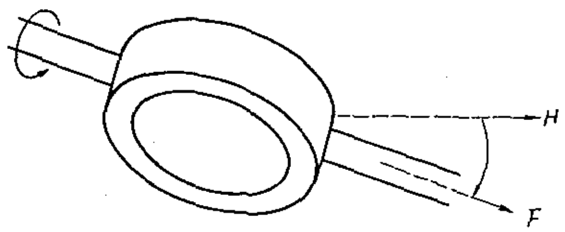


Fig. 15 Dip Circle.

With the use of an earth inductor, I is measured up to the accuracy of 0.'1. We use a coil which can be rotated around its diametral axis. First the coil is placed so that its axis is in the magnetic meridian, in which the direction of the axis can be changed. When the direction of the axis coincides with that of the earth's field the induced current in the coil produced by rotating it will vanish. The change of the current is indicated by a galvanometer connected to the coil with aid of brushes. The inclination of the axis from the horizontal plane in this case is taken as I (Fig. 16).



When no current in the coil, the axis points to F.

Fig. 16 Earth indicator.

For measuring H, a magnetometer is used. A classical method by Gauss consists of two experiments: oscillation and deflection.

A bar magnet is suspended horizontally by a wire, and the period of its free oscillation in the horizontal plane is measured. Denoting the period and the moment of inertia of the magnet by T and i respecitvely, we have

$$T = 2\pi \sqrt{\frac{i}{MH}} ,$$

where M is the magnetic moment. If an unmagnetized body having a simple form, say, cylinder is attached to the above bar magnet, the period T' of the compound is given

$$T' = 2\pi \sqrt{\frac{i + i'}{MH}} ,$$

where i' is the moment of inertia of the attached body and it is easily calculated on account of its simple form. Knowing T and T', and eliminating i from the above two equations, we can calculate MH (Fig. 17).

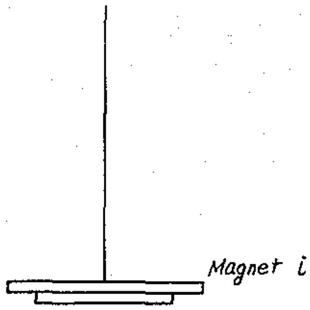


Fig. 17 Oscillation experiment.

Next, another magnet is suspended instead of the original one, which is horizontally placed on a long bar attached horizontally to the magnetometer in the direction parpendicular to the magnetic meridian, as in Fig. 18.

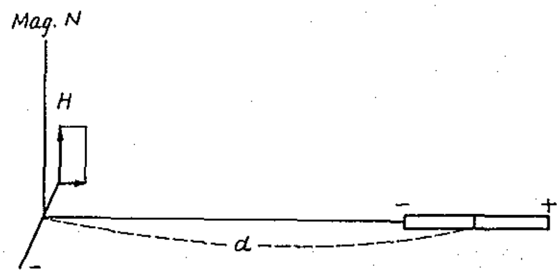


Fig. 18 Deflection experiment (Top view).

The direction of the suspended magnet is deflected by an angle θ from the magnetic north direction.

$$\tan \theta = \frac{2M/d^3}{H} ,$$

where d is the horizontal distance between the centres of the two magnets. From this deflection experiment, M/H is found. So from MH and M/H, H is calculated.

If we use an electrical instrument, the measurement is easier, and more accurate value of H can be obtained. In favourable cases H is determined up to 1γ.

A small magnet is suspended horizontally by a fine wire and is placed in the middle of a double coil which produces a magnetic field F in the direction perpendicular to the coil plane. A helmholtz coil is generally used. If the coil is rotated around a vertical axis which is parallel to the coil plane, the magnet deflects from the meridian. By adjusting the rotation of the coil, we obtain the position of the magnet of which N - pole points to the direction perpendicular to F. From the current in the coil and the dimensions of the coil, the applied field F is calculated. Therefore, from

$$\sin \theta = \frac{F}{H},$$

H can be determined, θ being the angle of rotation of the coil.

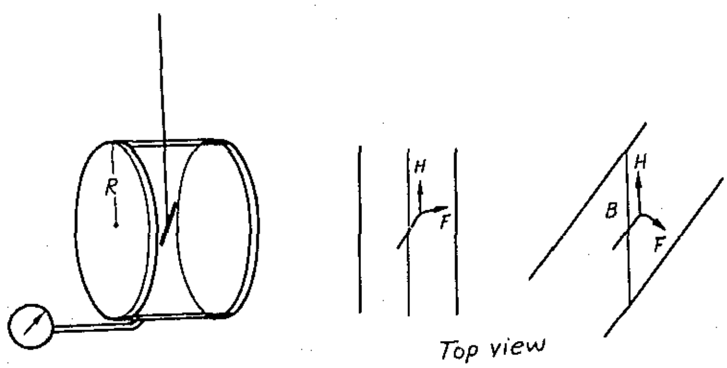


Fig. 19

A Helmholtz coil consists of two similar coils, of which the radius is equal to the distance between them. The magnetic field F at its centre is given:

$$F = \frac{64\ \pi N i}{\sqrt{5}\ R} \text{ gamma},$$

where i: milliamperes

N: number of turns in each of the coils

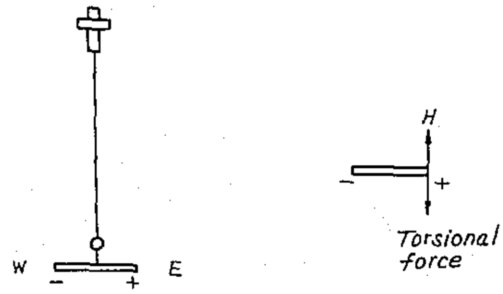
R: radius of the coil in cm.

b) Recording instruments.

Variometers are used for recording the small changes of the earth's field. Usually three variometers for D, H and V are provided at magnetic observatories.

