

REPUBLIC OF INDONESIA
MINISTRY OF COMMUNICATIONS
DIRECTORATE GENERAL OF LAND TRANSPORT
AND INLAND WATERWAYS

TENDER DOCUMENTS
FOR
NEW RAILWAY LINE FOR CENGKARENG AIRPORT
CONSTRUCTION PROJECT

STRUCTURAL CALCULATION SHEETS

PACKAGE II TRACK WORK

AUGUST 1984

JAPAN INTERNATIONAL COOPERATION AGENCY
(JICA)



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1. STRESS CALCULATION FOR SLEEPER

PRETENSION METHOD

(STRAIGHT LINE)

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1 : Design condition

Sleeper length	2 0 0 0 mm
Track gauge	1 0 6 7 mm
Axle load	1 6 ton
Pressure of rail	8 ton
Rail	R 5 4
Rail fastening	Elastic fastening
Type	Pretension methods

Material strength

A). Concrete

Compressive strength (age of 28 days)

Test piece : $\phi 10 \text{ cm} \times 20 \text{ cm} \dots 500 \text{ kg/cm}^2$

Compressive strength of prestressing

Test piece : $\phi 10 \text{ cm} \times 20 \text{ cm} \dots 400 \text{ kg/cm}^2$

Allowable bending compressive stress intensity

180 kg/cm^2

Allowable tensile stress intensity $\dots \dots - 18 \text{ kg/cm}^2$

B). P.C steel strand (3-strand) $\dots \phi 2.9 \times 3$

Ultimate strain $P_u \geq 3900 \text{ kg}$

Yield strain $P_y \geq 3450 \text{ kg}$

Allowable effective tensile load $P_a \geq 2340 \text{ kg}$

2 : Pressure of rail on sleeper

$$P = 1/2 \times W \times D_1 \times (1 + i)$$

P : Pressure of rail on sleeper

W : Axle load

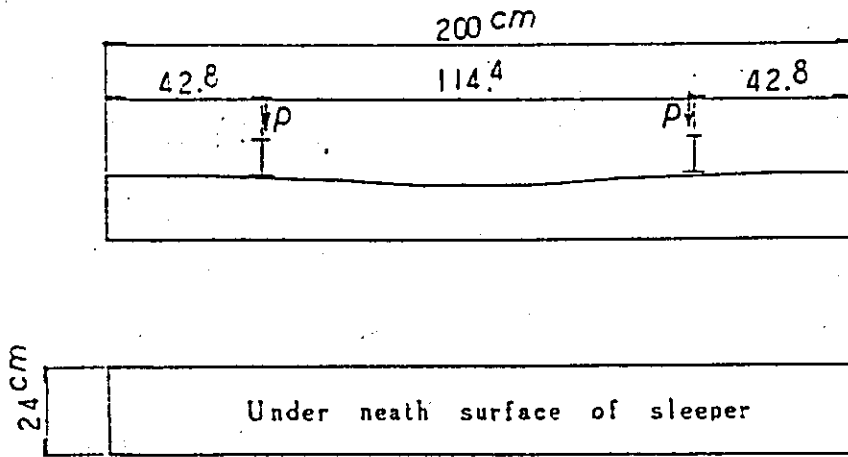
D₁ : Distributed coefficient

i : Impact coefficient

$$P = 1/2 \times 16 \times 0.5 \times (1 + 1.0) = 8.0 \text{ ton}$$

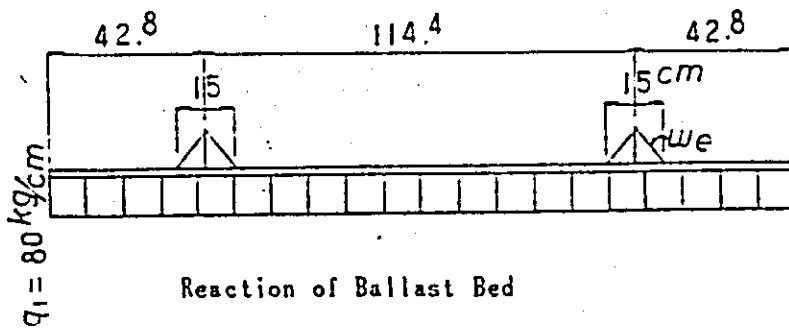
3 : Bending moment

3-1 Supporting condition & hypothesis



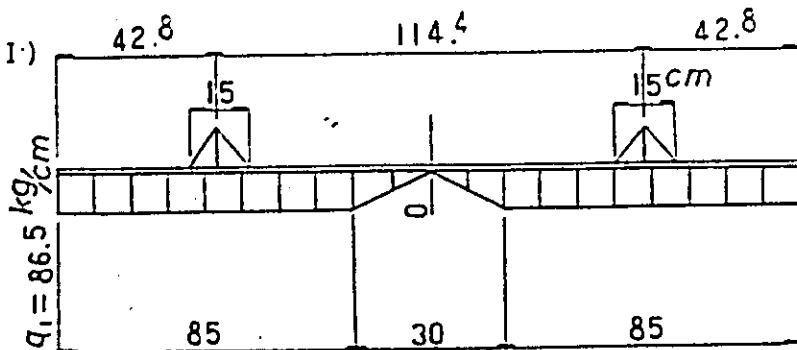
$P = 8000 \text{ kg}$

load (I)

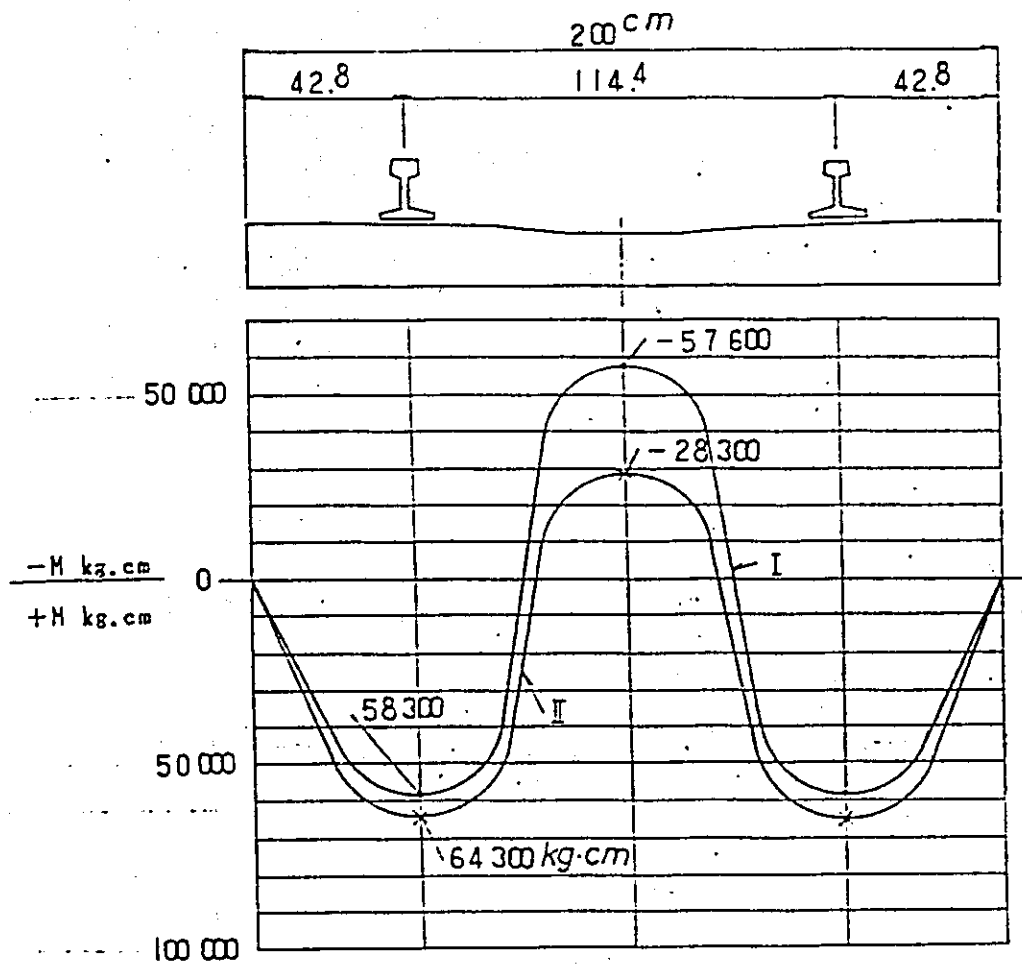


$w_e = 8000 \text{ kg}$
 15 cm
 $= 533 \text{ kg/cm}$

load (I I)