The principles of the H-variometer is as follows:
A small magnet is suspended horizontally by a fine quartz fibre. Then the fibre is twisted with the use of a torsion head until the axis of the magnet lie in the magnetic prime vertical. The change of the direction of the axis is recorded by the photographic device with a rotating drum (Fig. 20).

For the vertical intensity, a small magnet is suspended in the magnetic prime vertical by two horizontal quartz fibres, each of which is stretched by a spring. By twisting the fibres, let the magnet be horizontal. In this position the small change of V can be measured by the small inclination of the magnet (Fig. 21).



Side view *Top view*

Fig. 20 H-variometer.

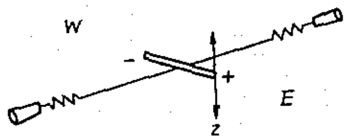


Fig. 21 Z-variometer.

A Helmholtz coil is always set in this kind of instrument for the calibrations of the scale value, which has to be periodically determined.

c) Schmidt-type magnetic field balance.

Let Z_1, H_1, be the vertical and horizontal components of the earth's field at a point P_1, and Z_2, H_2 be those at another point P_2, the difference

$$\Delta Z = Z_2 - Z_1$$

$$\Delta H = H_2 - H_1$$

can be measured by the vertical balance and the horizontal balance respectively. Under favourable condition, the balances have the accuracy of 1γ.

i) Vertical field balance.

A specially made magnetic system is used. It consists of two magnets which are connected in parallel to one another (Fig. 22).

Set the magnet almost horizontally in the magnetic prime vertical. Let ψ indicate a small deflection of the magnet from the horizontal direction. When the instrument is moved to another point, ψ will change by $\Delta\psi$ which measures the change of vertical intensity.

The magnet will be balanced when the moment due to gravity and that due to magnetic vertical intensity have an equal value (Fig. 23).

$$\text{Gravitational torque} = a.mg \cos\psi + d.mg \sin\psi$$

$$\text{Magnetic torque} = 2L.PZ \cos\psi$$

$$= MZ \cos\psi \quad (2PL = M)$$

$$\therefore \; MZ = mg\,(a + d \tan\psi)$$

$$\therefore \; \tan\psi = \frac{MZ - mga}{mgd} \; .$$

When the reading of the magnetometer is a for the deflection ψ, we have

$$s - s_0 = f \tan 2\psi,$$

where f is the distance between the scale and the mirror of the magnet, and s_0 is the reading for $\psi = 0$.

While, from trigonometry

$$\tan 2\psi = \frac{2 \tan\psi}{1 - \tan^2\psi} \; ,$$

and as ψ is small,

$$\tan 2\psi = 2 \tan\psi.$$

$$\tan\psi = \frac{s - s_0}{2f} = \frac{MZ - mga}{mgd} \; .$$

If $Z = Z_1$, $s = s_1$, $\psi = \psi_1$ at a base point, and they are respectively E_2, s_2, Q_2 at a field point,

$$\frac{s_1 - s_0}{2f} = \frac{MZ - mga}{mgd}$$

$$\frac{s_2 - s_0}{2f} = \frac{MZ_2 - mga}{mgd}$$

$$S_2 - S_1 = (Z_2 - Z_1)\,\frac{2fM}{mgd}$$

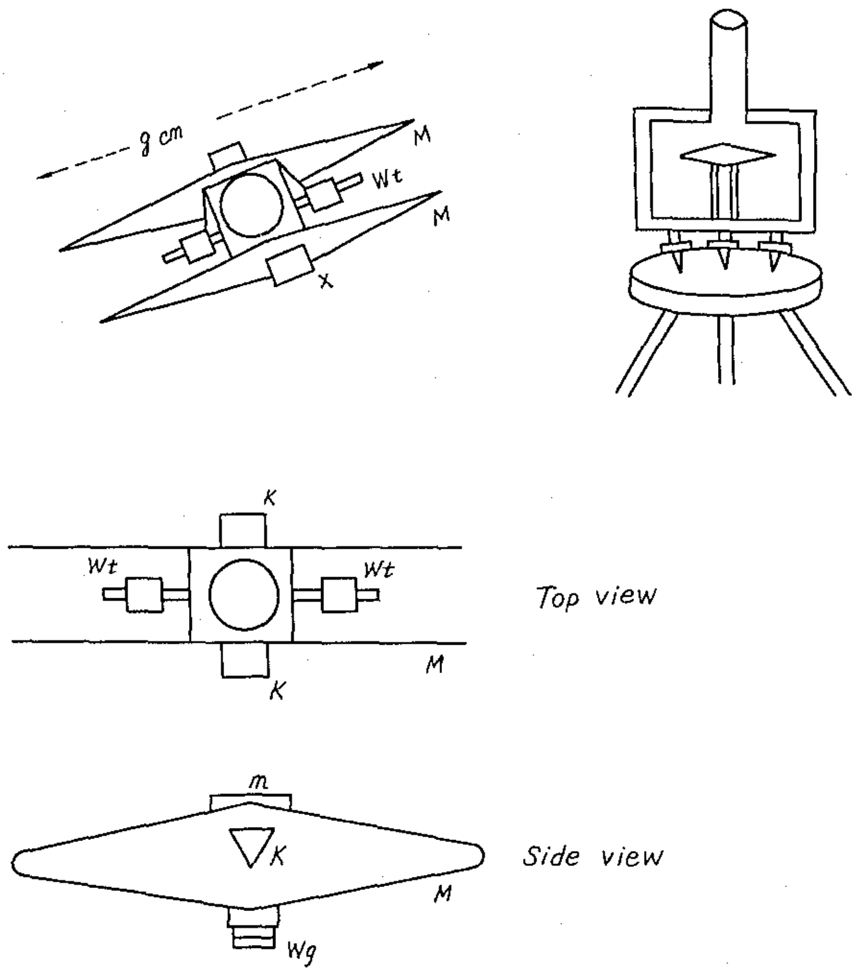
$$\Delta Z = Z_2 - Z_1 = (S_2 - S_1)\,\frac{mgd}{2fM}$$

$$= K\,(S_2 - S_1)\ ,$$

where $K = \frac{mgd}{2fM}$,

which is the scale constant of the instrument. The last equation is used for calculating ΔZ.