3-2 Design moment



Under rail of sleeper

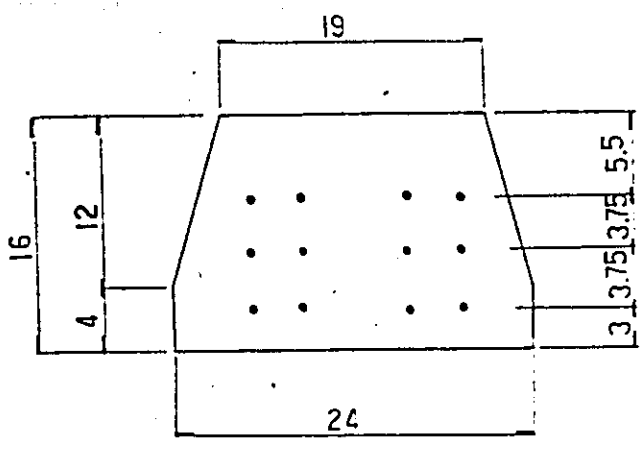
$$+M = 64,300 \text{ kg-cm}$$

Center of sleeper

$$-M = -57,600 \text{ kg-cm}$$

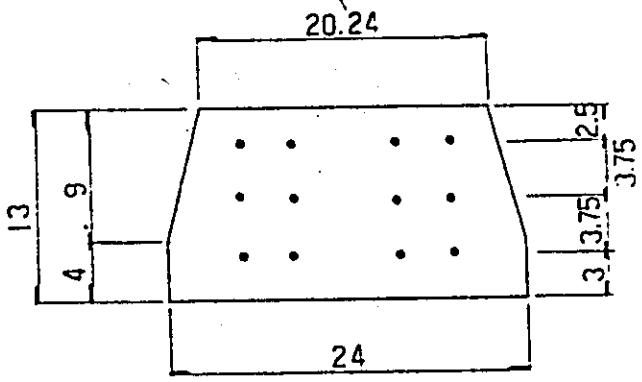
4 : Modulus of concrete section

4-1 Under rail of sleeper



$A = 354 \text{ cm}^2$
 $Y_1 = 8.34 \text{ cm}$
 $Y_2 = 7.66 \text{ cm}$
 $I = 7431 \text{ cm}^4$
 $Z_1 = 891 \text{ cm}^3$
 $Z_2 = 970 \text{ cm}^3$
 $E_p = 0.91 \text{ cm}$

4-2 Center of sleeper



$A = 295 \text{ cm}^2$
 $Y_1 = 6.70 \text{ cm}$
 $Y_2 = 6.30 \text{ cm}$
 $I = 4099 \text{ cm}^4$
 $Z_1 = 612 \text{ cm}^3$
 $Z_2 = 651 \text{ cm}^3$
 $E_p = - 0.45 \text{ cm}$

- A : Cross sectional area
- Y_1 : Height of center gravity from Upper
- Y_2 : Height of center gravity from Bottom
- I : Moment of inertia
- Z_1 : Section modulus (Upper)
- Z_2 : Section modulus (Bottom)
- E_p : Eccentricity between the center of concrete section the P.C. steel strand.

5 : Bending stress intensity

5-1 Design load

$$\sigma = \frac{M}{Z}$$

O: Bending stress intensity

M: Bending moment

Z: Section modulus

5-1-1 Under rail of sleeper

$$\sigma_{cu} = \frac{64300}{891} = 72.2 \text{ kg/cm}^2$$

$$\sigma_{ce} = \frac{-64300}{970} = -66.3 \text{ kg/cm}^2$$

5-1-2 Center of sleeper

$$\sigma_{cu} = \frac{-57600}{612} = -94.1 \text{ kg/cm}^2$$

$$\sigma_{ce} = \frac{57600}{651} = 88.4 \text{ kg/cm}^2$$

σ_{cu} : Extreme fiber stress (Upper)

σ_{ce} : Extreme fiber stress (Lower)

5-2 Calculation of the prestress

Total initial prestressing force

$$P_i = (2930 \times 12 = 35160 \text{ kg})$$

Effective prestressing force

$$P_e = 0.65 \times 35160 = 22850 \text{ kg}$$

5-2-1 Prestressing of stress intensity

$$\sigma_{ce} = \frac{P_e}{A} \pm \frac{P_e \cdot e_p}{Z_1 \text{ or } Z_2}$$

Under rail of sleeper

$$\sigma_{ceu} = \frac{22850}{354} - \frac{22850 \times 0.91}{891} = 41.2 \text{ kg/cm}^2$$

$$\sigma_{cel} = \frac{22850}{354} + \frac{22850 \times 0.91}{970} = 85.9 \text{ kg/cm}^2$$

Center of sleeper

$$\sigma_{ceu} = \frac{22850}{295} + \frac{22850 \times 0.45}{612} = 94.3 \text{ kg/cm}^2$$

$$\sigma_{cel} = \frac{22850}{295} - \frac{22850 \times 0.45}{651} = 61.7 \text{ kg/cm}^2$$

5-3 Total stress intensity

5-3-1 Under rail of sleeper

$$\sigma_u = \sigma_{ceu} + \sigma_{cu} = 41.2 + 72.2 = 113.4 \text{ kg/cm}^2 \leq 180 \text{ kg/cm}^2$$

$$\sigma_e = \sigma_{cel} + \sigma_{ce} = 85.9 - 66.3 = 19.6 \text{ kg/cm}^2 \geq -180 \text{ kg/cm}^2$$

5-3-2 Center of sleeper

$$\sigma_u = \sigma_{ceu} + \sigma_{cu} = 94.3 - 94.1 = 0.2 \text{ kg/cm}^2 \geq -18 \text{ kg/cm}^2$$

$$\sigma_e = \sigma_{cel} + \sigma_{ce} = 61.7 + 88.4 = 150.1 \text{ kg/cm}^2 \leq 180 \text{ kg/cm}^2$$

2. STRESS CALCULATION FOR SLEEPER

PRETENSION METHOD

(CURVED SECTION)

1 : Design condition	
Sleeper length	2 0 0 0 mm
Track gauge	1 0 6 7 mm
Axle load	1 6 ton
Pressure of rail	8 ton
Lateral rail pressur	4. 5 ton
Rail	R 5 4
Rail fastening	Elastic fastening
Type	Pretension methods

Material strength

A). Concrete

Compressive strength (age of 2 8 days)	
Test piece : $\phi 10 \text{ cm} \times 20 \text{ cm}$	500 kg/cm ²
Compressive strength of prestressing	
Test piece : $\phi 10 \text{ cm} \times 20 \text{ cm}$	400 kg/cm ²
Allowable bending compressive stress intensity	
	180 kg/cm ²
Allowable tensive stress intensity	- 18 kg/cm ²

B). P.C steel strand wire $\phi 2.9 \times 3$ strand

Ultimate strain	$P_u \geq 3900 \text{ kg}$
Yield strain	$P_y \geq 3450 \text{ kg}$
Allowable effective tensile load	$P_a \geq 2340 \text{ kg}$

2 :

2-1 Pressure of rail on sleeper

$$P = 1/2 \times W \times D_1 \times (1 + i_1)$$

P : Pressure of rail on sleeper

W : Axle load

D₁ : Distributed coefficienti₁ : Impact coefficient

$$P = 1/2 \times 16 \times 0.5 \times (1 + 1.0) = 2.0 \text{ ton}$$

2-2 Lateral rail pressure

$$Q_f = Q \times D_2 \times (1 + i_2)$$

Q_f : Lateral rail pressure

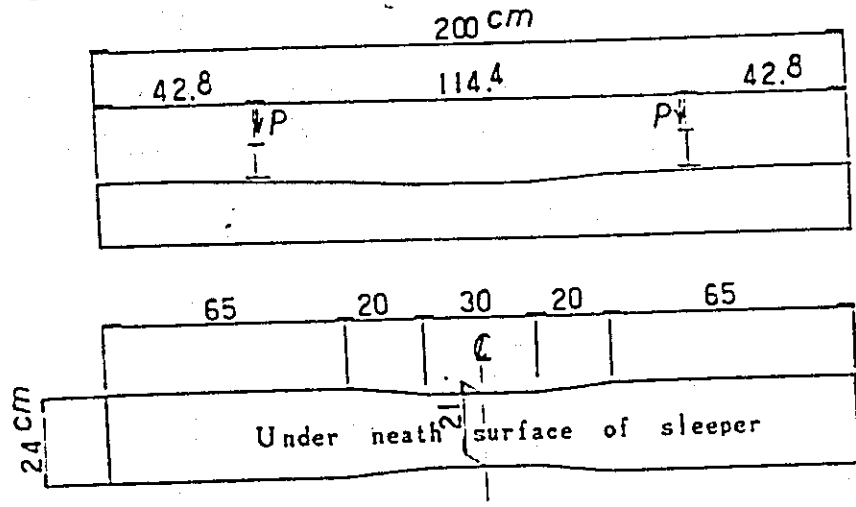
Q : Lateral force

D₂ : Distributed coefficienti₂ : Impact coefficient

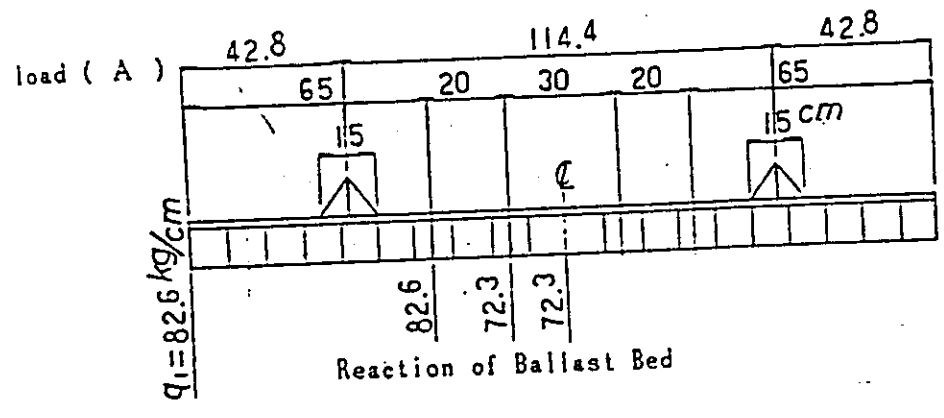
$$Q_f = 6 \times 0.5 \times (1 + 0.5) = 4.5 \text{ ton}$$

3 : Bending moment

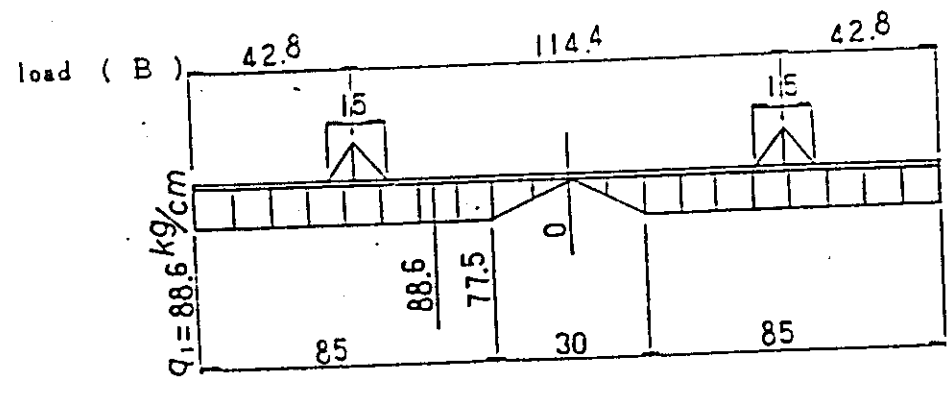
3-1 Supporting condition & hypothesis



$P = 8000\text{kg}$

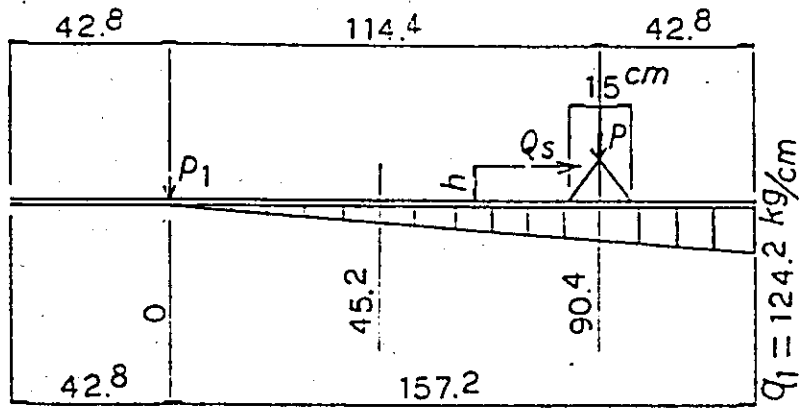


$w_e = \frac{8000\text{kg}}{15\text{ cm}}$
 $= 533\text{kg/cm}$



Reaction of Ballast Bed

Lateral rail pressure and pressure of rail on sleeper
load (C)