K is determined with the use of a Helmholtz coil. If the magnetic field, ΔV gamma, is given to the magnet by the coil, and the change of scale reading due to the field is Δs.



M : Magnet

m : mirror

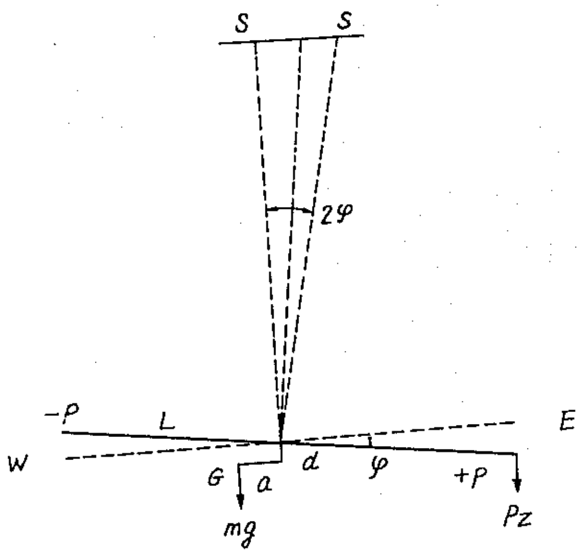
K : Quartz knife edge

W_L : Latitude adjustment counterweight, made of invar

W_t : Temperature compensating counterweight, made of brass, which can be moved along an aluminium screw

W_s : Sensitivity adjustment counterweight

Fig. 22 V-balance



G : Centre of gravity of magnet

a, d : Displacement of G

L : Held length of magnet

m : Mass of magnet

Fig. 23

$$K \Delta s = \Delta V$$

$$K = \frac{\Delta V}{\Delta s} \quad \text{gamma/s.d.}$$

The smaller the K is, the more the instrument is sensitive. To make K smaller, d should be made smaller. The sensitivity adjustment counterweight is used for this purpose. If it is screwed in, d becomes smaller and the instrument becomes more sensitive.

For general use, K may be adjusted 10 - 20 γ/s.d. or 20 - 30 γ/s.d. But it can be changed to 2 - 3 γ/s.d. if necessary.

If the temperature of the magnet changes, M will change and so K changes. To avoid the effect, temperature compensating counterweight W_t is attached. It is made of AI bar and a brass weight. AI has a somewhat large thermal expansion coefficient. When temperature rises, moment M decreases. Then N pole of the magnet is displaced upward. But W_t extends so as to pull the N pole downward and balance the magnetic system.

W_ℓ is made of invar, so has almost no effect with the change of temperature.

ii) Horizontal field balance.

A magnetic system of the same type as that of vertical balance is used vertically in the magnetic maridian (Figs. 24, 25).

Gravitational torque to the clockwise direction:

$$mg \cos \psi . d + mg \sin \psi . a$$

Magnetic torque to the counter-clockwise direction:

$$PH \cos \psi . 2L = PZ \sin \psi . 2L$$

or $MH \cos \psi - MZ \sin \psi$, where $M = 2L.P$

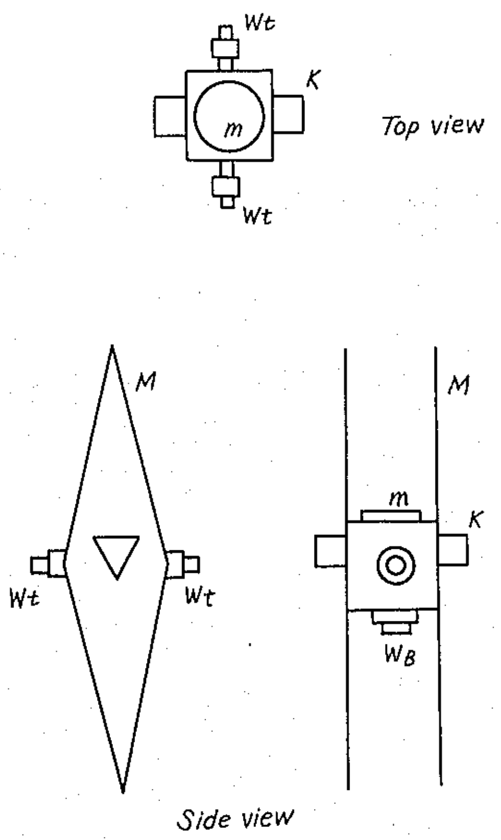


Fig. 24 H-balance

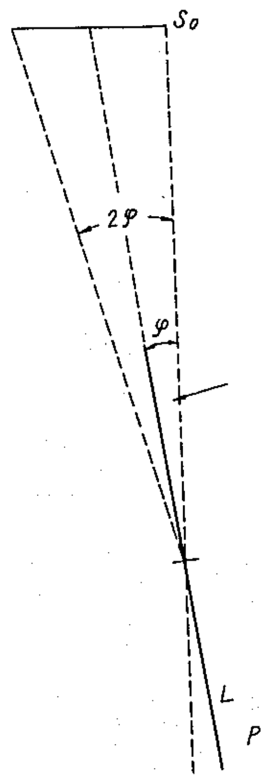


Fig. 25

If the magnet is in equilibrium,

$$MH - MZ \tan \psi = mg\ (d + a \tan \psi).$$

$$\tan \psi = \frac{MH - mga}{mga + MZ}$$

Since $\tan \psi = \dfrac{s - s_0{}'}{2f}$

$$\frac{S - S_0{}'}{2f} = \frac{MH - mgd}{mga + MZ}$$

Let the horizontal intensity be H_0 at a point where the magnetic axis is vertical or the scale reading is S_0.