Reaction of Ballast Bed

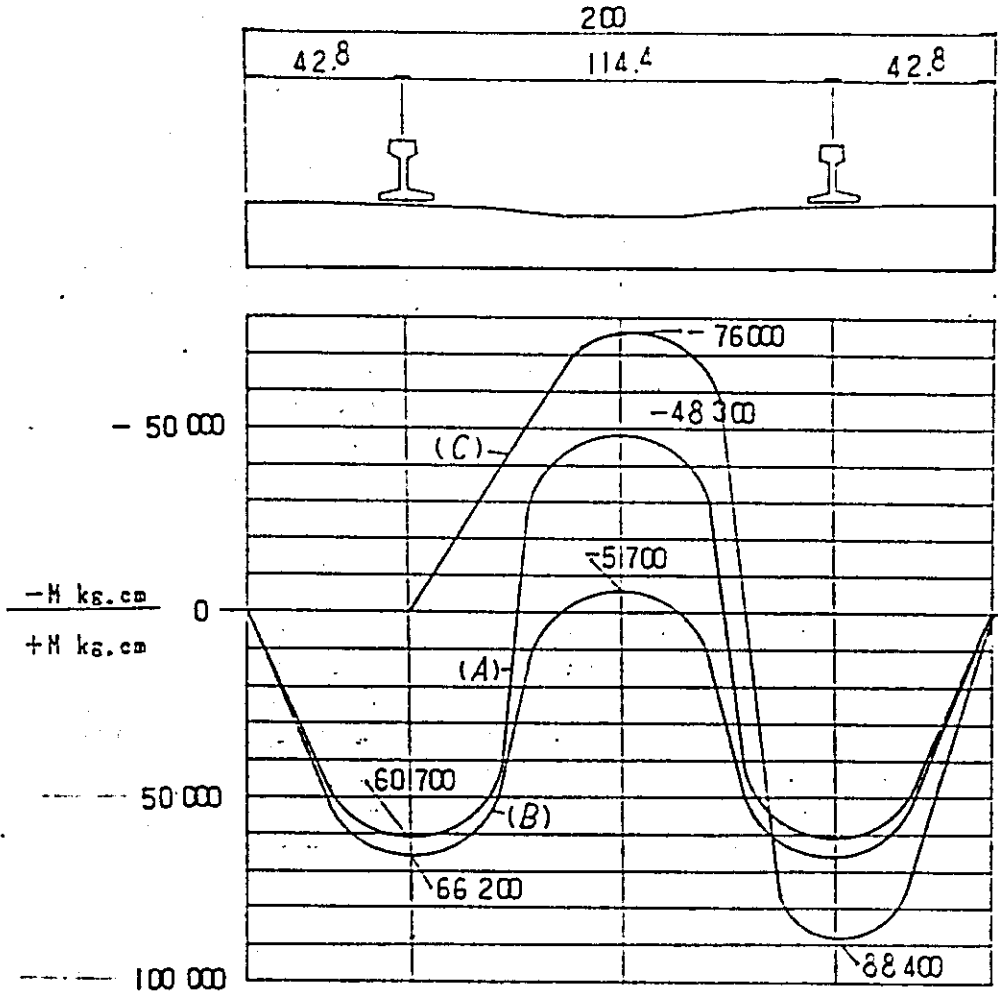
$Q_s = 4500 \text{ kg}$

$h = 24 \text{ cm}$

$P = 8000 \text{ kg}$

$P_1 = 1760 \text{ kg}$

3 - 2 Design moment



Under rail of sleeper

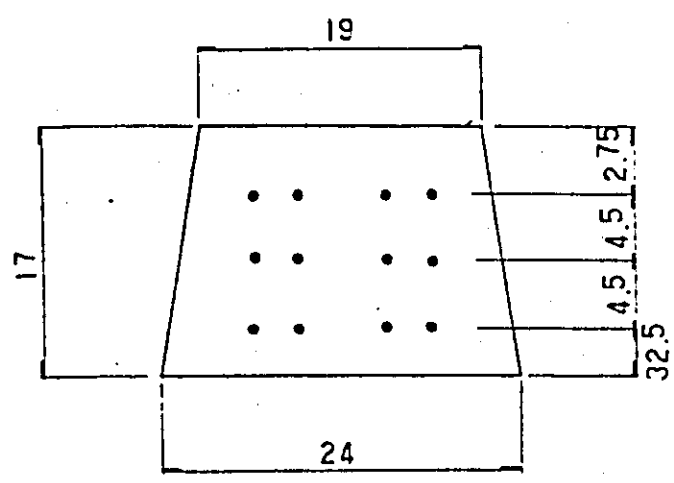
$$+M = 88,400 \text{ kg.cm}$$

Center of sleeper

$$-M = -76,000 \text{ kg.cm}$$

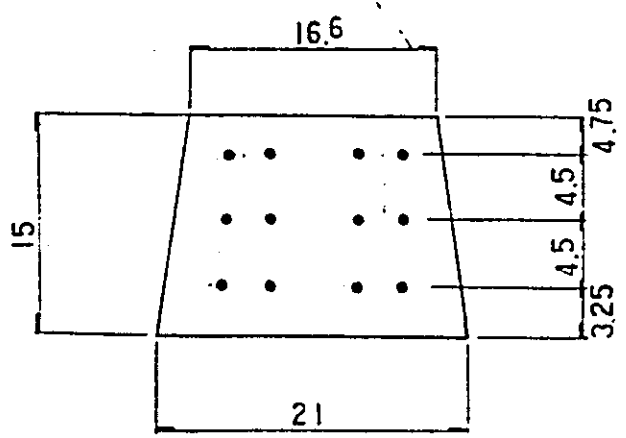
4 : Modulus of concrete section

4-1 Under rail of sleeper



$A =$	366 cm^2
$Y_1 =$	8.83 cm
$Y_2 =$	8.17 cm
$I =$	8763 cm^4
$Z_1 =$	992 cm^3
$Z_2 =$	1072 cm^3
$E_p =$	0.42 cm

4-2 Center of sleeper



$A =$	282 cm^2
$Y_1 =$	7.79 cm
$Y_2 =$	7.21 cm
$I =$	5263 cm^4
$Z_1 =$	675 cm^3
$Z_2 =$	730 cm^3
$E_p =$	$- 0.54 \text{ cm}$

- A : Cross sectional area
- Y_1 : Height of center gravity from Upper
- Y_2 : Height of center gravity from Bottom
- I : Moment of inertia
- Z_1 : Section modulus (Upper)
- Z_2 : Section modulus (Bottom)
- E_p : Eccentricity between the center of concrete section the P.C. steel strand wire.

5 : Bending stress intensity

5 - 1 Design load

$$\sigma = \frac{M}{Z}$$

- O: Bending stress intensity
- M: Bending moment
- Z: Section modulus

5 - 1 - 1 Under rail of sleeper

$$\sigma_{cu} = \frac{88400}{992} = 89.1 \text{ kg/cm}^2$$

$$\sigma_{ce} = \frac{-88400}{1072} = -82.5 \text{ kg/cm}^2$$

5 - 1 - 2 Center of sleeper

$$\sigma_{cu} = \frac{-76000}{675} = -112.6 \text{ kg/cm}^2$$

$$\sigma_{ce} = \frac{76000}{730} = 104.1 \text{ kg/cm}^2$$

- σ_{cu} : Extreme fiber stress (Upper)
- σ_{ce} : Extreme fiber stress (Lower)

5-2 Calculation of the prestress

Total initial prestressing force

$$P_i = 2930 \times 12 = 35160 \text{ kg}$$

Effective prestressing force

$$P_e = 0.65 \times 35160 = 22850 \text{ kg}$$

5-2-1 Prestressing of stress intensity

$$\sigma_{ce} = \frac{P_e}{A} \pm \frac{P_e \cdot e_p}{Z_1 \text{ or } Z_2}$$

Under rail of sleeper

$$\sigma_{ceu} = \frac{22850}{366} - \frac{22850 \times 0.42}{992} = 52.7 \text{ kg/cm}^2$$

$$\sigma_{cel} = \frac{22850}{366} + \frac{22850 \times 0.42}{1072} = 71.4 \text{ kg/cm}^2$$

Center of sleeper

$$\sigma_{ceu} = \frac{22850}{282} + \frac{22850 \times 0.54}{675} = 99.3 \text{ kg/cm}^2$$

$$\sigma_{cel} = \frac{22850}{282} - \frac{22850 \times 0.54}{730} = 64.1 \text{ kg/cm}^2$$

5-3 Total stress intensity

5-3-1 Under rail of sleeper

$$\sigma_u = \sigma_{ceu} + \sigma_{cu} = 52.7 + 89.1 = 141.8 \text{ kg/cm}^2 \leq 180 \text{ kg/cm}^2$$

$$\sigma_e = \sigma_{cel} + \sigma_{ce} = 71.4 - 82.5 = -11.1 \text{ kg/cm}^2 \geq -18 \text{ kg/cm}^2$$

5-3-2 Center of sleeper

$$\sigma_u = \sigma_{ceu} + \sigma_{cu} = 99.3 - 112.6 = -13.3 \text{ kg/cm}^2 \geq -18 \text{ kg/cm}^2$$

$$\sigma_e = \sigma_{cel} + \sigma_{ce} = 64.1 + 104.1 = 168.2 \text{ kg/cm}^2 \leq 180 \text{ kg/cm}^2$$

3 PARTIAL TRACK STRESS CALCULATION

Stress at each part of the track structure, selected as explained in the foregoing, was studied in detail.

Conditions

Rail : R54
 Sleeper : PC sleeper 1760 units/km
 Ballast : 30 cm thickness

Symbols:

W = Wheel load 8,000 kg
 E = Elasticity coefficient of rail steel $2.1 \times 10^6 \text{ kg/cm}^2$
 I_x = Moment of inertia for rail 2,346 cm⁴
 Z = Section modulus for rail 313 cm³
 k = Spring constant for rail supporting system kg/cm²
 d = Ballast thickness 30 cm
 C = Ballast coefficient = $\frac{10}{15} d = 20 \text{ kg/cm}^3$
 b = Sleeper width 24 cm
 l = Sleeper length 200 cm
 a = Sleeper space 58 cm
 D₁ = Spring constant for pad 100 ton/cm
 D₂ = Coefficient of ballast settlement
 = $\frac{bl}{2} \cdot C = 48,000 \text{ kg/cm} = 48 \text{ ton/cm}$

Settlement coefficient for rail bearing structure

$$D = \frac{D_1 D_2}{D_1 + D_2} = 32.4 \text{ ton/cm}$$

$$k = \frac{D}{a} = 0.559 \text{ t/cm}^2 = 559 \text{ kg/cm}^2$$

$$\beta = 4 \sqrt{\frac{k}{4EI_x}} = 1.30 \times 10^{-2} / \text{cm}$$

Rail deflection y directly below wheels is:

$$y = \frac{W}{8 EI_x \beta^3} = 0.09 \text{ cm}$$

Bending moment M of rail directly below wheels is:

$$M = \frac{W}{4\beta} = 153,846 \text{ kg}\cdot\text{cm}$$

Rail stress σ

$$\sigma = \frac{M}{Z} = 492 \text{ kg}$$

Rail pressure P directly below wheels:

$$P = W(1 - e^{-\frac{a\beta}{2}} \cdot \cos \frac{2\beta}{2}) = 2,816 \text{ kg}$$

Ballast pressure P_b directly below wheels:

$$P_b = \frac{P}{1/2 \cdot b_l} = 1.17 \text{ kg/cm}^2$$

Roadbed pressure P_r directly below wheels

$$P_r = \rho_0 P$$

ρ_0 = Coefficient in relation to ballast thickness as shown in the following table

Ballast thickness (mm)	ρ_0 (kg/cm ² /ton)
120	0.589
150	0.454
200	0.350
250	0.288
300	0.232

Therefore,

$$P_r = 0.232 \times 2.816 = 0.65 \text{ kg/cm}^2$$

Generally, an increase in speed is required to cope with the dynamic increases in wheel load caused by irregularity of track surface, rolling motion of cars and flat tires.

The rail bending stress at 100 kg/h will be:

$$i = 1 + \frac{V}{100} = 2$$

For rail pressure:

$$i = 1 + \frac{0.6V}{100} = 1.6$$

Hence, finally,

$$\text{Rail stress } \sigma = 2 \times 492 = 984 \text{ kg/cm}^2$$

$$\text{Roadbed pressure } Pr = 1.6 \times 0.65 = 1.0 \text{ kg/cm}^2$$

Against these values, the allowable stress is:

Rail stress with its fatigue taken into account will be 1,800 kg/cm² and Roadbed pressure 2 kg/cm².

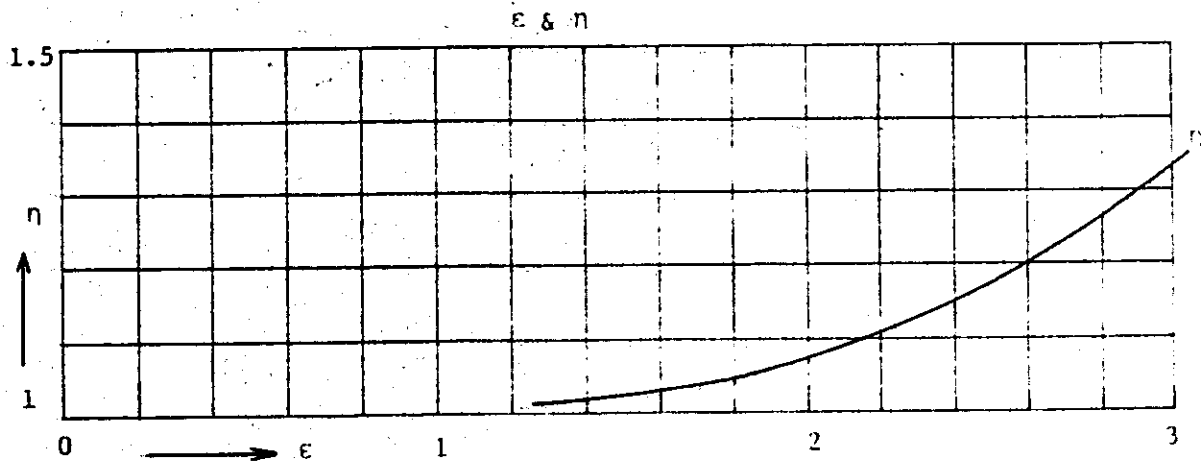
Those figures assure sufficient marginal track strength.

FIG. 1 FOR NARROW GAUGE

$$\ell = 200 \text{ cm}$$

$$g = 113 \text{ cm}$$

FOR PJKA



$$\epsilon = g \sqrt{\frac{cb}{4E'I'}}$$

E' : ELASTICITY COEFFICIENT OF SLEEPER

I' : MOMENT OF INERTIA FOR SLEEPER

g : $1067 \text{ mm} + 65 \text{ mm} = 1132 \text{ mm} \rightarrow 113 \text{ cm}$