Then $$MH_0 - mgd = 0$$

$$d = \frac{MH_0}{mg}$$

$$\frac{S - S_0}{2f} = \frac{MH - MH_0}{mga + MZ}$$

$$H - H_0 = (S - S_0)\ \frac{mga + MZ}{2fM}$$

$$= K' - (S - S_0)\ ,$$

where $K' = \dfrac{mga + MZ}{2fM} = \dfrac{mga}{2fM} + \dfrac{Z}{2f}$, K' is the scale constant of the horizontal balance, but strictly speaking, is not a constant. It depends on Z at the observation point, differing from the case of the vertical balance.

Let K'_1 be the scale constant at a base point P_1, where Z, H, s are Z_1, H_1, s_1 and K_2' that at field point P_2 having Z_2, H_2, s_2.

For P_1 $$K'_1 = \frac{mga}{2fM} + \frac{Z_1}{2f}$$

$$H_1 - H_0 = K'_1\ (s_1 - s_0)\ .$$

For P_2
$$K'_2 = \frac{mge}{2fM} + \frac{Z_2}{2f} = K'_1 + \frac{\Delta Z}{2f}\ , \text{ where } \Delta Z = Z_2 - Z_1\ .$$

$$H_2 - H_0 = K'_2(s_2 - s_0) = (K'_1 + \frac{\Delta Z}{2f})(s_2 - s_0)$$

$$\Delta H = H_2 - H_1 = K'_1(s_2 - s_1) + \frac{\Delta Z}{2f}(s_2 - s_0)\ .$$

The last formula is used for the calculation of the difference of H between a field point and a base point. K'_1 is measured at a base point P_1, where the vertical intensity is Z_1. For a magnetometer having a large f, the second term may be neglected, provided that ΔZ is not large. For magnetic prospecting the vertical balance is mostly used.

d) Torsion magnetometer.

For Z measurement, a small magnet is suspended by two horizontally stretched fibres so that the magnetic axis is perpendicular to the fibes. At a base point the magnetic axis is set in a horizontal direction by rotating the fibres. At a field point, the deflection angle of the axis is measured by the null method, that is, the position of the axis is restored to the original horizontal direction by twisting the fibres with the use of a graduated circle. The angle of the rotation is measured.

The scale constant of the instrument is determined by means of a Helmholtz coil. Since the magnet is always horizontal when readings are taken, the horizontal intensity has no effect on the reading whatever direction the fibres take. So it is easy to set up the instrument except paying attention to leveling it. Besides, advantages of this instrument are that a large range of Z-variation can be measured without any attachment and the instrumental drift is very small.

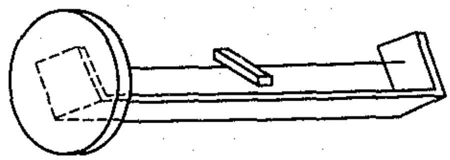


Fig. 26 Torsion magnetometer.

e) Fluxgate magnetometer.

A transformer is used as a magnetometer. In Fig. 27 C_1 and C_2 are ferromagnetic cores of high susceptibility, similarly made and placed together in parallel. P_1 and P_2 are primary coils of the same winding, but in the opposite direction, S_1 and S_2 are secondary windings of the same type but in the opposite direction each other.

By sending a large alternating current through the primary coils P_1 and P_2, the two cores of the transformer are magnetized to saturation. If there is no other magnetic field, the difference of voltage induced in the secondary coils, S_1 and S_2, must be zero. However, if the earth's magnetic field exist always in one direction of the coil, the difference shows a voltage, which is given by a voltmeter V. From the reading of the voltmeter, the intensity of the earth's magnetic field is obtained.

Usually the axis of the coil is directed along the line of force of the earth's magnetic field, and its total intensity is measured. To set the magnetometer in the line of force, a special device is used.

The fluxgate magnetometer is mostly used in airborns, but is used of course at sea or on the ground. At sea it is toward from ship. It is not to be mentioned that the magnetometer can measure the horizontal or vertical intensity, if it is set in the direction of H or Z.

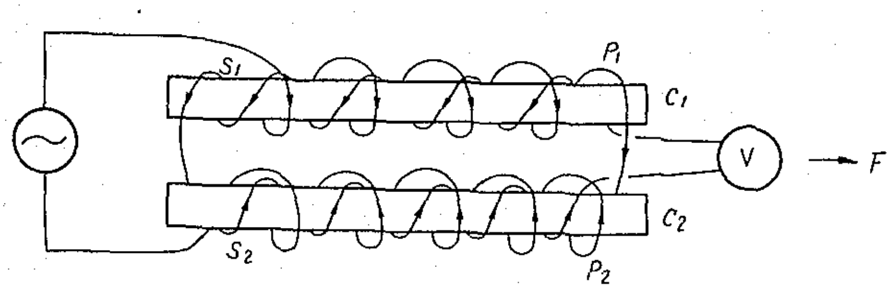


Fig. 27

f) Proton magnetometer.

Most of atomic nuclei have a magnetic moment and may be regarded as a spherical magnet spinning around its magnetic axis. In the earth's magnetic field the nuclei are oriented parallel to the field.

The simplest nucleus is of hydrogen, that is the proton. Water is composed of H and O. But oxygen has no magnetic moment. So, if we take water, only H may be taken into consideration as far as magnetic problem is concerned.

A bottle of water is wound by a coil, the axis of which is directed parpendicular to the earth's field F. By the coil a strong magnetic field, say 100 os, is generated. Owing to this field, the magnetic axes of proton which have been pointing mostly to F are aligned to the direction of the applied field, Fa. Then, suddenly the current is cut off. The axes of the protons may have the tendency to be restored at once to the original direction of F. But they will set up precessional motion round the direction of F with an angular veolocity w, which is proportional to F.

$$w = \gamma_P F,$$

here γ_P is a constant, called gyromagnetic ratio of the proton. Since w (angle per sec) is given by the frequency f (number of rotation per sec), as $f = w/2\pi$

$$f = \frac{\gamma_P F}{2\pi}$$

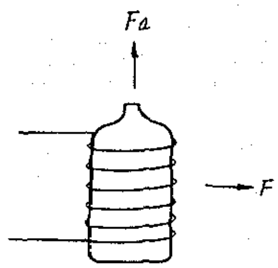


Fig. 28

The precessional oscillation wil induce an electromotive force in the coil. This a.m.f. will have the same frequency as that of the precession. From an accurate measurement of this frequency, the absolute strength of the field F can be determined by the above formula.

The nuclear magnetometer can be used on the ground as well as in the air or at see. In the latter case the detector, the principal part, is toward behind the airplane or the ship as to be free from the magnetic effect of the carrier.

g) Aeromagnetic survey.

Advantages and disadvantages of the aeromagnetic survey are summarized as follows.

Advantages:

i. A speedy survey of large area is carried out.

ii. A survey of several hundred km. is made per day. So the cost for one observation point is much less than the ground survey when a large scale of survey is planned.

iii. Measurements are made over water or rugged therein where there is no accessible motor road.

iv. Owing to high speed, effects of drift and diurnal variation of the earth's field are small.

v. As the airplane flies high, are eliminated the effects due to artificial magnetic materials such as railroad, building, etc.

vi. To find somewhat deep structures, the airborne magnetometer is useful, because magnetic rocks and other magnetic bodies near ground have almost no effect on the magnetic record.

Disadvantages:

i. Location of point has somewhat a large error. Even if a large positive anomaly would be found in the record, the site corresponding to the anomaly can not be determined exactly, owing to the height of the airplane.

ii. A small airplane is enough for carrying observers and instruments. Still it costs high. So due to the first investment the cost is much more than the surface survey when a survey of relatively small scale is to be done.

iii. For detailed survey, ground measurements are necessary. The inaccuracy comes not only from the uncertainty of position, but also from missing the detailed effect.

5. Interpretation of magnetic anomalies.

a) *Qualitative and quantitative interpretation.* All the observed values corrected for the effects above mentioned are plotted on a topographic map, and then isomagnetic lines are drawn on it with a fixed interval. Considering the geology and the topography of the area and other facts, we can deduce underground structures from the magnetic contour map.

As a rule, from Δz-map the general idea about the distribution of magnetic bodies can be found except the equatorial regions where the vertical induction of the earth's field is small.

It is note-worthy that where the anomely lines of equal interval run closely the source of the magnetic effect may be shallower. The criterion is understood from Fig. 29, which shows the magnetic traverse over iron quartzite of dyke shape in Kursk.

Where somewhat conspicuous maximum or minimum centres of magnetic anomely are found, special attention should be given about the cause of the extreme anomalies. Fig. 30 shows an example that the minimum centre of Δz corresponds to an underground salt dome. This example is taken from the well-known test measurements over a salt dome in Texas. The above two figures are roughly drawn from the famous results of magnetic measurements in order to illustrate the criteria for the interpretation.

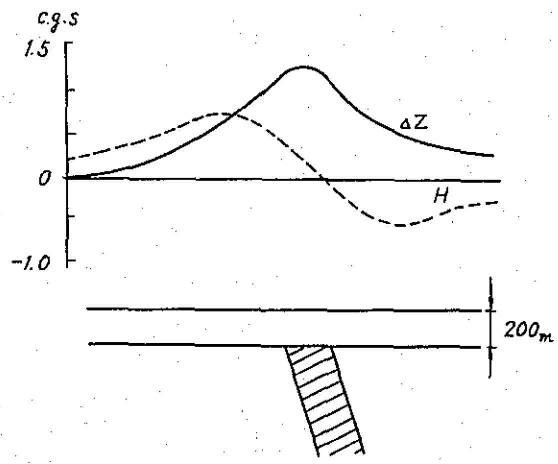


Fig. 29

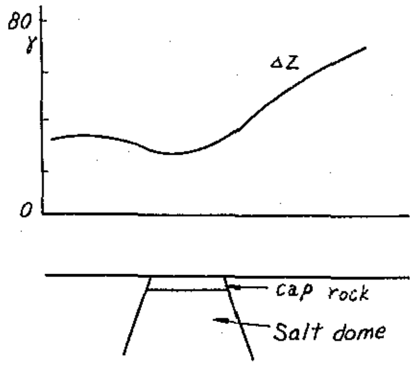


Fig. 30

Quantitative treatment of the results of magnetic surveys depends on the following factors:

1st Susceptibility of the body to be sought and that of its surrounding rocks.

2nd Direction of magnetization of rocks. Even if the magnetization is by induction, its direction is in general not the same as of the inducing field.

3rd Bipolar character of magnetic bodies.

4th Remanent magnetization having given by the ancient magnetic field.

Quantitative interpretations of simple cases are dealt with below.

b) Simple Bodies.

i) Vertical intensity due to a Pole.

We suppose a long magnet, whose negative pole is near the ground urface and positive pole is located deep so that its effect on observation points can be neglected.

Due to the induction by the earth's field, magnetic bodies are magnetized negative at their upper parts and positive at the lower parts in the Northern hemisphere.

A long body as above can be assumed to be a magnetized body having a simple negative pole.