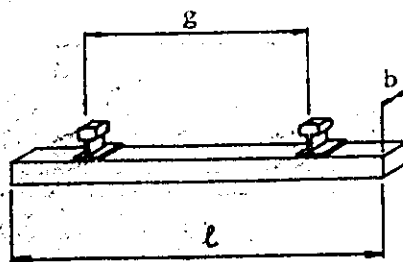


FIG. 2 ma & d

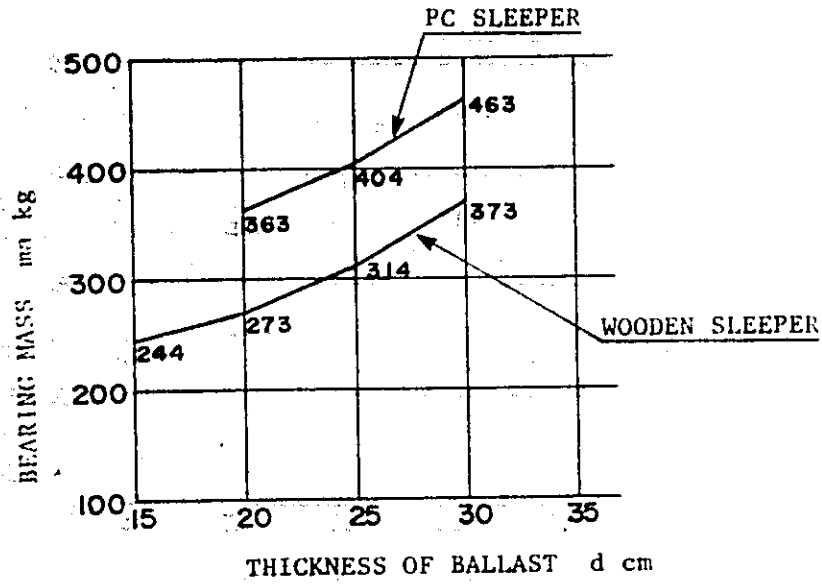


FIG. 3 TRACK MODEL FOR ANALYSIS

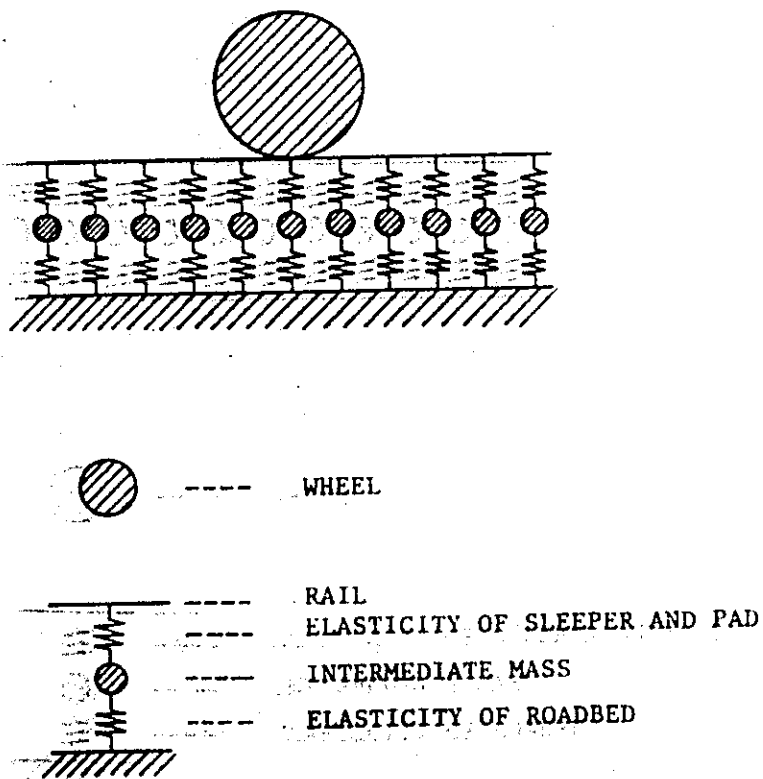
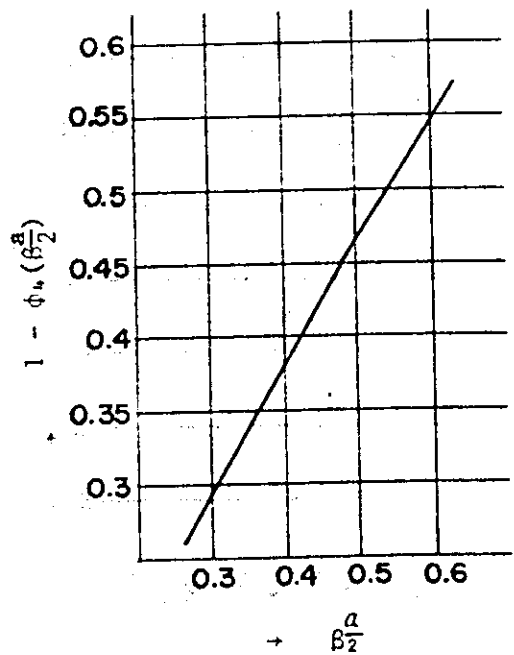


FIG. 4 $1 - \phi_v \left(\frac{a \xi}{2} \right)$



**4 CALCULATION OF RAIL EXPANSION AND JOINT GAPS OF
100 m LENGTH R54 RAILS**

Joint gaps must be properly selected after giving due consideration to keeping the track from buckling at maximum temperatures of rail and relieving the joint bolts from excessive stress at minimum temperatures of rail.

In order to ensure the setting of the correct joint gap, rail length, rail temperature variation, joint gap and rail axle force must be carefully studied.

The following symbols are used for the above analyses.

t	= Temperature of rail	°C
t _o	= Rail laying temperature	°C
t _{max}	= Maximum temperature of rail	60°C
t _{min}	= Minimum temperature of rail	20°C
Δ t _{max}	= t _{max} - t _{min}	40°C
e	= Expansion gap	cm
e _o	= Rail-laid gap	cm
e _{max}	= Maximum gap	2.2 cm
β	= Gross expansion coefficient of rail steel ...	1.14 x 10 ⁻⁵ /°C
E	= Young's modulus of rail steel	2.1 x 10 ⁶ kg/cm ²
L	= Rail length	10,000 cm
A	= Section area of rail	69.34 cm
R	= Friction force of joint bar	9,000 kg
r	= Resistance of rail creepage	6 kg/cm
P _a	= Minimum buckling axle force	49,000 kg
P _{max}	= Maximum compressive axle force of rail	kg
Δ t _r	= Temperature variation corresponding to joint bar friction force R	°C

Assuming that rail temperature is 't_o' when it is laid and the expansion gap is 'e_o', and the temperature would rise to 't - e' loop.

$$\Delta t_R = \frac{R}{E \cdot A \cdot \beta} = \frac{9000}{2.1 \times 10^6 \times 69.34 \times 1.14 \times 10^{-5}} = 5.4^\circ\text{C}$$

A - M

From A to the maximum temperature 't_{max}' the temperature difference is Δ t_m.

$$\Delta t_m = t_{\max} - t_o - \Delta t_r = 60 - 40 - 5.4 = 14.6^\circ\text{C}$$

The rail length X_m which lies in the section with the capacity for expansion for the temperature range Δt_m is:

$$X_m = \frac{E.A.B.\Delta t_m}{\gamma} = \frac{2.1 \times 10^6 \times 6 \times 9.34 \times 1.14 \times 10^{-5} \times 14.6}{6} = 4,039 \text{ cm}$$

In short, there is, in the central part of the rail, an immobile gap of:

$$100 - 40.39 \times 2 = 19.22 \text{ m}$$

Expansion gap Δe_m from Δt_m , x_m is:

$$\begin{aligned} \Delta e_m &= 2\beta.\Delta t_m.x_m - \frac{\gamma.x_m^2}{E.A.} = 2 \times 1.14 \times 10^{-5} \times 14.6 \times 4039 - \frac{6 \times (4039)^2}{2.1 \times 10^6 \times 69.34} \\ &= 1.34 - 0.67 = 0.67 \text{ cm} \end{aligned}$$

Therefore,

$$e_o = 0.67 \text{ cm}$$

As shown in the calculations above, it is advisable to lay rail at 40°C with gap of joint of 0.67 cm.

If the temperature falls,

M - D

$$2\Delta t_R = 5.4 \times 2 = 10.8^\circ\text{C}$$

D - N

Because contraction slowdown caused by residual axle force, resistance to rail creepage is apparently doubled.

Temperature variation Δt_N from point D to minimum temperature t_{\min} is calculated as follows:

$$\Delta t_N = t_{\max} - t_{\min} - 2\Delta t_R = 2\Delta t_M = 2 \times 14.6 = 29.2^\circ\text{C}$$

Gap variation Δe_N is:

$$\Delta e_N = 2 e_M = 2 \times 0.67 = 13.4 \text{ cm}$$

If the temperature rises again, $t - e$ loop will trace the NHM locus. Maximum compressive axle force P_{\max} in the central part of the rail allows for a tolerance of 2 mm plus or minus at fixed joint gaps.

Temperature variation Δt corresponding to a 2 mm gap may be calculated as follows:

$$\Delta t = 0.2 / \beta L = 0.2 / 1.14 \times 10^{-5} \times 10^4 = 1.8^\circ\text{C}$$

$$P_{\text{max}} = R + \gamma \cdot x_M + E \cdot A \cdot \beta \cdot \Delta t = 9000 + 6 \times 4039 + 2.1 \times 10^6 \times 69.34 \times 1.14 \times 10^{-5} \times 1.8$$

$$= 36,222 \text{ kg} < P_a = 48,000 \text{ kg} \quad (R = 300, \text{ Safety ratio } 20\%)$$

Gap e is equal to 13.4 mm at lowest temperature of 20°

Even if the tolerance of plus or minus 2 mm for joint gap is taken into account in addition to the maximum structural gap of $e_{\text{max}} = 22$ mm, there still will remain a marginal surplus of $22 - 13.4 - 2 = 6.6$ mm. The joint bolts may therefore be kept free from any bending stress.

The result of the above study is shown in Fig. t - e Loop.