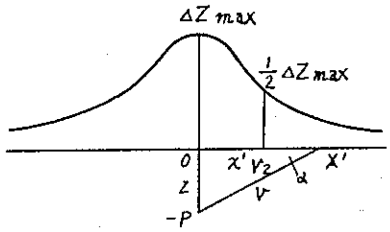


Fig. 31

Vertical intensity Δz at x^1 is given as follows (Fig. 31). The magnetic force toward a point (x', 0) is $\frac{P}{\gamma^2}$, where $r^2 = x'^2 + z^2$ and P is the Pole strength of the magnetic body.

$$\Delta z = \frac{P}{r^2} \sin \alpha = \frac{P}{r^2} \frac{z}{r} = \frac{P}{z^2} \frac{1}{(1 + \frac{x^2}{z^2})^{3/2}} , \qquad (1)$$

When x' = 0, $\Delta z_{max} = \frac{P}{z^2}$

$$\Delta z = \Delta z_{max} \frac{1}{(1 + \frac{x'^2}{z^2})^{3/2}} \qquad (2)$$

For $\Delta z = \frac{1}{2} \Delta z_{max}$ at $x' = x'_{1/2}$,

$$\frac{1}{2} = \frac{1}{(1 + \frac{x'^2}{z^2})^{3/2}}$$

$$1 + \frac{x'^2_{1/2}}{z^2} = 2^{2/3} = 1.588$$

$$\frac{x'_{1/2}}{z} = 0.767 \qquad \text{or} \qquad z = 1.306 \ x'_{1/2} \qquad (3)$$

Thus the depth z is given with the mutiplication of 1.305 and the distance between the two points having Δz max and 1/2 Δz max.

ii) Bar Magnet

When a bar magnet exists vertically underground, its vertical intensity Δz at a point (x', 0) is given from (1):

$$\Delta z = \frac{P_{z1}}{(x'^2 + z_1^2)^{3/2}} - \frac{P_{z2}}{(x'^2 + z_2^2)^{3/2}}, \qquad (4)$$

where z_1 and z_2 are the depths of the negative and the positive poles respectively (Fig. 32).

The pole strength P is found from the intensity of magnetization I and the area A of the cross-section of the magnet: thus

$P = IA$, where $I = kZ_o$.

$$P = kZ_oA,$$

here Zo denotes the vertical intensity of the earth's magnetic field at the place where the bar magnet exists.

From (4), the effect of the magnet on a point is calculated, provided susceptibility k, the area A of the cross-section, the depth z_1 and z_2 are known. By means of the trial and error method z_1, z_2 etc. may be determined.

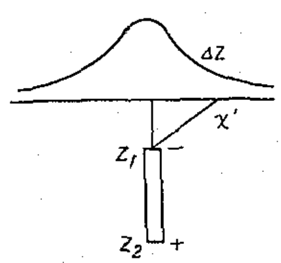


Fig. 32

iii) Dipole

When a bar magnet in (ii) is regarded as a dipole,

$$z_2 - z_1 = \ell \ll z .$$

z indicates the depth of the dipole. Then (4) becomes

$$\Delta z = \frac{M(2z^2 - x'^2)}{r^5} = \frac{M}{z^2} \frac{2 - (\frac{x'}{z})^2}{\{1 + (\frac{x'}{z})^2\}^{5/2}} , \qquad (5)$$

where $M = PL$ and $r^2 = x'^2 + z^2$.

(5) can be derived as follows. The second term of (4) is transformed as

$$-\frac{P(z + \ell)}{\{x'^2 + (z + \ell)^2\}^{3/2}} = -\frac{P(z + \ell)}{(x'^2 + z^2)^{3/2}} - \frac{1}{(1 + \frac{2z\ell + \ell^2}{x'^2 + z^2})^{3/2}}$$

$$= -\frac{P(z_2 + \ell)}{(x'^2 + z^2)^{3/2}} \left(1 - \frac{3}{2}\frac{2z\ell + \ell^2}{x'^2 + z^2} + \dots\right)$$

$$\left[(1 + X)^{-3/2} = 1 - \frac{3}{2}X - \frac{3.5}{2.4} X^2 \dots \right]$$

$$= -\frac{P(z + \ell)}{(x'^2 + z^2)^{3/2}} \cdot \frac{x'^2 + z^2 - 3z\ell}{x'^2 + z^2} ,$$

$$z = \frac{Pz(x'^2 + z^2)}{(x'^2 + z^2)^{5/2}} - \frac{P(2x'^2 + 2^3 - 2z^2\ell + x'^2\ell)}{(x'^2 + z^2)^{5/2}}$$

$$= \frac{M(2z^2 - x'^2)}{r^5}$$

where higher terms $-P\ell$ are neglected.

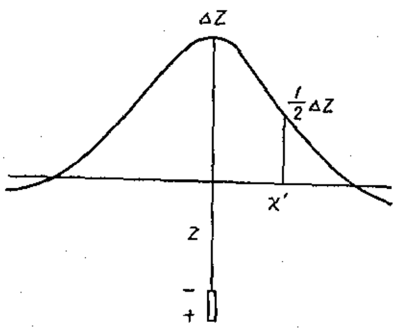


Fig. 33

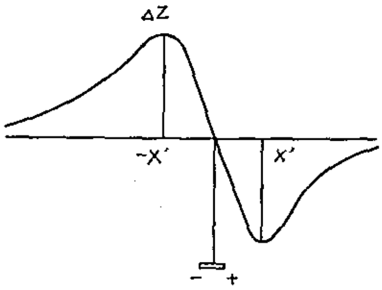


Fig. 34

when $x^1 = 0$,

$$\Delta z_{max} = \frac{2M}{z^3} .$$

(5) is transformed as follows:

$$\Delta z = \Delta z_{max} \frac{1 - \frac{1}{2}(\frac{x'}{z})^2}{\{1 - (\frac{x'}{z})^2\}^{5/2}} \tag{6}$$

If $x'_{1/2}$ expresses the abscissa for which Δz has 1/2 Δz_{max}, we have from (6)

$$\frac{1}{2} = \frac{1 - \frac{1}{2}(\frac{x'_{1/2}}{z})^2}{\{1 + (\frac{x'_{1/2}}{z})\}^{5/2}}$$

Solving this equation with respect to $(\frac{x'_{1/2}}{z})^2$ we have $\frac{x'_{1/2}}{z} = 1/4$.

$$z = 2x'_{1/2} \tag{7}$$

With the horizontal distance between the two points of z max and 1/2 z max, the depth z of the dipole can be found.