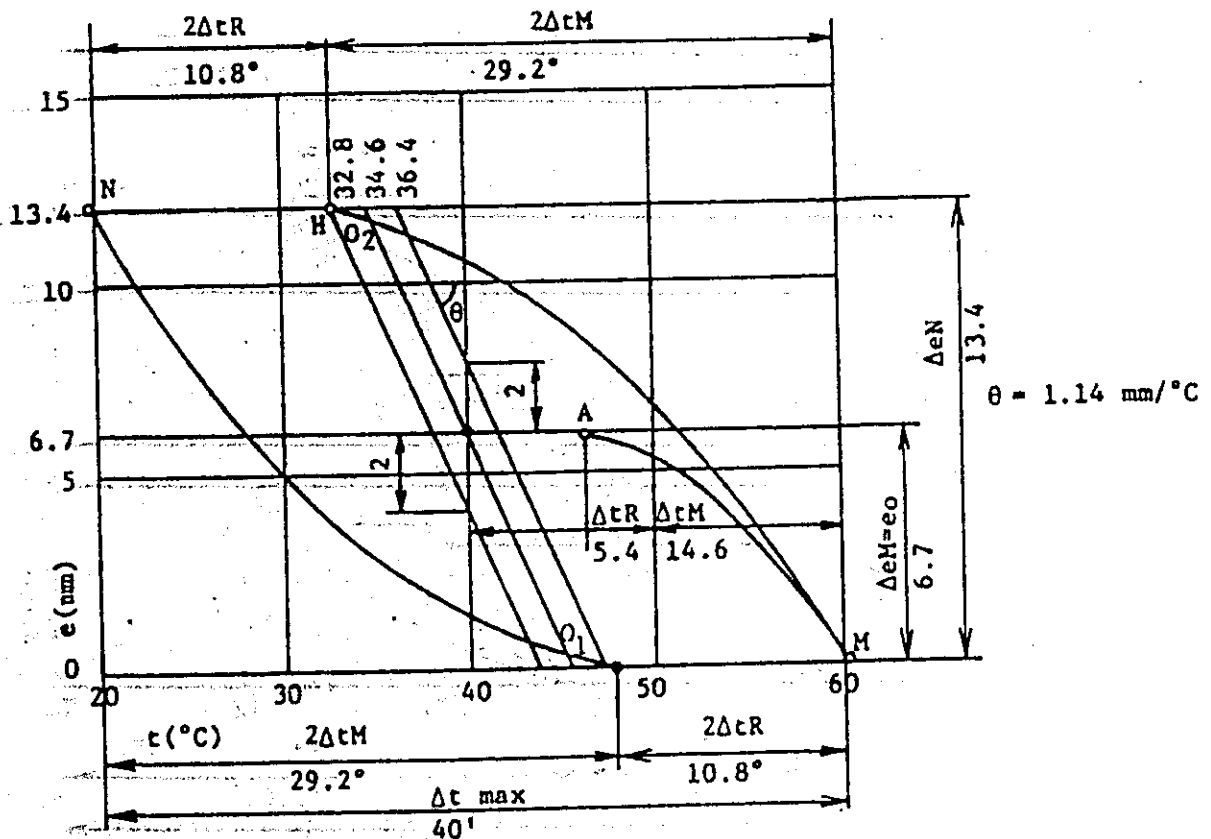


Fig. t - e LOOP

5 CALCULATION OF STRUCTURE COEFFICIENT M

1. Structure Coefficient M

Settlement of the ballast will vary depending on the type of track structures even under the same loaded conditions. Settlement of the ballast is expressed by the product of sleeper pressure and ballast vibration acceleration. Ballast vibration acceleration can be expressed by the product of vibration acceleration when the mass below the spring fallen from designated height and impact occurred during the wheels are in motion. Track structure strength M therefore, can be expressed by the product of $P_b \times \ddot{y} \times S$, taking into account the following:

- (i) Ballast pressure (P_b) caused on the sleeper directly below the wheels with a certain load imposed,
- (ii) Ballast vibration acceleration (\ddot{y}) against a certain wheel impact, and,
- (iii) Impact coefficient (S) which varies depending on the type of track structure.

However, practically it is expressed as below by a ratio to a certain standard structure, and called as structure coefficient M.

$$M = \frac{P_b \times \ddot{y} \times S}{P_b \times y \times S(\text{Standard Structure})}$$

Accordingly, comparison is made to the strength of other track structures on the basis of structure coefficient M of the standard structure as one (1). Therefore, various factors that will determine the strength of the track are considered as follows:

- 1) Type of rails
- 2) Type of sleepers
- 3) The number of sleepers laid
- 4) Use of track pads
- 5) Use of tie plates
- 6) Type of ballast material
- 7) Thickness of ballast

The smaller M is, the stronger track is, contrary, the bigger M is, the weaker track is.

2. Ballast Pressure P_b

2.1 Symbols:

D_1 : Sleeper Compressive Coefficient (t/cm)

Experimentally, 100 t/cm for wooden and PC sleepers, while 50 t/cm for wooden sleepers with the use of pad (rubber or nylon).

C: Ballast Coefficient

Experimentally, ballast coefficient C is 10 kg/cm³ when ballast depth (thickness) d = 15 cm and is proportional to ballast depth d. Accordingly,

$$C = \frac{10}{15}d \text{ (kg/cm}^3\text{)}$$

D_2 : Spring Constant including Ballast

Where sleeper width is b and sleeper length is ℓ

$$D_2 = \frac{b\ell}{2} \cdot C \text{ (t/cm)}$$

D_3 : Settlement Coefficient due to Bending of Sleeper

Sleeper will cause bending affected by reaction from the ballast.

Because of this, reaction of the sleeper will be indefinite. Accordingly, settlement coefficient will vary depending on the positions of the sleepers. D_3 expresses the settlement coefficient beneath the rail and it is expressed by the following formula:

$$D_3 = \frac{D_2}{\eta - 1} \text{ (t/cm)}$$

η is a coefficient expressing its degree, and will vary depending on bending strength of sleeper E'I' sleeper length ℓ , distance between rail centers g, and ballast coefficient C.

$$\epsilon = g \sqrt[4]{\frac{C_b}{4E'I'}}$$

relation between η and ϵ is as shown in Fig. 1.

In the case of rigid sleeper such as Pc sleeper, $\eta = 1$.

D_1 : Composite spring coefficient due to bending of sleeper

$$D_1 = \frac{D_1' \cdot D_3}{D_1' + D_3} \quad (\text{t.cm})$$

a : Interval between sleepers (sleeper spacing)

D : Settlement Coefficient

$$D = \frac{D_1 \cdot D_2}{D_1 + D_2} \quad (\text{t/cm})$$

k : Settlement Coefficient of Rail Supports

$$k = \frac{D}{a} \quad (\text{kg/cm}^2)$$

β :

$$\beta = 4 \sqrt{\frac{k}{4EI}} \quad (\text{cm}^{-1})$$

EI : Bending Strength of Rail (kg/cm^2)

2.2 Calculation

Rail pressure P below the wheel, when W is wheel load

$$P = \left\{ W \left[1 - e^{-\frac{a\beta}{2}} \cdot \cos \frac{a\beta}{2} \right] \right\} = W \left\{ 1 - \phi \left(\frac{a\beta}{2} \right) \right\}$$

is shown in Fig. 4.

Ballast Pressure P_b

$$P_b = \frac{P}{\frac{1}{2} b \cdot l} \quad \eta = 1$$

$$\therefore P_b = \frac{2P}{b \cdot l}$$

3. Ballast Vibration Acceleration \ddot{y}

Ballast Vibration Acceleration \ddot{y} in Track model in Fig. 3.

is expressed as $\ddot{y} \propto \sqrt{D_1} \cdot \frac{1}{\sqrt{ma}}$

ma : Ballast Bearing Mass

Result of calculation of bearing mass is shown in Fig. 2. Further, taking into account pressure distribution in the ballast calculation was made to a certain degree of the mass which is assumed to be vibrated uniformly.