When z = 0 in (5) is used,

$$x'^2 = 2z^2$$

$$z = \frac{1}{\sqrt{2}} x'$$

From the distance x'o between the two points giving Δz max and $\Delta z = 0$, z can also be determined. But to know the exact point of $\Delta z = 0$ is somewhat ambiguous.

When a dipole is placed horizontally, its vertical effect at x' is given from (4) as follows:

$$z = \frac{P_2}{\{(x'^2 + \frac{\ell}{2})^2 + z^2\}^{3/2}} - \frac{P_2}{\{(x' - \frac{\ell}{2})^2 + z^2\}^{3/2}}$$

$$= \frac{P_2}{(x'^2 + z^2)^{3/2}} \left\{ \frac{1}{(1 + \frac{2x' \frac{1}{4} + \frac{\ell^2}{4}}{x'^2 + z^2})^{3/2}} - \frac{1}{(1 + \frac{-2x' \frac{1}{4} + \frac{\ell^2}{4}}{x'^2 + z^2})^{3/2}} \right\}$$

$$= \frac{P_2}{(x'^2 + z^2)^{3/2}} \{(1 - \frac{3}{2} \frac{x'\ell}{x'^2 + z^2}) - (1 - \frac{3}{2} \frac{-x'\ell}{x'^2 + z^2})\}$$

$$= -\frac{2Mx'z}{r^5}$$

$$= -\frac{3M}{z^3} \frac{\frac{x'}{z}}{\{1 + (\frac{x'}{z})^2\}^{5/2}} , \qquad (8)$$

where M = P .

For the point where Δz is maximum or minimum,

$$\frac{\partial \Delta z}{\partial x'} = 0$$

$$\frac{\partial \Delta z}{\partial x'} = -3Mz\ (x'^2 + z^2)^{-5/2}\ \{1 - 5x'^2\ (x'^2 + z^2)^{-1}\} = 0$$

$$1 = \frac{5x'^2}{x'^2 + z^2}$$

$$x' = \pm \frac{z}{2} \qquad \text{or} \qquad z = 2x' .$$

The depth of the dipole is equal to the distance between the two points giving two extreme values of Δz.

The effect of a dipole varies with its direction of magnetization (Fig. 35).

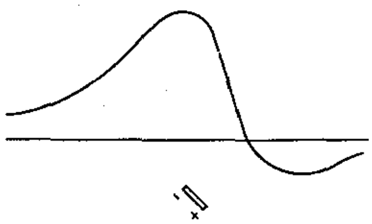


Fig. 35

iv) Horizontal Infinite Plans

When a surface having a polarity -P is extended horizontally to infinity, its vertical effect Δz at a point O(oo) can be obtained by integrating the equation (1) for unit area:

$$\delta z = \frac{Pz}{(\rho^2 + z^2)^{3/2}} \quad .$$

Using the cylindrical coordinates, we have an elementary surface as $\rho d\phi d\rho$, which is shown in Fig. 36.

$$\therefore \Delta z = \int_{\phi=0}^{2\pi} \int_{\rho=0}^{\infty} \frac{Pz\rho d\phi d\rho}{(\rho^2 + z^2)^{3/2}}$$

$$= 2\pi Pz \left| \frac{-1}{(\rho^2 + z^2)^{1/2}} \right|_0^{\infty}$$

$$= 2\pi P.$$

Here P is the pole strength for unit area, and since I = P/A, P = I.

$$\Delta z = 2\pi I = 2\pi\ kz_o \qquad (9)$$

which is independent on z. Zo indicates the vertical intensity of the earth's field.

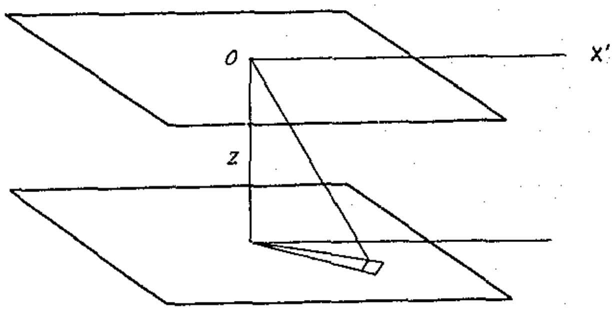


Fig. 36

From the equation (9), we can estimate k by knowing Δz and Zo.

v) Vertical Contact of Two sheets.

Two different basement rocks contact each other at a vertical plans and their upper surfaces are horizontal planes extending infinitely to the directions opposite to the contact line. When these rocks are covered with non-magnetic sediment, we can estimate the difference of the two susceptibilities k_1 and k_2 of the rocks from the measurements of their vertical effect Δz at the surface.

Suppose that the magnetization of the rocks are by the induction of the earth's field of large dip, and the effect of the vertical contact plane is neglected.

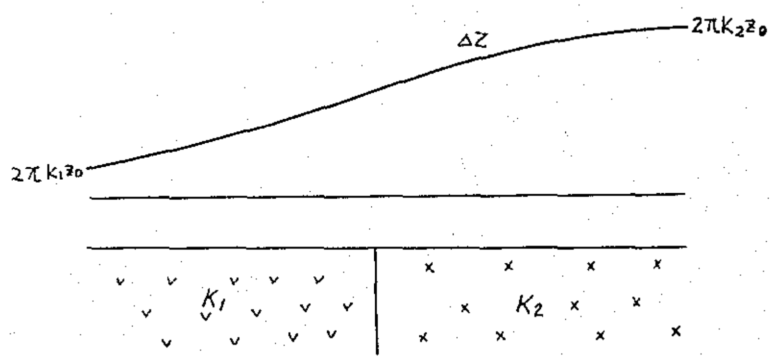


Fig. 37

The surface of one basement rock is considered to be magnetized k_1 Zo per unit area and the other k_2 Zo. The difference between the largest and the smallest Δz in the field may be approximately given by $2\pi(k_2 - k_1)$ Zo from (9).

$$k_2 - k_1 = \frac{\Delta z_{max} - \Delta z_{min}}{d\pi\ Z_o}$$

b) Calculation of Magnetic Potential from Gravitational Potential.

i) Poisson's Law between Gravitational and Magnetic Potentials.

Let U and W be the gravitational and magnetic potentials due to a body which is magnetized in a direction i with an intensity of magnetization I. The Poisson's law is

$$W = \frac{1}{G\sigma}\frac{\partial U}{\partial i}, \tag{10}$$

σ being the density of the body.

The magnetic force in a direction a is given:

$$F_s = \frac{\partial W}{\partial s} = \frac{I}{G\sigma}\frac{\partial}{\partial s}\left(\frac{\partial U}{\partial i}\right). \tag{11}$$

ii) Fault.

Suppose that a slab resulted by a fault is magnetized in or nearly in the vertical direction. The vertical intensity due to the slab is calculated as follows:

$$W = \frac{I}{G\sigma}\frac{\partial U}{\partial z}$$

$$\Delta z = \frac{\partial W}{\partial z} = \frac{I}{G\sigma}\frac{\partial^2 U}{\partial z^2}. \tag{12}$$

If the mass of the slab is supposed to be concentrated to a median sheet with a thickness of t, we have from gravitational calculation.

$$\frac{\partial U}{\partial z} = 2G\sigma t \quad = 2G\sigma t \left(\frac{\pi}{2} + \tan^{-1}\frac{x}{z}\right)$$

$$\frac{\partial^2 U}{\partial z^2} = 2G\sigma\ t \cdot \frac{1}{1 + \frac{x^2}{z^2}} \cdot \frac{x}{z^2} = 2G\sigma t \frac{x}{x^2 + z^2} \quad .$$

Accordingly from (11)

$$z = 2I \frac{tx}{x^2 - z^2} \quad .$$

Putting I = k Zo,

$$z = 2kZ_o \frac{tx}{x^2 - z^2} = \frac{2kZ_o}{z} \frac{t\cdot(x/z)}{(x/z)^2 + 1} \quad . \qquad (13)$$

By this formula the effect of the thin slab can be calculated.

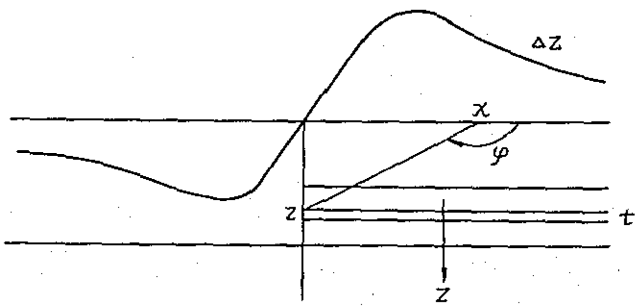


Fig. 38

The depth z may be estimated by the method of trial and error.

iii) Sphere.

Gravitational potential of a sphere is

$$U = G\frac{m}{r}$$

where $r = (x^2 + z^2)^{1/2}$

$$m = \frac{4}{3}\pi R^3\sigma \ ,$$

R being the radius and σ the density.

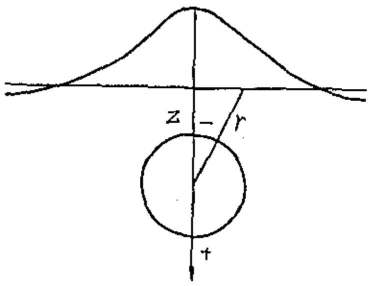


Fig. 39

When the share is vertically magnetized, from (12)

$$\frac{\partial U}{\partial z} = +Gmz\ (x^2 + z^2)^{-3/2}$$

$$\frac{\partial^2 U}{\partial_z{}^2} = Gm\ \{-(x^2 + z^2)^{-3/2} + \frac{3}{2}z \cdot 2z\ (x^2 + z^2)^{-5/2}\}$$

$$= Gm\ \frac{2z^2 - x^2}{r^5} \qquad (14)$$

$$\Delta z = \frac{4}{3}\ \pi R^3 I\ \frac{2z^2 - x^2}{(x^2 - z^2)^{5/2}}$$

$$= \frac{4}{3}\ \pi R^3 I\ \frac{1}{z^3}\ \frac{2 - (\frac{x}{z})^2}{\{1 + (\frac{x}{z})^2\}^{5/2}}\ , \qquad (15)$$

where I = k Zo.

If R, k, Z are known, Δz at any x can be calculated.

(15) is the same as the formula (5) for a dipole when $\frac{4}{3}\ \pi R^3 I$ is applied instead of M. The curve of Δz is similer to the dipole.

When the sphere is magnetized horizontally, its magnetic potential at (x, 0) is

$$W = \frac{I}{G\sigma}\ \frac{\partial U}{\partial x}\ ,$$

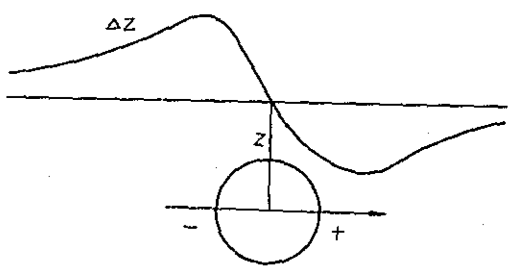
where $$U = Gm\,(x^2 + z^2)^{-1/2}$$

$$\therefore \Delta z = \frac{\partial W}{\partial z} = + \frac{I}{G\sigma}\,\frac{\partial^2 U}{\partial x \partial z} \; .$$

$$\frac{\partial U}{\partial x} = Gmx\,(x^2 + z^2)^{-3/2}$$

$$\frac{\partial^2 U}{\partial x \partial z} = - Gm \frac{3xz}{(x^2 + z^2)^{5/2}}$$

$$\Delta z = \frac{4}{3}\pi R^3 I \frac{1}{z^3} \frac{3(\frac{x}{z})}{\{1 + (\frac{x}{z})^2\}^{5/2}} \qquad (16)$$



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