For convenience of calculation, \ddot{y} can be expressed by the following equation.

$$\ddot{y} = \sqrt{D_1} \cdot \frac{1}{\sqrt{ma}}$$

4. Impact Coefficient S

Impact coefficient S is expressed as $S = \frac{1}{EI\beta^2}$

It is assumed that the above relation composed from the experimental result by JNR.

5. Sample of Calculation

5-1 Track Structure: R54 Rail, PC sleeper 44 per 25 m length, Ballast thickness 25 cm

a) D_1'

As PC sleepers used, $D_1' = 100$ t/cm

b) C and D_2

Ballast thickness $d = 25$

$$C = \frac{10d}{15} = \frac{10}{15} \cdot 25 = 16.7 \text{ kg/cm}^3$$

$$b = 24 \text{ cm}, \ell = 200 \text{ cm}$$

$$D_2 = \frac{b\ell}{2} \cdot C = \frac{24 \times 200}{2} \times 16.7 = 40,080 \text{ kg/cm} = 40.08 \text{ t/cm}$$

c) D_3

Where rigid sleeper such as PC sleeper, $\eta = 1$

$$D_3 = \frac{D_2}{\eta - 1} = \infty$$

d) D_1

$$D_3 = \infty$$

$$D_1 = \frac{D_1' \cdot D_3}{D_1' + D_3} = \frac{D_1'}{D_1'/D_3 + 1} = D_1' = 100 \text{ t/cm}$$

e) D

$$D = \frac{D_1 \cdot D_2}{D_1 + D_2} = \frac{100 \times 40.08}{100 + 40.08} = 28.6 \text{ t/cm}$$

f) k

When 44 sleepers used for a rail length of 25 meter

$$a = 58 \text{ cm}$$

$$k = \frac{D}{a} = \frac{28.6}{58} = 0.493 \text{ t/cm}^2 = 493 \text{ kg/cm}^2$$

g) β and P

$$\beta = 4 \sqrt{\frac{k}{4EI}} = 4 \sqrt{\frac{493}{4 \times 2,100,000 \times 2,346}} = 1.26 \times 10^{-2} / \text{cm}$$

$$\frac{a}{2} \times \beta = \frac{58}{2} \times 1.26 \times 10^{-2} = 0.365$$

Unit weight $W = 1$ ton from Fig. 4

$$P = W \left(1 - \phi \left(\frac{a\beta}{2} \right) \right) = 1,000 \times 0.351 = 351 \text{ kg}$$

h) P_b

$$P_b = \frac{2P}{b \cdot l} = \frac{2 \times 351}{24 \times 200} = 0.146 \text{ kg/cm}^2$$

i) m_a and \ddot{y}

From Fig. 2

$$m_a = 404 \text{ kg}$$

$$\ddot{y} = \sqrt{D_1} \cdot \frac{1}{\sqrt{m_a}} = \sqrt{100} \cdot \frac{1}{\sqrt{404}} = 0.500$$

j) S

$$S = \frac{1}{EI\beta^2} = \frac{1}{2,100,000 \times 2,346 \times (1.26 \times 10^{-2})^2} = 1.279 \times 10^{-6}$$

$$k) M P_b \times \ddot{y} \times S = 0.146 \times 0.500 \times 1.279 \times 10^{-6} = 0.934 \times 10^{-7}$$

l) M/M_0

Standard track structure when composed of R50, PC 44/25m,
and ballast thickness 25 cm

structure coefficient M is 1.066×10^{-7}

$$M/M_0 = 0.934 \times 10^{-7} / 1.066 \times 10^{-7} = 0.88$$

5-2 Track Structure: R50 rail, wooden sleeper 44/25 m,
ballast depth 20 cm

a) D_1'

As pads are used for wood sleepers,

$$D_1' = 50 \text{ t/cm}$$

b) C and D_2

as $d = 20$ cm

$$C = \frac{10}{15} \cdot 20 = 13.3 \text{ kg/cm}^3$$

as $b = 22$ cm, $l = 200$ cm

$$D_2 = \frac{b \cdot l}{2} \cdot C = \frac{22 \times 200}{2} \times 13.3 = 29,260 \text{ kg/cm} = 29.26 \text{ t/cm}$$

c) D_3

$$\epsilon = 8 \sqrt[4]{\frac{C_b}{4E'I'}} = 113 \quad \sqrt[4]{\frac{13.3 \times 22}{4 \times 100,000 \times 3,168}} = 2.477$$

From Fig. 1

$$\eta = 1.176$$

$$D_3 = \frac{D_2}{\eta - 1} = \frac{29.26}{1.176 - 1} = 166 \text{ t/cm}$$

d) D_1

$$D_1 = \frac{D_1' \cdot D_3}{D_1' + D_3} = \frac{50 \times 166}{50 + 166} = 38.4 \text{ t/cm}$$

$$e) D = \frac{D_1 \cdot D_2}{D_1 + D_2} = \frac{38.4 \times 29.26}{38.4 + 29.26} = 16.6 \text{ t/cm}$$

f) k

$$k = \frac{D}{a} = \frac{16.6}{58} = 0.286 \text{ t/cm}^2 = 286 \text{ kg/cm}^2$$

g) β and P

$$\beta = \sqrt[4]{\frac{k}{4EI}} = \sqrt[4]{\frac{286}{4 \times 2,100,000 \times 1,960}} = 1.15 \times 10^{-2} / \text{cm}$$

$$\frac{a}{2} \times \beta = \frac{58}{2} \times 1.15 \times 10^{-2} = 0.334$$

Unit load $W = 1 \text{ ton}$ From Fig. 4

$$P = W \left\{ 1 - \phi \left(\frac{a\beta}{2} \right) \right\} = W \left\{ 1 - e^{-\frac{a\beta}{2}} \cdot \cos \frac{a\beta}{2} \right\}$$

$$= 1000 \left\{ 1 - e^{-0.334} \cos 0.334 \right\} = 1000 \left\{ 1 - \frac{\cos 0.334}{e^{0.334}} \right\} = 323 \text{ kg}$$

h) P_b

$$P_b = \frac{P}{\frac{1}{2} b \cdot l} = \frac{1.176 \cdot 323}{\frac{1}{2} \cdot 22 \cdot 200} = 0.173 \text{ kg/cm}^2$$

i) \ddot{y}

From Fig. 2

$$m_a = 273 \text{ kg}$$

$$\ddot{y} = \sqrt{D_1} \cdot \frac{1}{\sqrt{m_a}} = \sqrt{38.4} \cdot \frac{1}{\sqrt{273}} = 0.375$$

j) S

$$S = \frac{1}{EI\beta^2} = \frac{1}{2,100,000 \times 1,960 \times (1.15 \times 10^{-2})^2} = 1,837 \times 10^{-6}$$

k) M

$$M = P_b \times \ddot{y} \times S = 0.173 \times 0.375 \times 1,837 \times 10^{-6} = 1,192 \times 10^{-7}$$

l) M/M_0

$$M/M_0 = 1.192 \times 10^{-7} / 1.066 \times 10^{-7} = 1.12$$

STRUCTURE COEFFICIENT M

R 50	PC (44/25 ^m)	200mm	1.10
"	"	250	1.00
"	"	300	0.90
R 54	"	200	0.97
"	"	250	0.88
"	"	300	0.80
R 14A	"	200	1.43
"	"	250	1.27
"	"	300	1.17
R 50	Wooden (44/25 ^m)	200mm	1.12
"	"	250	1.05
"	"	300	0.97
R 54	"	200	0.98
"	"	250	0.92
"	"	300	0.85
R 14A	"	200	1.45
"	"	250	1.36
"	"	300	1.26

Note: Figures show the value of the structure coefficient expressed in the ratio against that of standard track structure using R 50, PC 44, 250 mm.

calculation sheets ; package II : track work

